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Learning Advanced Mathematics Through Engineering Design (Resource Exchange)

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PURDUE POLYTECHNIC | Engineering-Technology Teacher Education

M-Arch: Building an Arch bridge using Trigonometry

Designed by Euisuk Sung, Scott Bartholomew, Greg Strimel, and Seokyoung Kwon

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Lesson Overview

This project aims to integrate STEM content with advanced mathematical concepts through a hands-on activity. The lesson, titled *M-Arch*, is designed to teach concepts of trigonometry as students construct an arch bridge using specific mathematic formulas and approaches. In this unit, a team of students will design an arch bridge using provided pieces of wood. Before building an arch bridge, students will learn the way to calculate the length of both the bottom and top panels of each piece through the application of a trigonometry formula. Then, students will cut out designated pieces of wood using the results from their mathematical calculations; these pieces will be assembled to make an arch bridge. Through this design activity, students will learn concepts of trigonometry in a practical hands-on activity which incorporates an authentic design task.

Lesson Information

1. **Time: 3 hours**

2. **Lesson Objectives: Students will be able to**
   - use the distance formula to find out the outer length of a triangle using Trigonometric Parallax.
   - design a prefabricated arch bridge from an odd number of wooden blocks.
   - build a prototype of an arch bridge that spans two abutments.

3. **Related Academic Standards:**

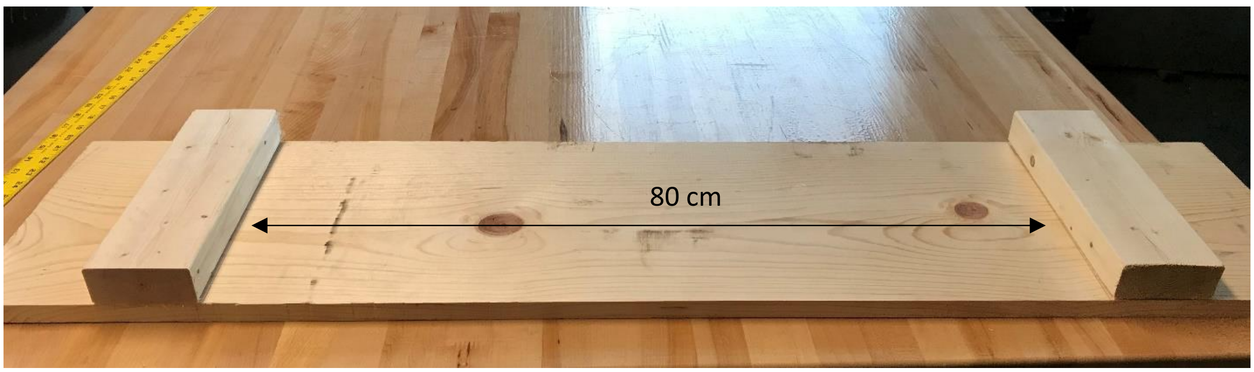
- Indiana Academic Standards Mathematics Geometry (2014)
  - G.T.1: Prove and apply theorems about triangles, including the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point; a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem, using triangle similarity; and the isosceles triangle theorem and its converse.

- G.T.7: State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle. Understand and use the geometric mean to solve for missing parts of triangles.
- G.T.8: Develop the distance formula using the Pythagorean Theorem. Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems. Find measures of the sides of polygons in the coordinate plane; apply this technique to compute the perimeters and areas of polygons in real-world and mathematical problems.

- Standards for Technological Literacy (ITEA/ITEEA, 2003)
  - 3.W. Systems thinking applies logic and creativity with appropriate compromise in complex real-life problems.
  - 3. Y. The stability of a technological system is influenced by all the components in the system, especially those in the feedback loop.
  - 20.K. Structures are constructed using a variety of processes and procedures.
  - 20.L. The design of structures includes a number of requirements.

4. Required Materials and tools

- Wood block 100 W × 8.4 D × 3.5 H
- Band saw or jigsaw
- Sandpaper
- Supporting panel with 80 cm span (abutment base for the arch bridge)



Lesson 1: Parallax Trigonometry

To measure the distance from the earth to a nearby star, scientists devised a simple method, called Trigonometric Parallax. Astronomers can this approach to observe a star in two different months or seasons (e.g., January and June), and then use the triangulation formula provided in Figure 1, to calculate the distance between the earth and the star.

$$\frac{Earth - Sun\ distance}{star - Sun\ circumstance} = \frac{parallax}{360}$$

$$\frac{d}{2\pi D} = \frac{p}{360}, so\ D = \frac{360\ d}{2\pi p}$$

P in arc seconds, d in parsecs, ad ab in AU

$$d = \frac{ab}{p}$$

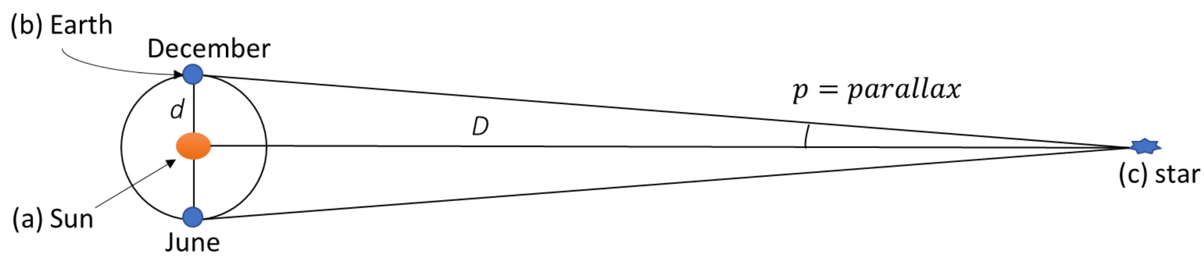


Figure 1. Triangulation formula.

This idea can also be extended to calculate the distance between two objects when given the distance from an observer. For example, the distance, denoted as L in Figure 2, can be obtained through the following procedures.

1. $\sin\frac{\theta}{2} = \frac{r}{l}$
2. $l = r \times \sin\frac{\theta}{2}$
3. $L = 2 \times r \sin\frac{\theta}{2}$

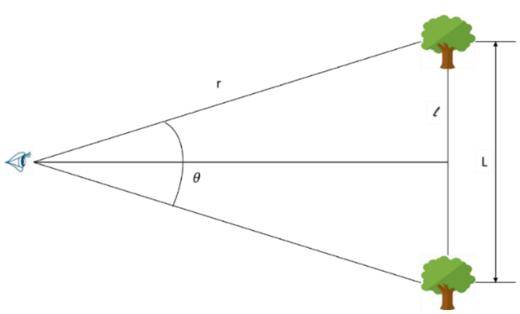


Figure 2. Example triangulation scenario.

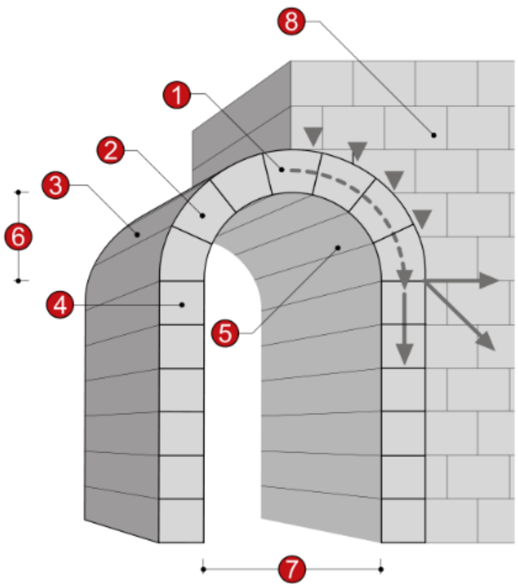
Additional Instructional Resources can be Found at:

- https://www.e-education.psu.edu/astro801/content/l4_p3.html
- http://astronomy.swin.edu.au/cosmos/T/Trigonometric+Parallax

Lesson 2: Arch bridge in construction

S T E

- What is arch bridge?
  - An arch bridge is a bridge shaped as a curved arch with abutments at each end. The basic components of an arch include voussoirs, abutments, and a keystone.
- Why build arch bridge?
  - Arch bridges work by transferring the weight of the bridge and its' loads partially into a horizontal thrust restrained by the abutments at either side.



1. Keystone
2. Voussoir
3. Back
4. Impost
5. Intrados
6. Rise
7. Clear span, "Bay"
8. Abutment

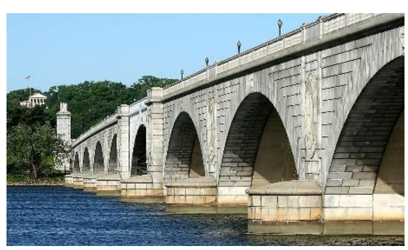
https://commons.wikimedia.org/wiki/File:Arch_illustration.svg

- How does arch bridge work?
  - The basic principle of arch bridge is its curved design, which does not push load forces straight down, but instead conveys these forces along the curve of the arch to the supports on each end.

Example of arch bridges

Bridge of Arta in Arta, Greece

https://commons.wikimedia.org/wiki/File:Stari_Most22.jpg



Arlington Memorial Bridge, Washington, D.C, USA

https://commons.wikimedia.org/wiki/File:Memorial_Bridge_sunrise.jpg

Pointed arch of the Puente del Diablo, Spain

https://commons.wikimedia.org/wiki/File:Puente_del_Diablo,_Martorell,_Catalonia,_Spain._Pic_01.jpg

Lesson 3: Design an arch bridge using Trigonometric Parallax

S T E M

Design Brief

https://upload.wikimedia.org/wikipedia/commons/e/e4/Canal_Lake_Arch_Bridge_NHS.jpg

You are a civil engineer at *Math Inc*, an engineering company located in the state of Indiana. Your job is to design and build an arch bridge across the Wabash River in West Lafayette. To build this bridge, the company wants to use prefabrication engineering which is a construction practice that fabricates wood sections in advance and assembles these on site. Your task is to work in teams to design, build, and test a prefabricated arch bridge prototype to span the distance of 80 cm. Please read the following criteria and constraints carefully and design your bridge.

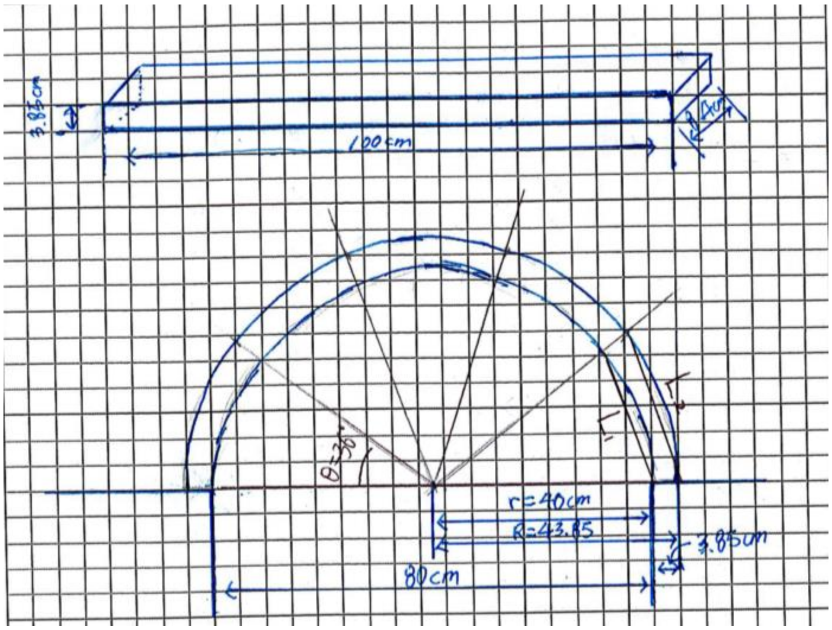
Criteria

- Determine the size of the prefabricated arch bridge components, also known as the voussoirs and keystone, using trigonometry.
- Each bridge component must be the same size.
- The total number of bridge components must be an odd number (e.g., 5, 7, 9, etc.).
- Create the prefabricated components arch bridge components and assemble the bridge to test its functionality.

Constraint

- The prefabricated arch bridge should connect between two provided abutments that are placed 80 cm apart.
- Your team must present the mathematical calculations for the bridge component dimensions prior to making the individual components.
- Each prefabricated bridge component should be made from wood and manufactured using the equipment found in the production laboratory (i.e. band saw and jigsaw).

Appendix: Design Example. 5 pieces of wood



Mathematical calculation

$$\theta = \frac{180°}{5} = 36°$$

$$L1 = 2 \times r \times sin\frac{2}{\theta} = 2 \times 40 \times sin18°, sin18° = 0.309$$

$$\mathrm{L1} = 2 \times 40 \times 0.309 = 24.72\mathrm{cm}$$

$$\mathrm{L2} = 2 \times \mathrm{R} \times \sin\frac{2}{\theta} = 2 \times 43.85 \times 0.309 = 27.077cm$$

Student outcome