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Learning Enhancement of Systems Dynamics via Laboratory Demonstrations

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Abstract

Introduced since Spring 2004 into the MSOE's mechanical engineering curriculum is a four-credit sophomore level course, Dynamics of Systems. This course is a perfect sequel to the calculus sequence that culminates in differential equations and the mechanics sequence (statics-dynamics) and a crucial prelude to the numerical modeling and analysis and a host of mechanical engineering courses such as thermodynamics, fluid mechanics, dynamics of machinery and automatic controls offered in the junior year. The co-author, Dr. Kumpaty coordinated the course offering and charted out laboratory demonstrations at crucial stages of the course material. The student learning has been tremendously increased as experiments are performed, data is gathered, experimental results are compared to the theory and reports are prepared. The similarity of systems and the characteristics of first-order and second-order systems are fully emphasized and clearly grasped. The overall experience with this integrated teaching has been very rewarding to both faculty and students. The details of the experience, the laboratory demonstrations developed covering mechanical, electrical and thermal systems and the effective utilization of the data gathered and the results obtained are presented.

Introduction

Milwaukee School of Engineering is dedicated to excellence in undergraduate education. The goal of the undergraduate curriculum is to produce well-rounded engineers, which is achieved through strong emphasis in a) excellent technical preparation, b) strong laboratory orientation with faculty teaching labs in small size sections and c) required Senior Design projects. Accordingly, MSOE graduates are highly sought by industry (over 99% placement). The mechanical engineering students are introduced to MATLAB programming in the freshman year itself and are taught numerical modeling and analysis in the junior year. Bridging the gap is our four-credit class in Systems Dynamics in the spring (last) quarter of sophomore year. Up until winter quarter of sophomore year, the students would be taking a five-course calculus sequence culminating in differential equations, and they would also complete statics and dynamics in the fall and winter of sophomore year, setting up Dynamics of Systems to be a perfect sequel to these courses in the spring quarter of the sophomore year. Currently, the Woods and Lawrence text[1] is employed for this course while the faculty teaching are equally approving of the Close, Frederick and Newell text[2] as an alternate for future offerings. Dr. Kumpaty, as the course coordinator, charted out various laboratory demonstrations at crucial stages of the material being covered; he used his prior experience in handling the Vibrations course and was ably supported by Prof. Ficken who had many years of experience in handling Controls courses. The demonstrations could not have been successful without excellent cooperation of our senior laboratory technician, Mr. Richard Philips. The student feedback has been very affirming and this paper intends to present the sweet story of successful integration of laboratory demonstrations that has impacted student learning immensely. The bulk of the paper concentrates on the demonstrations in the order introduced in the course. The integration of labs and the student reaction/feedback in our first implementation is also addressed briefly.

Laboratory Demonstration 1: First order Model

The purpose of the demonstration was to learn how to calculate the time constant for a system that fit the first order differential equation model. First we started with a digital thermometer in a cup of ice so that it was close to 32 F. Then the thermometer was taken out and the temperature was read every 5 seconds as it rose towards room temperature (T_∞) which was 61 F.

The general expression relating the rate of temperature change to the difference between the current temperature and the final temperature is shown in equation (1) with k being the proportionality constant.

$$\frac{dT}{dt} = k\left(T_\infty - T(t)\right) \qquad \text{with } T(t=0) = T_0 = 32 \text{ F} \tag{1}$$

This equation can be solved by the method of separation of variables resulting in the following:

$$e^{-kt} = \frac{T_\infty - T(t)}{T_\infty - T_0} \tag{2}$$

By rearranging the equation (2) and substituting the experimental data for t and $T(t)$, the proportionality constant k was calculated and averaged for the entire data. The reciprocal of k is the time constant, τ, which was found to be 58.6s. This means that after 58.6s the temperature would be 63% of the way to 61 F. Using τ the following graph (Figure 1) was generated; the analytical solution is quite similar to the data that was collected. The exponential nature of the solution is also highlighted in the demonstration.

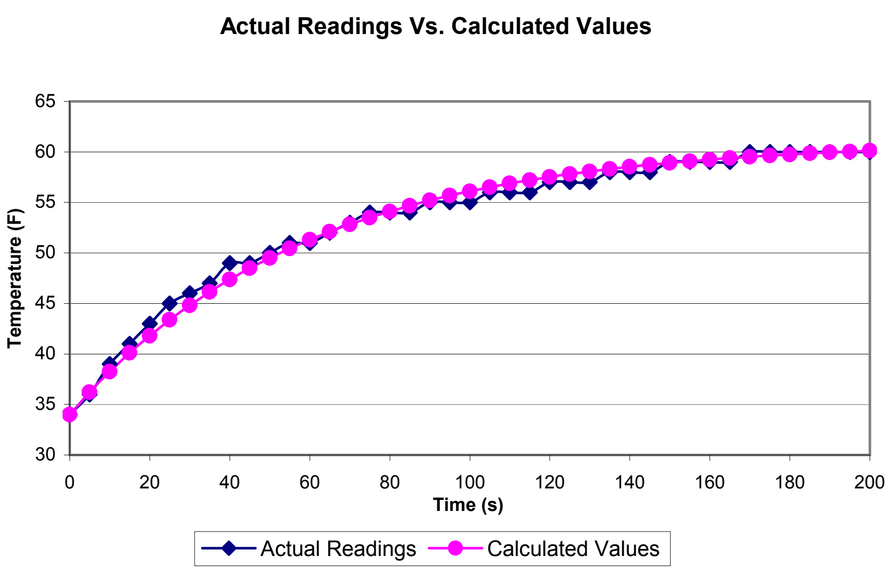


Figure 1 Temperature history of a thermocouple

The students recognize that the first order differential equations could be used for other models besides temperature of a thermocouple, such as capacitor charge, radioactive decay, and many other systems. Therefore, learning how to fit the data to a model as was done in this lab is a very useful tool. Not much later in the course, an RC circuit (1.2 KΩ, 0.5 μF) is demonstrated by charging and discharging the capacitor and verifying the time constant from experiment matching the product of R and C, 0.6s. Also seen was the fact that in 4 to 5 time constants, the voltage across the capacitor reached the steady state value.

Laboratory Demonstration 2: Free Vibration

This demonstration was conducted to show how the damping coefficient (*b*) and spring constant (*k*) could be found for a second order system from the acceleration data. More importantly, this demonstration helped students learn about the important characteristics of second order differential equations: the natural frequency (ω_n) and the damping ratio (ζ). An accelerometer was attached to a bar that had a spring and damper on it. This accelerometer was then connected to an oscilloscope which gave the acceleration data. The bar was given an initial input and allowed to oscillate. From the readout on the oscilloscope the data was collected: the amplitudes of consecutive cycles and the time between them. Figure 2 shows the setup for free vibration of a bar along with the data from the accelerometer on the oscilloscope. Figure 3 shows the schematic diagram of the mechanical system along with the values of the elements used.

Figure 2 The designed mechanical system with accelerometer and its free vibration response

The following second order differential equation for this rotational system is obtained by summing the moments about the point where the system is pivoted and applying Newton's law.

$$Mp^2\ddot{\theta} + bq^2\dot{\theta} + kr^2\theta = 0 \qquad (3)$$

The mass *M* in Figure 3 refers to effective mass of the rotor and the bar at distance *p* from the pivot. Equation (3) can be written as the more general form shown in equation (4).

$$\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0 \qquad (4)$$

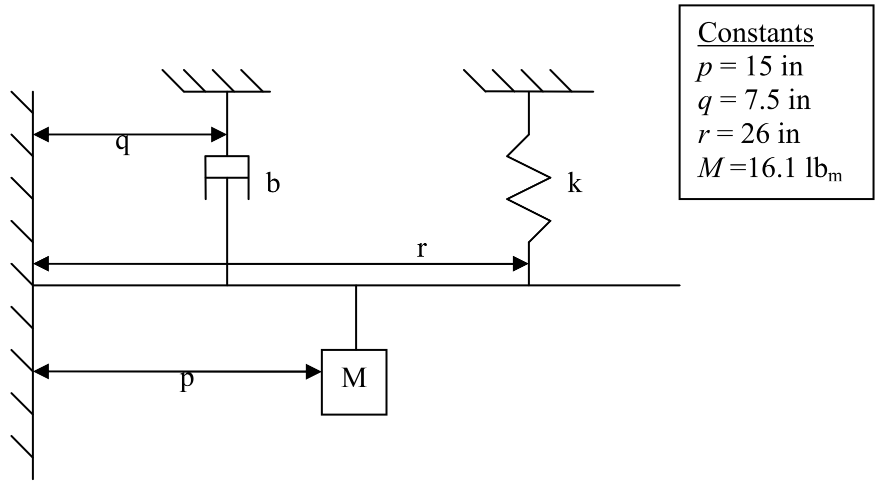


Figure 3 Sketch of the mechanical system and its parameters

The damping ratio is calculated from the logarithmic decrement, δ which is the natural logarithm of the ratio of two consecutive peaks. The two are related by the equation $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$. Then using the time period of the damped cycle, damped natural frequency is obtained ($\omega_d = \frac{2\pi}{t_d}$) and natural frequency is gleaned from $\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$. Equation (3) upon division by the acceleration coefficient Mp^2 becomes $\ddot{\theta} + \frac{bq^2}{Mp^2}\dot{\theta} + \frac{kr^2}{Mp^2}\theta = 0$ and its comparison with equation (4) lets students calculate b and k. Also, the students get to see the experimental data (see Table 1 below) agree with the following analytical solution for free vibration where A and B are constants that can be obtained from initial conditions, $\theta(0)$ and $\dot{\theta}$ (0).

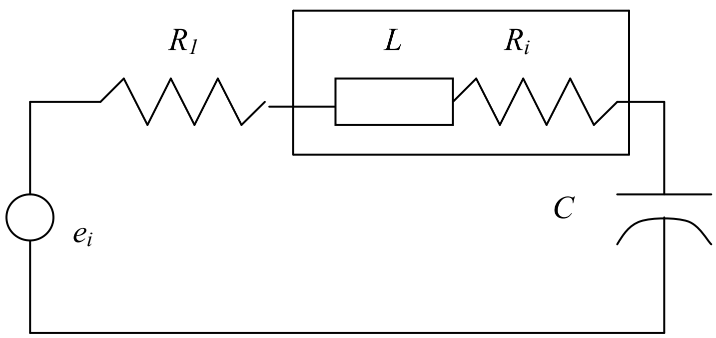
$$\theta(t) = e^{-\zeta\omega_n t}(A \sin \omega_d t + B \sin \omega_d t) \qquad (5)$$

Table 1. Log decrement data and associated calculations for mechanical system elements

	Amplitude	Ratio	Ln(Ratio)		ζ		k	
1	**4.9375**	**1.2155**	**0.19518**		**0.0298**		**21.4058**	**Lbf/in**
2	**4.062**	**1.2036**	**0.18528**				b	
3	**3.375**	**1.1998**	**0.18214**		ω_d		**0.39073**	**Lbf/in/s**
4	**2.813**				**39.27**	**rad/s**		
	Average	1.2063	0.18755	δ				
					ω_n		Frequency	
	τ_d				**39.287**	**rad/s**	**6.25278**	**Hz**
	Time Period	160 ms						

Laboratory Demonstration 3: Step response of a RLC circuit

This demonstration was meant to show the class how an RLC circuit would respond to a step input voltage (10 V dc). It also introduced students to overshoot, peak time, and steady state values. As with the lab demo 2, this was a second order differential equation and so students could look at the natural frequency and damping ratio of the circuit. The natural frequency and damping ratio were calculated before the lab started, thus giving students an opportunity to compare with the lab results. The test was conducted, and the overshoot, peak time, steady state value etc. were read. Figure 4 describes the circuit used and the values of elements.



Elements
$R_1 = 220\ \Omega$
$L = 30$ mH
$R_i = 130\ \Omega$
$C = 0.01\ \mu F$
$e_i = 10$ V

$R=R_1+R_i$

Figure 4 Sketch of the electrical system and its parameters

The governing differential equation is $(LCD^2 + RCD + 1)e_0 = e_i$ where D is the differential operator (d/dt), R includes the inductor resistance and e_0 is the output across the capacitor. Comparing with the general second order differential equation such as equation (4) and substituting the values of the circuit elements, one can obtain the natural frequency and damping ratio from $\omega_n = \sqrt{1/LC}$ = 57735 rad/s and $\zeta = \frac{RC\sqrt{1/LC}}{2} = \frac{RC\omega_n}{2}$ = 0.101. Solving analytically for the output voltage, we get:

$$e_0(t) = e_i\left[1 - e^{-\zeta\omega_n t}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\omega_d t) + \cos(\omega_d t)\right)\right] \quad (6)$$

The plot of this analytical solution is very similar to the oscilloscope trace of the actual system while it was running. The experimental readings from step response of the RLC circuit are the first peak of 16 V at 50 ms, the second peak of 12.2 V at 170 ms and steady state value of 10 V at 298 ms. The first peak measured (overshoot of 6 V) was very close to the analytical result (17 V). Similarly the time period between the first two peaks was read as 120 μs as opposed to the analytical value of 110 μs. The steady state value was measured as expected (10 V). It took about 3 time periods for the voltage to settle to the steady state value. The experiment was a grand success in teaching second order characteristics and step response using an electrical system, namely RLC circuit. Figure 5 shows the general picture of the actual experimental setup with the oscilloscope readout (not very clear in the shot taken).

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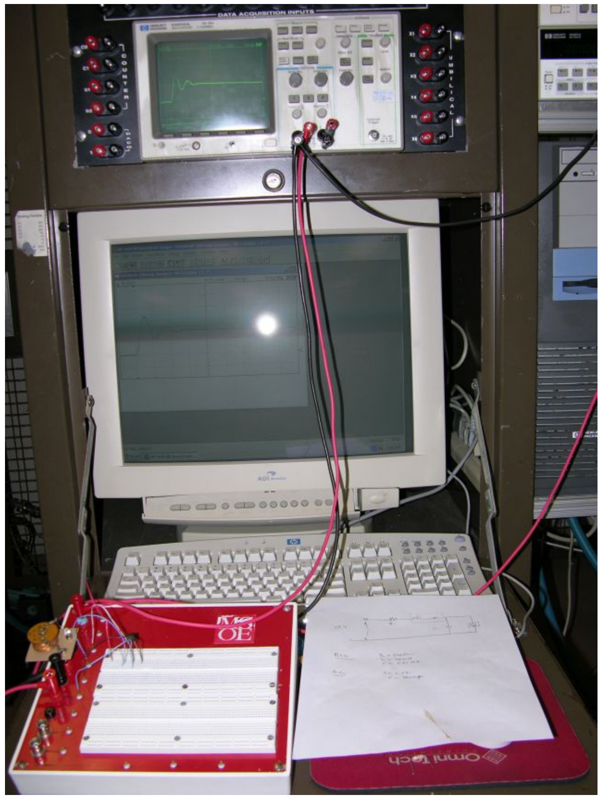

Figure 5 RLC Circuit and its step response

Laboratory Demonstration 4: Response of a RLC circuit under harmonic input

The goal of this demonstration was to show the frequency response of an RLC circuit. The experimental setup was the same used in laboratory demonstration 3 except that input voltage was sinusoidal (10 V ac). With the same values for the resistor, inductor, and capacitor, the natural frequency and the damping ratio were the same as in the previous demo. The output amplitude was measured as the input frequency was adjusted from 500 Hz to 15000 Hz, covering both sides of the natural frequency (9190 Hz). The output was harmonic in tune with the harmonic input. The output-input amplitude ratio or gain is given by the equation

$$\frac{E_o}{E_i} = \frac{1}{\sqrt{\left(1-\varpi^2\right)^2 + \left(2\zeta\varpi\right)^2}} \qquad (7)$$

where ϖ is the normalized frequency (ω/ω_n). The experimental data collected is shown along with the analytical solution in Figure 6. The peak output occurs at or very near the normalized frequency of 1, which is when the input frequency equals the natural frequency of the circuit (resonance). The measured output-input ratio peak was 3.8 while the analytical peak is 4.95. The lower peak measured may be due to resistance in the wires or slight errors in the resistance, capacitance or inductance values of the elements of the circuit. However, the frequency of the peak was measured to be 9200 Hz, which is within 0.1% of the theoretical. Thus, a lot of concepts on harmonic excitation are assimilated via this appropriate demonstration.

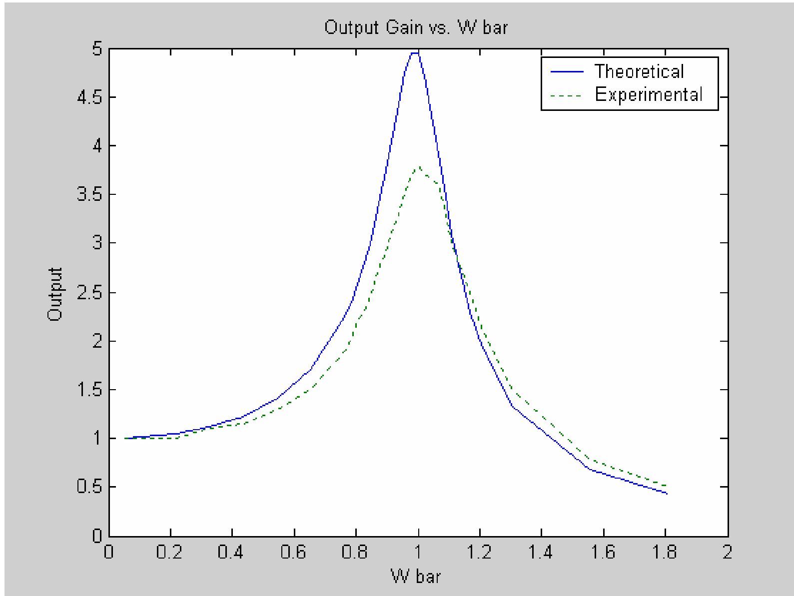

Figure 6 Frequency response of RLC circuit

Laboratory Demonstration 5: Time constant of a thermal system

This demonstration was designed to show what makes up the time constant of a thermal system. The thermocouples were measuring the temperature of two basic shapes, a steel ball and an iron cylinder that were initially heated up to the temperature of boiling water and then were allowed to cool in the air. The experiment was similar to the laboratory demonstration 1; however, data acquisition system was employed to record the temperature history. The time constant was related to mass of the object (density and volume), specific heat of the material, surface area and convection coefficient which was assumed as 10 W/m^2 K. The time constant ($\tau = \frac{mC_p}{hA_s}$) in the particular demonstration was calculated to be 265s and 460s respectively for the steel ball and iron cylinder respectively which were within 5% of the theoretical. The main intent of the demonstration was to become familiar with thermal properties that influence the time constant.

Laboratory Demonstration 6: Forced Vibration

The purpose of this demonstration was to show how a mechanical system reacts to a sinusoidal input. Just like an electrical system, the spring-mass-damper system has a frequency response that is related to the ϖ. For this lab the same mechanical system that was utilized in the second demonstration was employed. Instead of just giving the system an initial input, forced sinusoidal input in the form of a rotating unbalance (m=0.0614 lbm; e=2.25 in.) was provided. The speed of this mass was controlled by a precise harmonic oscillator and the frequency and the acceleration of the system were measured on the oscilloscope.

Table 2. Experimental Data and Theoretical Calculations (Forced Vibration)

Frequency, Hz	Input, rad/s	wbar	Theoretical Values Xth, in	Normalized	p-p mV	Acceleration in/s^2	Experimental Values Xexp	Xexp cor., in	Normalized
4.63	29.09	0.7405	0.0104	**0.5702**	76.80	15.14	0.0179	0.0096	**0.5271**
5.1	32.04	0.8156	0.0169	**0.9283**	137.50	27.11	0.0264	0.0141	**0.7778**
5.68	35.69	0.9084	0.0387	**2.1282**	343.80	67.78	0.0532	0.0285	**1.5679**
5.747	36.11	0.9191	0.0440	**2.4221**	525.00	103.50	0.0794	0.0425	**2.3388**
6.211	39.02	0.9933	0.1395	**7.6744**	1906.00	375.75	0.2467	0.1322	**7.2697**
6.329	39.77	1.0122	0.1350	**7.4240**	1438.00	283.49	0.1793	0.0960	**5.2821**
6.666	41.88	1.0661	0.0647	**3.5612**	931.00	183.54	0.1046	0.0560	**3.0827**
6.849	43.03	1.0954	0.0490	**2.6936**	675.00	133.07	0.0719	0.0385	**2.1172**
6.944	43.63	1.1106	0.0436	**2.3998**	575.00	113.36	0.0595	0.0319	**1.7545**
7.092	44.56	1.1342	0.0375	**2.0627**	525.00	103.50	0.0521	0.0279	**1.5358**

In this lab, the theoretical frequency response of the system was compared to the experimental data collected. The amplitude is related to the input frequency by the equation $X = \dfrac{\frac{me\omega^2}{M\omega_n^2}}{\sqrt{(1-\varpi^2)^2+(2\zeta\varpi)^2}}$ where the mass of the main system is M and the small rotating mass was m, the input frequency was ω, and the radius of the rotating mass was e. Since the oscilloscope read the acceleration and not the amplitude, several conversions were needed. First, the peak to peak voltage had to be divided by two to get the amplitude, and then it was necessary to divide by instrument sensitivity 98mV/g and then by accelerometer and oscilloscope gains. Then g s were converted into appropriate units and displacement was calculated by dividing by ω^2 and then multiplied by 15/28 to move it from where the accelerometer was located to the position where the mass was located. The experiment verified the theoretical response quite well as depicted in Figure 7. The calculations are shown in Table 2 (above).

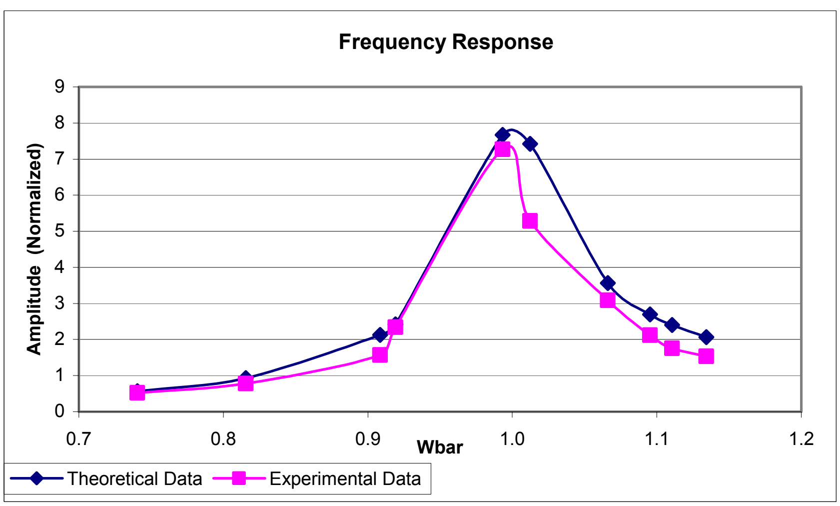

Figure 7 Normalized frequency response of the mechanical system

Integration of laboratory demonstrations and Student Reaction/Feedback

The laboratory demonstrations described thus far could very well be the laboratory experiments (student run) if the course is offered as 3-2-4 (lecture-lab-credit hours) instead of 4-0-4 in our curriculum. For this class, we have 40 class periods of 50 minutes each (10 weeks) in the quarter system. As the course was developed, dealing with many weeks of formulating differential equations for mechanical and electrical systems was viewed as a drag on students. So Dr. Kumpaty, the course coordinator felt it was in the best interest of student comprehension that the demonstrations be incorporated along the way by introducing analytical solutions and verifying the models experimentally while dealing with the formulation of differential equations for mechanical and electrical systems. The time constant of a thermocouple (Demo 1) is handled in the very first week and so the analytical solution is shown and the experiment is conducted right in the classroom. The first week of introduction to systems culminates in this Demo 1 and its analysis. The students are instructed ahead of time about bringing the laptop to the classroom on particular class periods. The classrooms are wired and so students can get to the MATLAB on the network even if they don't own a student's copy and analyze the systems (calculate or plot) during the class and discuss the results. The mechanical systems are taught in the next few weeks and by the end of Week 3, the free vibration of a mechanical system is demonstrated (Demo 2). The preparation for the Demo is done in the classroom: the second-order differential equation of motion is derived, the analytical solution is discussed (initial conditions only) and the second order characteristics are highlighted. The experiment/demo enhances the student comprehension. The students have reacted positively by recognizing this response at any given point throughout the course offering. By the end of Week 5, the electrical systems are introduced and to some extent, the solution of differential equations- first and second order systems (time response), and hence, Demo 3 fits perfectly in introducing an RLC circuit with step input (dc voltage). In fact, an RC circuit is also utilized to show the first order system characteristics along with Demo 3. By this time, students could identify the difference between impulse response and step response. Weeks 6 and 7 are used for further treatment on system behavior (time response and frequency response) culminating in the frequency response of an RLC circuit with harmonic input (ac voltage) which is Demo 4. The thermal systems are presented in Week 8 and the time constant is revisited with more detailed study of parameters that make up the time constant through Demo 5. During Weeks 9 and 10, the electromechanical systems and a brief introduction to fluid systems (since there is not much time to address nonlinearity in this course) are presented. Demo 6 brings out similar ideas as Demo 5 but for mechanical engineering students, revisiting the system they have been familiar with in Demo 2 and applying the concepts of Demo 5 are quite a treat and a fitting end to the course in Week 10. We have observed the student reports to indicate a high level of comprehension by the students and an appreciation for making theory come alive and meaningful.

"The labs helped me understand and apply the theory very well."
"It is a hard class but the demos were a great help in making sense of the theory."
"I liked the course a lot because I learnt about mechanical, electrical, thermal, and fluid systems, and how they are related." "Good modeling course."
"This course helped me understand mathematics behind mechanical systems."
"Tied everything together from previous courses; Good introduction to future courses."

"I liked learning about how to apply differential equations. Labs provided the depth."
"Labs reflected what we learned in class."

The above are just a few statements that describe the positive influence of employing laboratory demonstrations on student learning of System Dynamics concepts. Out of 120 students (in six sections of 20) that have been introduced to mechanical, electrical and thermal systems via laboratory demonstrations by the authors and their colleagues in Spring 2004, all contained only positive comments. Several commented that the course must be given an official laboratory credit. It can be safely stated that the incorporation of laboratory demonstrations will continue to enhance student learning of System Dynamics at MSOE. The authors envision a follow-up paper on the student reaction/feedback and the integration/implementation updates upon teaching the class several times and assessing the overall impact in the curriculum.

Conclusion

The laboratory demonstrations have proved to be an effective tool in enhancing the learning environment for the Systems Dynamics course in the presenters' classrooms at Milwaukee School of Engineering. The authors presented various demonstrations in thermal, electrical and mechanical systems to familiarize students with variety of systems while at the same time, system characteristics could be observed, verified and impressed upon the budding minds. Such an integrated course offering will go a long way in students' perception of concepts in junior and senior level classes. The favorable reaction by students during the course and their positive feedback in the course evaluations affirm the assessment of the presenters. More importantly, the authors are continuing to receive appreciation from students after taking the class as they are finding the junior courses easier because of the exposure received in the Dynamics of Systems class. In conclusion, integration of laboratory demonstrations in Systems Dynamics is highly recommended since it will facilitate the student learning become a rewarding experience for all involved- both faculty and students.

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Biographical Information

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