Session 1566

Learning from a Golf Ball

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Overview

Projectile motion of objects, in the absence of air friction, is studied in dynamics classes and textbooks[1, 2, 3]. In this study students learn about the effect of air drag on the projectile's height and range. In the process of achieving an understanding of the effect of air friction on projectile motions, students, in dynamics class, learn how to utilize programming features of Math-Cad® software to solve a system of non-linear, first-order, time dependent, ordinary differential equations. Students also learn about the application of difference equations and Runge-Kutta method to obtain a solution for a system of non-linear, first-order, time dependent, ordinary differential equations. Physical understanding of effects of air resistance on a golf ball's trajectory is achieved by comparing the results of these different approaches with the result of motion in the absence of the air drag.

Formulation

The drag force due to air friction on a golf ball can be estimated by:

$$D = \frac{1}{2} C_D \rho A V^2 \quad (1)$$

Where D is the drag force on the ball, V is the speed of the ball, ρ is the density of the air, A is the projected area of the ball normal to the air flow, and C_D is the drag coefficient. Neglecting the surface roughness of the ball, the drag coefficient C_D depends on, air viscosity, air density, ball speed, and ball diameter. That is:

$$C_D = f(\mu, \rho, V, d) \quad (2)$$

Where μ is the air viscosity, ρ is the air density, V is the speed of the ball, and d is the diameter of the ball. The effect of all these parameters on the drag coefficient can be lumped into a single dimensionless parameter known as Reynolds number (Re). That is:

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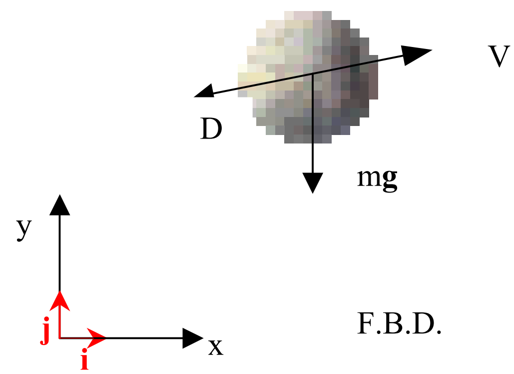
$$C_D = f(\mathrm{Re}) \quad (3)$$

Where
$$\mathrm{Re} = \frac{\rho V d}{\mu} \quad (4)$$

The drag force, being of frictional nature, acts in the opposite direction of the velocity of the golf ball. Application of Newton's second law to the projectile motion of the ball renders the governing equations of motion. One can write:



F.B.D.

$$\sum \vec{F}_{ext} = m\vec{a} \quad (5)$$

Observing the free body diagram above, equation (5) renders:

$$m\vec{g} + \vec{D} = m\vec{a} \quad (6)$$

Where m is the mass of the ball, $\vec{g}$ is the gravitational acceleration,

$$\vec{a} = \frac{d\vec{V}}{dt} \quad (7)$$

$\vec{a}$ is the acceleration of the ball, and $\vec{V}$ is the ball velocity. Defining the unit vector $\vec{\lambda} = \frac{\vec{V}}{V}$, one could obtain:

$$\vec{D} = -\vec{\lambda} D = -\frac{\vec{V}}{V} D \quad (8)$$

Substituting equation (1) into equation (8) and the outcome of that into equation (6), one obtains:

$$-mg\vec{j} - \frac{1}{2} C_D \rho A V \vec{V} = m\vec{a} \quad (9)$$

Substituting (7) into (9) and noticing that the cross-section of the ball is $A = \frac{\pi}{4} d^2$, equation (9) becomes:

$$-mg\vec{j} - \frac{\pi}{8} C_D \rho V d^2 \vec{V} = m \frac{d\vec{V}}{dt} \quad (10)$$

Writing the velocity of the ball in terms of its x and y components $\vec{V} = u\vec{i} + v\vec{j}$ and after some algebraic manipulation equation (10) becomes:

$$\frac{d\left(u\vec{i} + v\vec{j}\right)}{dt} = -g\vec{j} - \frac{C_D}{8} \frac{\mathrm{Re}}{\left(\frac{m}{\pi d \mu}\right)} \left(u\vec{i} + v\vec{j}\right) \quad (11)$$

Considering that u = dx/dt, and v = dy/dt, where x is the horizontal distance and y is the vertical distance traveled by the ball, and noticing that the term $\frac{m}{\pi d \mu} = \tau$ in equation (11) is a constant and has dimension of time, one can arrive at the governing equations of motion upon writing (11) in its x and y components:

$$\begin{aligned} u &= \frac{dx}{dt} \\ v &= \frac{dy}{dt} \\ \frac{du}{dt} &= -\frac{C_D \,\mathrm{Re}}{8\tau} u \\ \frac{dv}{dt} &= -\frac{C_D \,\mathrm{Re}}{8\tau} v - g \end{aligned} \quad (12)$$

Equation (12) subject to initial conditions below:

$$\begin{aligned} x(0) &= 0 \\ y(0) &= 0 \\ u(0) &= V_0 \cos\theta \\ v(0) &= V_0 \sin\theta \end{aligned} \quad (13)$$

Make the system of non-linear, time dependent, ordinary differential equations, which describes the flight of the golf ball. In equation (13) V_0 is the launching speed of the golf ball and θ is the launching angle of the ball with respect to horizon. The drag coefficient in the differential equations of motion is calculated using a correlation equation for a smooth sphere[4].

$$C_D = \frac{24}{\mathrm{Re}}(1 + 0.15\,\mathrm{Re}^{0.687}) + \frac{0.42}{1 + 4.25 \times 10^4\,\mathrm{Re}^{-1.16}} \quad (14)$$

The above correlation deviates from the standard drag curve for spheres by 4% to 6% for Reynolds numbers up to 3×10^5. At the lower end of the Reynolds spectrum the above correlation renders drag coefficient of a sphere in Stoke's flow, that is $C_D = \frac{24}{\text{Re}}$.

The above correlation will not incorporate the effect of surface roughness. Surface roughness is to induce turbulence, which in turn reduces the pressure drag. To simulate the effect of dimples on the golf ball, we calculate the drag coefficient by considering Reynolds number at 3×10^5 and using the following equation.

$$C_D = \frac{24}{\text{Re}}(1 + 0.15\,\text{Re}^{0.687}) \qquad (15)$$

To show the significant role, which dimples play on the range and height of the golf ball projectile, we compare the results of rough surface calculations with those of smooth surface balls.

Solution

A closed form solution to the system of non-linear, time-dependent, ordinary differential equations (12) subject to initial conditions (13) is not possible. Three different numerical approaches are employed in this study to calculate the path of a golf ball; namely, difference equations scheme, Runge-Kutta method, and a Math-Cad® built-in function. This is the first time that the students try to use numerical methods to solve system of differential equations in the curriculum. Pedagogically, it is very useful if students try different approaches and compare the results of these different approaches, for a practical application such as the golf ball motion. To simulate the motion in Math-Cad® we consider the flight of a ball of m = 43 g, d = 45 mm in air of $\mu = 1.79\times10^{-5}$ N.s/m^2, ρ = 1.225 kg/m^3. The launching speed and launching angle of the golf ball are 36 m/s and 30º with horizon respectively.

Approach I- Difference Equations Scheme

In this approach we approximate the variables and their derivatives in equations (13) as follows:

$$\begin{aligned} \frac{dx}{dt} &\approx \frac{x(t) - x(t-\Delta t)}{\Delta t} \\ \frac{dy}{dt} &\approx \frac{y(t) - y(t-\Delta t)}{\Delta t} \qquad & u &\approx \frac{u(t) + u(t-\Delta t)}{2} \\ \frac{du}{dt} &\approx \frac{u(t) - u(t-\Delta t)}{\Delta t} \qquad & v &\approx \frac{v(t) + v(t-\Delta t)}{2} \\ \frac{dv}{dt} &\approx \frac{v(t) - v(t-\Delta t)}{\Delta t} \end{aligned} \qquad (16)$$

Substituting equations (15) into the governing equations of motion, one arrives at the difference equations of motion as:

$$
\begin{aligned}
x(t) &= x(t-\Delta t) + \frac{\Delta t}{2}\left[u(t-\Delta t) + u(t)\right] \\
y(t) &= y(t-\Delta t) + \frac{\Delta t}{2}\left[v(t-\Delta t) + v(t)\right] \\
u(t) &= u(t-\Delta t) - \frac{C_D \operatorname{Re}}{8\tau}\left[\frac{u(t)+u(t-\Delta t)}{2}\right]\Delta t \\
v(t) &= v(t-\Delta t) - \frac{C_D \operatorname{Re}}{8\tau}\left[\frac{v(t)+v(t-\Delta t)}{2}\right]\Delta t - g\Delta t
\end{aligned}
\qquad (17)
$$

Solving the above system of equations for the variables at time "t", one obtains:

$$
\begin{aligned}
u(t) &= \frac{u(t-\Delta t) - \left(\dfrac{C_D \operatorname{Re} u(t-\Delta t)}{16\tau}\right)\Delta t}{1 + \dfrac{C_D \operatorname{Re}\Delta t}{16\tau}} \\
v(t) &= \frac{v(t-\Delta t) - \left(\dfrac{C_D \operatorname{Re} v(t-\Delta t)}{16\tau}\right)\Delta t - g\Delta t}{1 + \dfrac{C_D \operatorname{Re}\Delta t}{16\tau}} \\
x(t) &= x(t-\Delta t) + \frac{\Delta t}{2}\left(u(t-\Delta t) + u(t)\right) \\
y(t) &= y(t-\Delta t) + \frac{\Delta t}{2}\left(v(t-\Delta t) + v(t)\right)
\end{aligned}
\qquad (18)
$$

The above system of equation is then solved by marching through time from initial t = 0, when the ball was launched, until the time t when the ball hits the ground. A time step of $\Delta t = 0.01$ sec is adopted in this simulation. Appendix A represent the Math-Cad® program used to implement this approach.

Approach II- Fourth-Order Runge-Kutta Method

A fourth-Order Runge-Kutta[5] solution technique is employed to solve the system of non-linear time-dependent first-order differential equations. Appendix B shows the detail Math-Cad® program to perform this integration.

Approach III- Math-Cad® Built-in Function rkfixed

The Math-Cad® built-in function rkfixed[6], which is an implementation of fourth-order Runge-Kutta method, is used to solve the system of non-linear differential equations (12). The detail of calling this function in Math-Cad® is also shown in Appendix B.

Results and Discussions

Figure I, taken from Appendix A, shows the projectile motion of the golf ball for both smooth and rough surface cases. The simulation results for smooth ball and rough ball trajectories were achieved by solving the difference equations (18) of the governing differential equations of motion. The rough ball case results in a height of trajectory of 15.52 m and a range of 101.7 m. The corresponding values for the smooth ball in Figure I are 11.792 m and 89.14 m respectively. It is worth noting that the trajectories for both cases are identical up to almost 8% of the smooth ball flight. The high Reynolds number at the start of the projectile motion of the smooth ball renders a very low drag coefficient, which results in a trajectory of the ball similar to that of a rough ball. Notice also that the effect of air resistance is more pronounced on the height of the golf ball than on its range. The reason being that the ball decelerates more in the vertical direction than in the horizontal direction due to gravity.

Figure II, taken from Appendix B, depicts the results of the golf ball simulation for smooth and rough ball surfaces, using approaches II and III to solve the governing differential equations of motion. Figure II also shows the result of the drag free simulation. Comparison was made between the Runge-Kutta approach and Math-Cad® rkfixed approach for smooth ball surfaces. The results were identical as expected. Incidentally, we get the same identical results when we compare the results of 3 different approaches for smooth ball simulation. That is to say that the 3 different solution strategies yield a range of 89.14 m and a height of 11.792 m for the smooth surface golf ball trajectory. Figure II indicates that rough balls have a trajectory much closer to the drag free one. The reason being that the rough surface induces turbulence which in turn sends the flow separation points further down-stream of the flow on the surface of the ball and reduces drag coefficient.

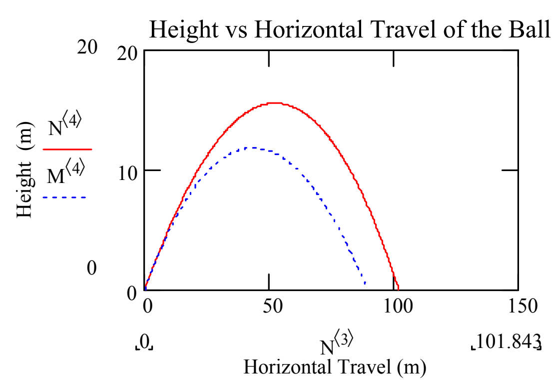


Figure I- Comparison of Approach I Result
With the Drag Free Ball

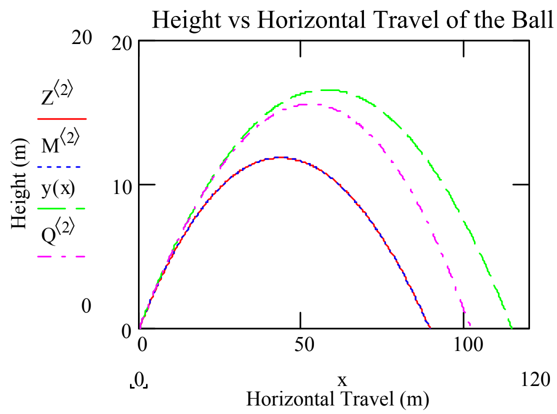


Figure II-Comparison of Approach II and III Results With Each Other for Smooth Ball case, and With the Results of Rough and Drag Free Balls

The numerical values of the range and the height of the projectile in case of the rough surface golf ball were 101.70 m, and 15.52 m respectively. The corresponding values for the drag free flight are 114.46 m, and 16.537 m respectively. This made it clear to the students as to why there are dimples on the surface of the golf balls. The Table below summarizes the results of all the simulated cases.

Comparison of Results of the Golf Ball Projectile Simulation

Projectile	Difference Equations Results for Smooth Ball	Runge-Kutta Results for Smooth Ball	Rkfixed Results for Smooth Ball	Runge-Kutta Results for Rough Ball	Drag Free Results
Height (m)	11.792	11.80	11.80	15.52	16.537
Range (m)	89.14	89.21	89.21	101.70	114.46

Table I

Conclusion

Golf ball trajectories for smooth and rough balls in the presence of air resistance were evaluated by employing difference equations schemes, Runge-Kutta method, and Math-Cad® rkfixed function to solve the governing differential equations of motion. It was realized that results from the 3 different solution techniques were the same (See Table I). It was also observed that by using a rough surface ball instead of a smooth one, one could achieve a 14% improvement in the range of the golf ball and a 31.5% improvement in the height of the ball (See Table I). Students learned about the important effect that dimples on the surface of the golf ball have on the ball's trajectory. Students learned how to solve a system of non-linear first-order time dependent differential equations in 3 different ways. They also learned about the programming features of Math-Cad® software. Overall the project was a success and students' feedback was that they learned substantially from this project.

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5. Ayyub, B.M., and R.H. McCuen: Numerical Methods for Engineers, Prentice Hall, 1996.
6. Math-Cad® 7 User's Guide, MathSoft Inc., 1997.

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APPENDIX A

Difference Equation Solution of the Projectile Motion of the Ball for Both Smooth and Rough Golf Ball Cases

Ball Diameter and Air Properties

$d := 0.045 \ \text{in}$ $\qquad \mu := 1.79 \cdot 10^{-5} \ \text{N} \cdot \frac{\text{sec}}{\text{m}^2}$ $\qquad \rho := 1.225 \ \frac{\text{kg}}{\text{m}^3}$

Drag Coefficient for Smooth Ball
J = 1

$$C_D(u,v) := \frac{24}{\rho \cdot \frac{\sqrt{(u)^2 + (v)^2} \cdot d}{\mu}} \left[1 + 0.15 \cdot \left[\rho \cdot \frac{\sqrt{(u)^2 + (v)^2} \cdot d}{\mu} \right]^{0.687} \right] + \frac{0.42}{1 + 4.25 \cdot 10^4 \cdot \left[\rho \cdot \frac{\sqrt{(u)^2 + (v)^2} \cdot d}{\mu} \right]^{-1.16}}$$

Drag Coefficient for the Rough Ball
J =2

It is Assumed that the Roughness of the Surface is Simulated by incorporating a High Value of Reynolds Number into a Correlation, Due to Roughness Induced Turbulence

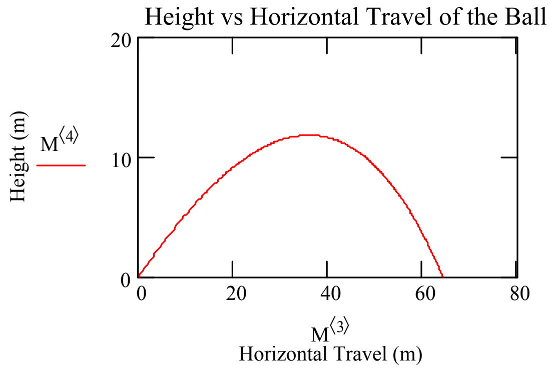
@ $\qquad Re := 3 \cdot 10^5 \qquad C_{DRough} := \frac{24}{Re}\left(1 + 0.15 \cdot Re^{0.687}\right)$

The Programming part, Which Implements the solution of System of Nonlinear, Time-Independent First Order Differential Equations of the Golf Ball Trajectory By Difference Approach

$$
f(\theta, V, j) := \left|\begin{array}{l}
m \leftarrow 0.043 \\
g \leftarrow 9.81 \\
i \leftarrow 0 \\
\theta \leftarrow \theta \cdot \dfrac{\pi}{180} \\
\tau \leftarrow \dfrac{m}{\pi \cdot d \cdot \mu} \\
u_i \leftarrow V \cdot \cos(\theta) \\
v_i \leftarrow V \cdot \sin(\theta) \\
x_i \leftarrow 0 \\
y_i \leftarrow 0 \\
t_i \leftarrow 0 \\
\Delta t \leftarrow 0.01 \\
s \leftarrow t_i \\
\text{while } y_i \geq 0 \\
\quad \left|\begin{array}{l}
Re \leftarrow \dfrac{\rho \cdot \sqrt{(u_i)^2 + (v_i)^2} \cdot d}{\mu} \\
C_D \leftarrow C_D(u_i, v_i) \quad \text{if } j = 1 \\
C_D \leftarrow C_{DRough} \quad \text{if } j = 2 \\
i \leftarrow i + 1 \\
u_i \leftarrow \dfrac{u_{i-1} - \left(\dfrac{C_D \cdot Re \cdot u_{i-1}}{16 \cdot \tau}\right) \cdot \Delta t}{1 + \dfrac{C_D \cdot Re \cdot \Delta t}{16 \cdot \tau}} \\
v_i \leftarrow \dfrac{v_{i-1} - \left(\dfrac{C_D \cdot Re \cdot v_{i-1}}{16 \cdot \tau}\right) \cdot \Delta t - g \cdot \Delta t}{1 + \dfrac{C_D \cdot Re \cdot \Delta t}{16 \cdot \tau}} \\
x_i \leftarrow x_{i-1} + \dfrac{\Delta t}{2} \cdot (u_{i-1} + u_i) \\
y_i \leftarrow y_{i-1} + \dfrac{\Delta t}{2} \cdot (v_{i-1} + v_i) \\
t_i \leftarrow s + i \cdot \Delta t
\end{array}\right. \\
H \leftarrow \text{augment}(t, u, v, x, y) \\
H
\end{array}\right.
$$

Calling Up the Difference Function
for the Smooth Ball Case
J = 1

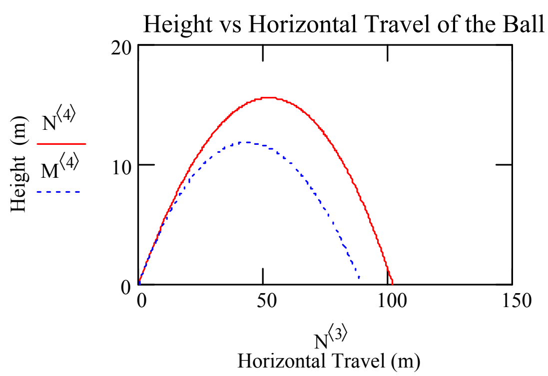
$$M := f(30,36,1)$$



Calling Up the Difference Function
for the Rough Ball Case
J = 2

$$N := f(30,36,2)$$

Comparison of the Height and Horizontal Trave
of the Golf Ball for Smooth and Rough Surface
Cases
Dashed Blue - Smooth Surface
Red - Rough Surface
In the Graph Below



APPENDIX B

4th. order Runge_Kuttaa System of
Differential Equations
Program

$$\mathrm{RK4\_sys}(f,h,xinitial,n) := \left| \begin{array}{l} x^{\langle 0\rangle} \leftarrow xinitial \\ \text{for } i \in 0..n \\ \quad t_i \leftarrow h\cdot i \\ \text{for } i \in 0..n-1 \\ \quad \left| \begin{array}{l} k1 \leftarrow h\cdot f\left(t_i, x^{\langle i\rangle}\right) \\ k2 \leftarrow h\cdot f\left(t_i + \frac{h}{2}, x^{\langle i\rangle} + \frac{k1}{2}\right) \\ k3 \leftarrow h\cdot f\left(t_i + \frac{h}{2}, x^{\langle i\rangle} + \frac{k2}{2}\right) \\ k4 \leftarrow h\cdot f\left(t_i + h, x^{\langle i\rangle} + k3\right) \\ x^{\langle i+1\rangle} \leftarrow x^{\langle i\rangle} + \frac{k1}{6} + \frac{k2}{3} + \frac{k3}{3} + \frac{k4}{6} \end{array} \right. \\ s \leftarrow \mathrm{augment}\left(t, x^T\right) \\ s \end{array} \right.$$

Application of RK4_sys to Projectile Problem

Input Data

$m := 0.043 \quad kg \qquad g := 9.81 \ \frac{m}{s^2} \qquad V_0 := 36 \ \frac{m}{sec}$

$d := 0.045 \quad m \qquad \mu := 1.79\cdot 10^{-5} \ \frac{N\cdot sec}{m^2} \qquad \theta := 30 \ de$

$h := 0.01 \quad sec \qquad \rho := 1.225 \ \frac{kg}{m^3} \qquad n := 1800$

Time Constant:

$$\tau := \frac{m}{\pi \cdot d\cdot \mu}$$

Golf Ball Surface Indicator
J = 1, for Smooth Surface, J =
2, for Rough Surface

$j := 1$

Initial Conditions

$$\text{xinitial} = \begin{pmatrix} 0 \\ 0 \\ V_0 \cdot \cos\left(\theta \cdot \frac{\pi}{180}\right) \\ V_0 \cdot \sin\left(\theta \cdot \frac{\pi}{180}\right) \end{pmatrix}$$

Drag Coefficient function
for Smooth Surface
J = 1

$$C_D(u,v) := \frac{24}{\rho \cdot \frac{\sqrt{(u)^2 + (v)^2} \cdot d}{\mu}} \left[1 + 0.15 \cdot \left[\rho \cdot \frac{\sqrt{(u)^2 + (v)^2} \cdot d}{\mu} \right]^{0.687} \right] + \frac{0.42}{1 + 4.25 \cdot 10^4 \cdot \left[\rho \cdot \frac{\sqrt{(u)^2 + (v)^2} \cdot d}{\mu} \right]^{-1.16}}$$

Drag Coefficient
for Rough Surface
J = 2

@ $\quad Re := 3 \cdot 10^5 \qquad C_{DRough} := \frac{24}{Re}\left(1 + 0.15 \cdot Re^{0.687}\right)$

The Derivative Functions

Correlation to calculate Drag coefficient is used in <u>if</u> Statement

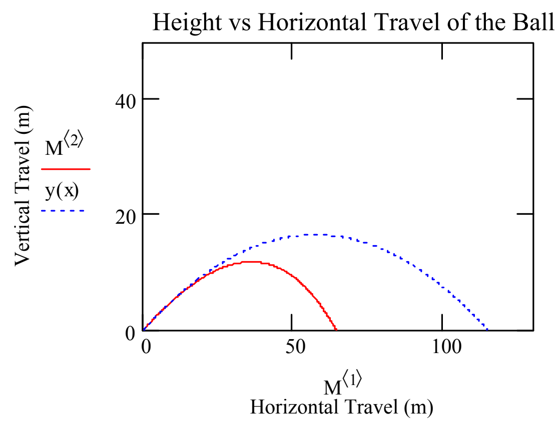
$$f(t,x) := \begin{bmatrix} x_2 \\ x_3 \\ \dfrac{-\text{if}\left(j = 1, C_D\left(x_2, x_3\right), C_{DRough}\right) \cdot \rho \cdot \dfrac{\sqrt{\left(x_2\right)^2 + \left(x_3\right)^2} \cdot d}{\mu}}{8 \cdot \tau} \cdot x_2 \\ \dfrac{-\text{if}\left(j = 1, C_D\left(x_2, x_3\right), C_{DRough}\right) \cdot \rho \cdot \dfrac{\sqrt{\left(x_2\right)^2 + \left(x_3\right)^2} \cdot d}{\mu}}{8 \cdot \tau} \cdot x_3 - g \end{bmatrix}$$

Calling Up The RK4 Program

$$M := RK4\_sys(f, h, xinitial, n)$$

Projectile Motion in
the Absence of Drag

$$x := M^{\langle 1 \rangle} \qquad y(x) := (\tan(0.524))\cdot x - \frac{g\cdot x^2}{2\cdot(36\cdot\cos(0.524))^2}$$



Using Built in function in Mathcad
rkfixed
To Solve System of nonlinear
O.D.E.

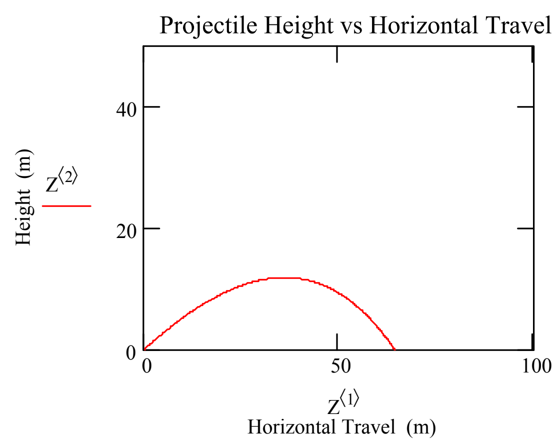
$$x := \begin{pmatrix} 0 \\ 0 \\ 36\cdot\cos\left(30\cdot\frac{\pi}{180}\right) \\ 36\cdot\sin\left(30\cdot\frac{\pi}{180}\right) \end{pmatrix}$$

$$D(t,x) := \begin{bmatrix} x_2 \\ x_3 \\ \dfrac{-\text{if}\left(j = 1, C_D(x_2,x_3), C_{DRough}\right)\cdot\rho\cdot\dfrac{\sqrt{(x_2)^2 + (x_3)^2}\cdot d}{\mu}}{8\cdot\tau}\cdot x_2 \\ \dfrac{-\text{if}\left(j = 1, C_D(x_2,x_3), C_{DRough}\right)\cdot\rho\cdot\dfrac{\sqrt{(x_2)^2 + (x_3)^2}\cdot d}{\mu}}{8\cdot\tau}\cdot x_3 - g \end{bmatrix}$$

Calling Up the rkfixed Function

$$Z := \text{rkfixed}(x, 0, 10, 1000, D)$$

rkfixed Result



Simulation of Rough Ball
J = 2
Using RK4 Function

$$Z^{\langle 1\rangle} := M^{\langle 1\rangle}$$

$$x := Z^{\langle 1\rangle}$$

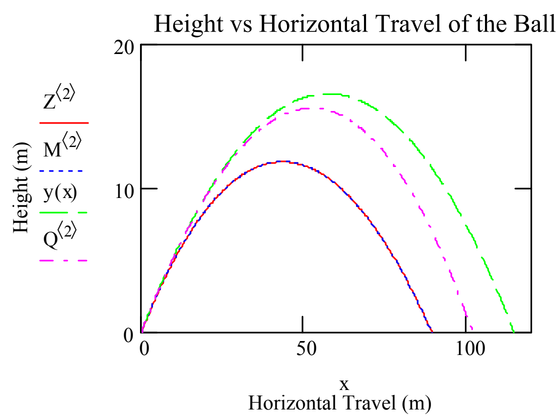
$$j := 2$$

$$f(t,x) := \begin{bmatrix} x_2 \\ x_3 \\ \dfrac{-\mathrm{if}\left(j = 1, C_D(x_2,x_3), C_{DRough}\right)\cdot\rho\cdot\dfrac{\sqrt{(x_2)^2+(x_3)^2}\cdot d}{\mu}}{8\cdot\tau}\cdot x_2 \\ \dfrac{-\mathrm{if}\left(j = 1, C_D(x_2,x_3), C_{DRough}\right)\cdot\rho\cdot\dfrac{\sqrt{(x_2)^2+(x_3)^2}\cdot d}{\mu}}{8\cdot\tau}\cdot x_3 - g \end{bmatrix}$$

$$Q := \mathrm{RK4\_sys}(f, h, xinitial, n)$$

$$x := Q^{\langle 1 \rangle}$$

Comparison of the Results of Rough and Smooth Ball:
,Using RK4 and rkfixed Functions, with Drag Free Cas



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