Learning Modules for Geometric Pattern Identification and Mathematical Modeling of Facade Systems

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Ali Eydgahi started his career in higher education as a faculty member at the Rensselaer Polytechnic Institute in 1985. Since then, he has been with the State University of New York, University of Maryland Eastern Shore, and Eastern Michigan University. During 2006-2010, he was Chair of the Department of Engineering and Aviation Sciences, Founder and Director of the Center for 3-D Visualization and Virtual Reality Applications, and Technical Director of the NASA funded MIST Space Vehicle Mission Planning Laboratory at the University of Maryland Eastern Shore. In 2010, he joined Eastern Michigan University as an Associate Dean in the College of Technology and currently is a Professor in the School of Engineering Technology. He has an extensive experience in curriculum and laboratory design and development. Dr. Eydgahi has served as a member of the Board of Directors for Tau Alpha Pi, as a member of Advisory and Editorial boards for many International Journals in Engineering and Technology, as a member of review panel for NASA and Department of Education, as a regional and chapter chairman of IEEE, SME, and ASEE, and as a session chair and as a member of scientific and international committees for many international conferences.

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Abstract

Over the past decade, responsive facade systems have emerged to improve user comfort, energy consumption, and cost efficiency as they are capable of responding and adapting to environmental stimuli. Design of a responsive facade system involves various fields such as engineering, architecture, robotics, material science, mathematics, physics, structure, fabrication, and geometry in the process of design. The geometry of facades as socio-cultural design parameters affects the visual performance of facade. However, existing responsive facades are socio-culturally inert.

This paper presents a set of educational activities that can be used in facade design courses. The proposed hands-on activities consist of different modules on pattern identification, mathematical modeling, shading function development, and design of the mechanism and simulation of the designed facade system.

The educational activities of each module are demonstrated by using a Persian pattern named "*SHAMSEH*" as a pattern of the facade system. Excel software was utilized to develop the mathematical model of the selected pattern. Grasshopper-for-Rhino software was used to create shading function, design a mechanism for motion, and simulate the facade system. The proposed hands-on activities assist students, educators, and architects in pattern identification and mathematical modeling, in mechanism and control design, and in simulation of a facade system.

Introduction

Mathematics serve as the foundation of design science (Kappraf, 1999). The importance of mathematics in architecture, as a scientific issue, is indubitable. Architecture can be considered a graphical interpretation of mathematics (Megahed, 2013). There are two approaches to integrate mathematics into architecture and architectural engineering curricula that include realistic mathematics education and mathematics as a service subject (Verner & Maor, 2006). Based on curricular priorities for architecture and architectural engineering programs, realistic mathematics education is a common approach as part of the first-year mathematics curriculum (Verner & Maor, 2003). While second approach considers mathematics as a part of a professional education and focuses on mathematical skill required for professional practices.

Review of various architecture and architectural engineering curricula shows a limited offering of courses on the application of mathematical modeling in architectural design programs (Megahed, 2013; Maor & Verner, 2007; Burry, 2007; Maor & Verner, 2006). Fischler (1976) has indicated that mathematical modeling and techniques, especially in connection with design and modeling are ignored in architectural curricula due to concentrating on the visual and aesthetic aspect of the subject. However, a few graduate courses as independent studies and capstone design have been offered with a concentration in geometry designs that could be used in the design of systems with different purposes such as responsive facades, deployable/kinetic structures, and kinetic sculptures (Kalantar & Borhani, 2015, 2016b; Kalantar & Zhou, 2016). These courses did not focus on mathematical modeling of patterns and were more considered conceptual design using trial and error approaches to build 3D-models.

Integrating mathematical modeling into architecture curricula can result in improvement of the students' abilities to apply mathematics to architectural design (Verner & Maor, 2006). There is a need to expose architecture students to the mathematical way of thinking through topics that are suited to their interests and activities (Barrallio & Sanchez-beitia, 2015).

There are advantages in introducing mathematical modeling to architecture students (Barrallo & Sanchez-beitia, 2015; Maor & Verner, 2007). Mathematical modeling assists students in characterizing design problem using the mathematical formula in order to understand the behavior of the designed systems. Developing a mathematical model and/or simulating a computer model improve students' capabilities in the understanding of advanced tools, in implementing further applications for the designed systems, and in proposing more innovative solutions (Krishnan, 2017; Krishnan & Li, 2018, 2019; Darlington & Bowyer, 2016; Megahed, 2013; Crikis, 2010; Burry & Burry, 2010; Burry, 2007).

Mathematical and geometric modeling has been a challenging experience for architecture students who typically do not have a background in mathematics and engineering (Fischler,1976). However, having a course that emphasizes mathematical and geometric modeling improves students' skills in analytical thinking and problem-solving.

The purpose of this paper is to present a set of hands-on modules that utilizes mathematical and geometric modeling as educational activities in a facade design course. The proposed modules can be utilized in various courses such as facade design, capstone design/ thesis, or independent study in architecture and/or interior design undergraduate or graduate programs.

The proposed design modules

The process of pattern modeling and design of pattern-based responsive facades can be considered a valuable learning experience in an interdisciplinary process that includes research, design, simulation, performance evaluation, optimization, testing, and documentation.

The proposed hands-on modules have been designed to provide a series of project-based exercises as educational activities for a facade course. These modules complement educational activities in a design course by introducing pattern modeling along with its contemporary applications in advanced facade systems such as responsive facades.

The proposed modules consist of activities on pattern identification, mathematical modeling, shading function development, and simulation of the mechanism and facade system. The relationship between the modules and the activities is demonstrated in figure 1.

The goals of the activities are:

- To enable students to understand the geometric principles of a pattern with an emphasis on Persian geometric patterns.
- To introduce use of mathematical modeling and geometry transformation in pattern modeling.
- To introduce use of mechanical movement for pattern modeling.
- To engage students in two and three-dimensional modeling and simulation using advanced parametric modeling software.
- To improve students’ skills in deploying mathematical modeling in designing a responsive facade system.

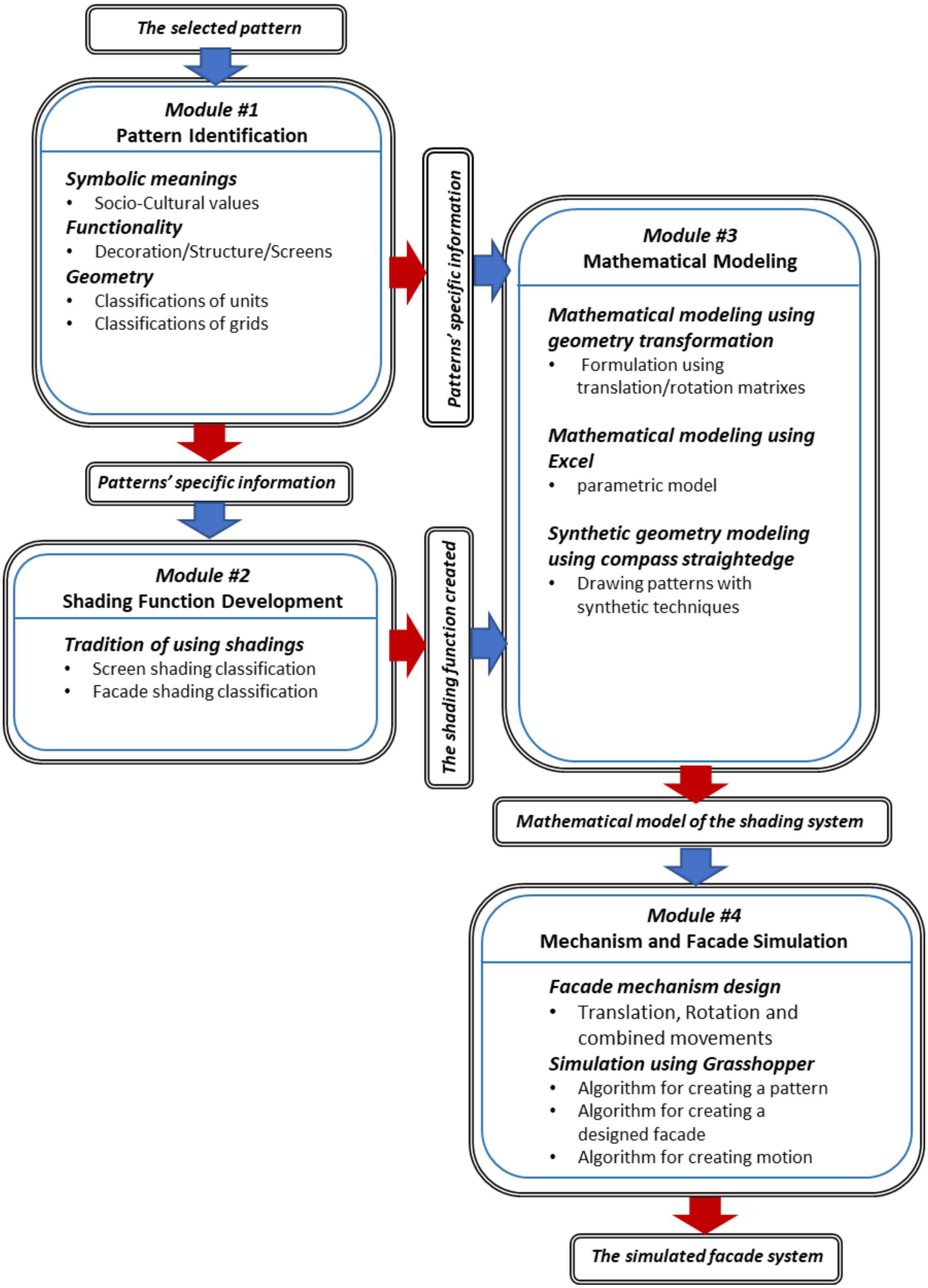


Figure 1: The relationship between proposed hands-on activity modules

Activity #1: Pattern Identification

Pattern identification is a process that studies pattern characteristics focusing on symbolic meaning of patterns, functionality of patterns, and geometric classification of patterns. Existing advanced facades are socio-culturally inert. Geometry as a socio-cultural design parameter can affect the performance of the facade (Anshuman, 2005). Geometric patterns along with calligraphy and floral patterns are the most renowned visual tradition in Persia and the surrounding regions socio-cultural values (Broug, 2013). The prohibition of anthropomorphic forms resulted from religious beliefs and created the potential for growing sets of geometric patterns (Abas & Salman, 1992; Emami & Giles, 2016). Geometric patterns can be classified based on both the practical experience of tiling and general geometric construction rules and mathematical facts (Abas & Salman, 1992, Abdullahi & Embi, 2013; Emami, Khodadadi, & Von Buelow, 2015, Sarhangi, 2012).

The goal of this activity is to encourage students to explore design concepts from regional socio-cultural values. To pursue this goal, in this activity students would identify socio-cultural geometric patterns that could be used in the design of responsive facades.

Through various existing patterns, students may be asked to focus on Persian patterns originated from a circle. As shown in figure 2, various circular patterns are constructed by dividing a circle into four, five, or six equal parts and they are known as four-fold, five-fold, and six-fold polygons, respectively (Broug, 2013). Other higher-fold polygons can be constructed from these basic polygons as shown in figure 2. For example, a twelve-fold polygon can be constructed from a six-fold polygon (Emami & Giles, 2016; Emami et al., 2015; Abdullahi & Embi, 2013; Kazempour & Mohammadzaheh, 2017).

Most of the Persian patterns are constructed using basic circular polygons as shown in figure 2. The center pattern of Persian domes, "*SHAMSEH*", is made from circular divisions. The term "*SHAMSEH*" stemmed from the Persian word meaning sun (Mehdi Nejad, Zarghami, & Sadeghi Habib Abad, 2016).

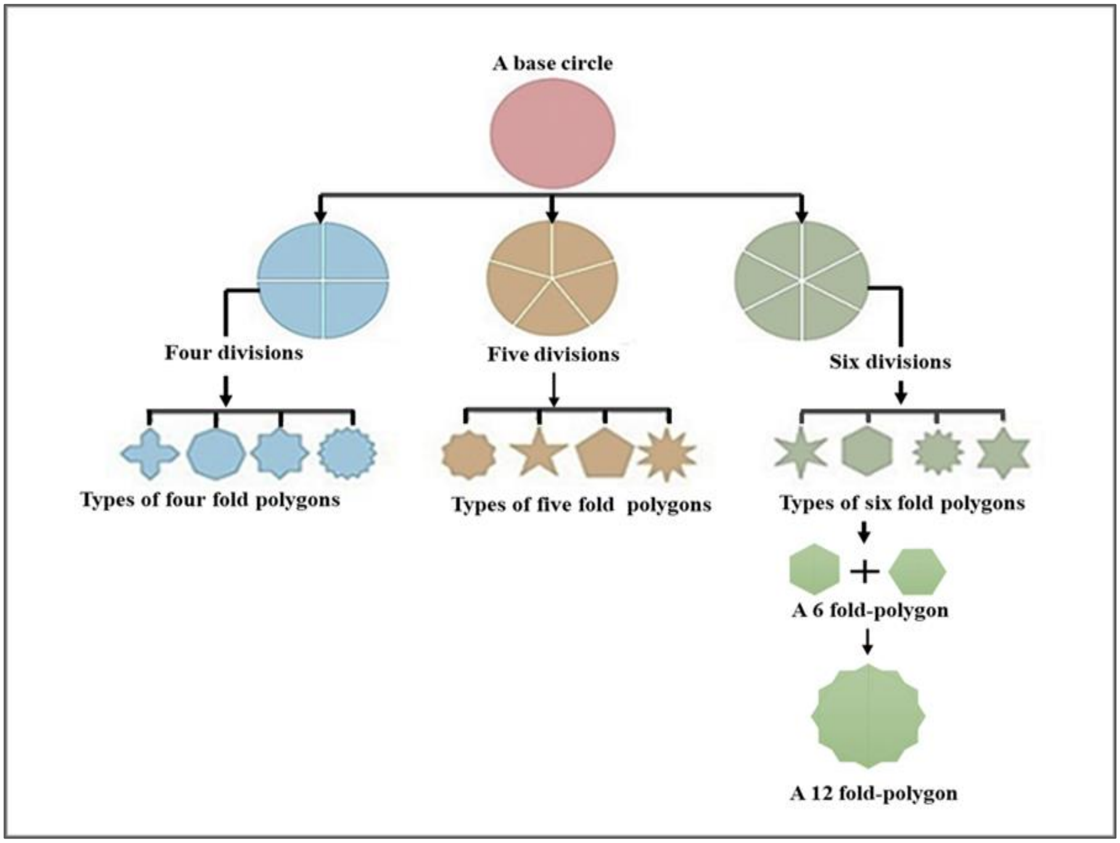
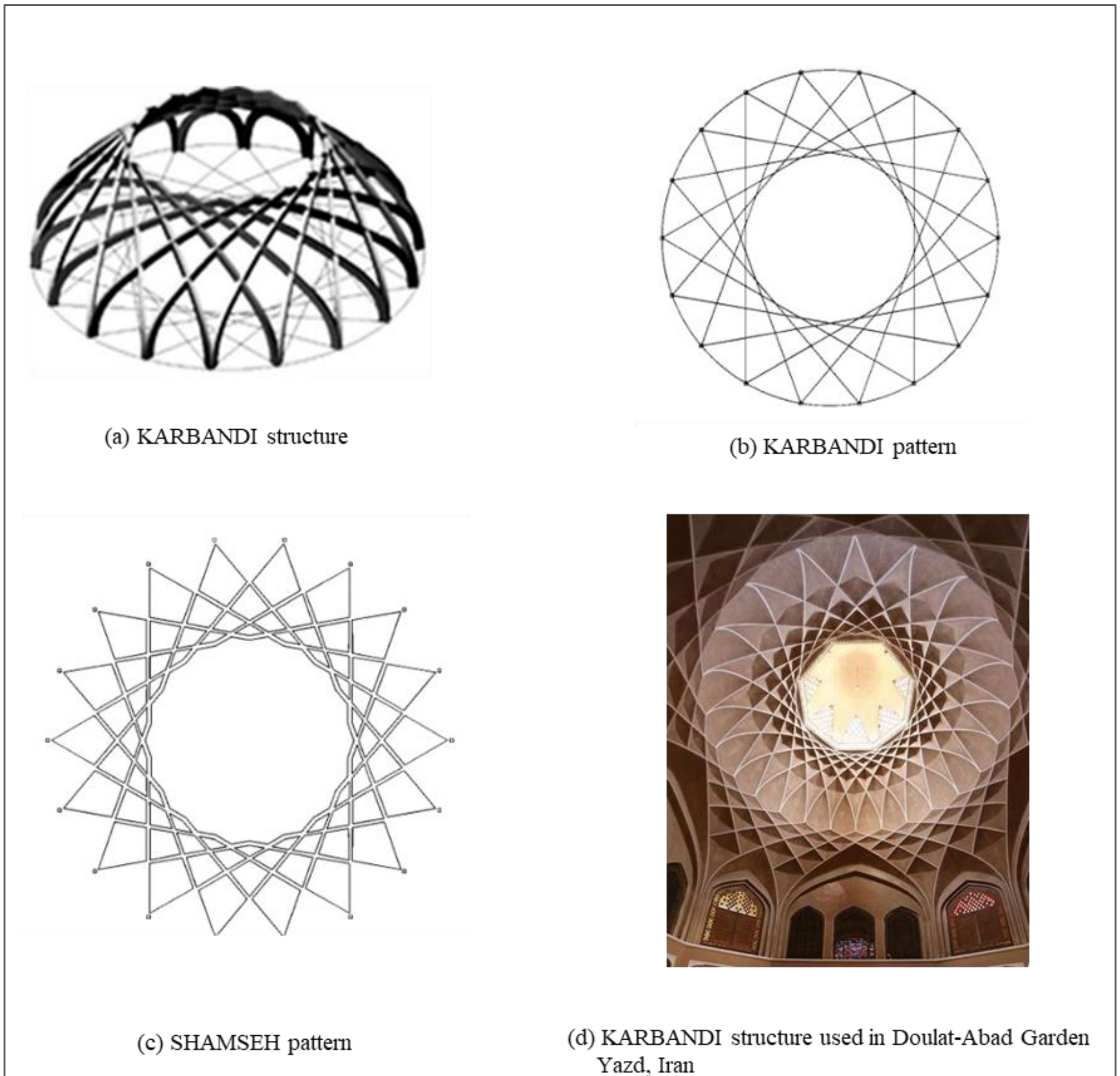


Figure 2: Different types of *n*-fold polygon

The 2D pattern of an 18-fold *"SHAMSEH"* and its corresponding 3D "*KARBANDI*" structure is shown in figure 3. The "*KARBANDI"* structure is constructed from a series of intersected arches in order to cover an interior dome (Ebrahimi, Aliabadi, & Aghaei, 2014; Broug, 2013; Mohamadianmansoor, Faramarzi, & Hatamimajd, 2012; Porahmadi, 2016).

Persian *"SHAMSEH"* is recognized as star polygon patterns in Islamic art and architecture (Sarhangi, 2000, Bourg, 2013, Abdullahi & Embi, 2013). Various "*SHAMSEH*" patterns that have been used as decorative elements are shown in figure 4 (Sarhangi, 2000).



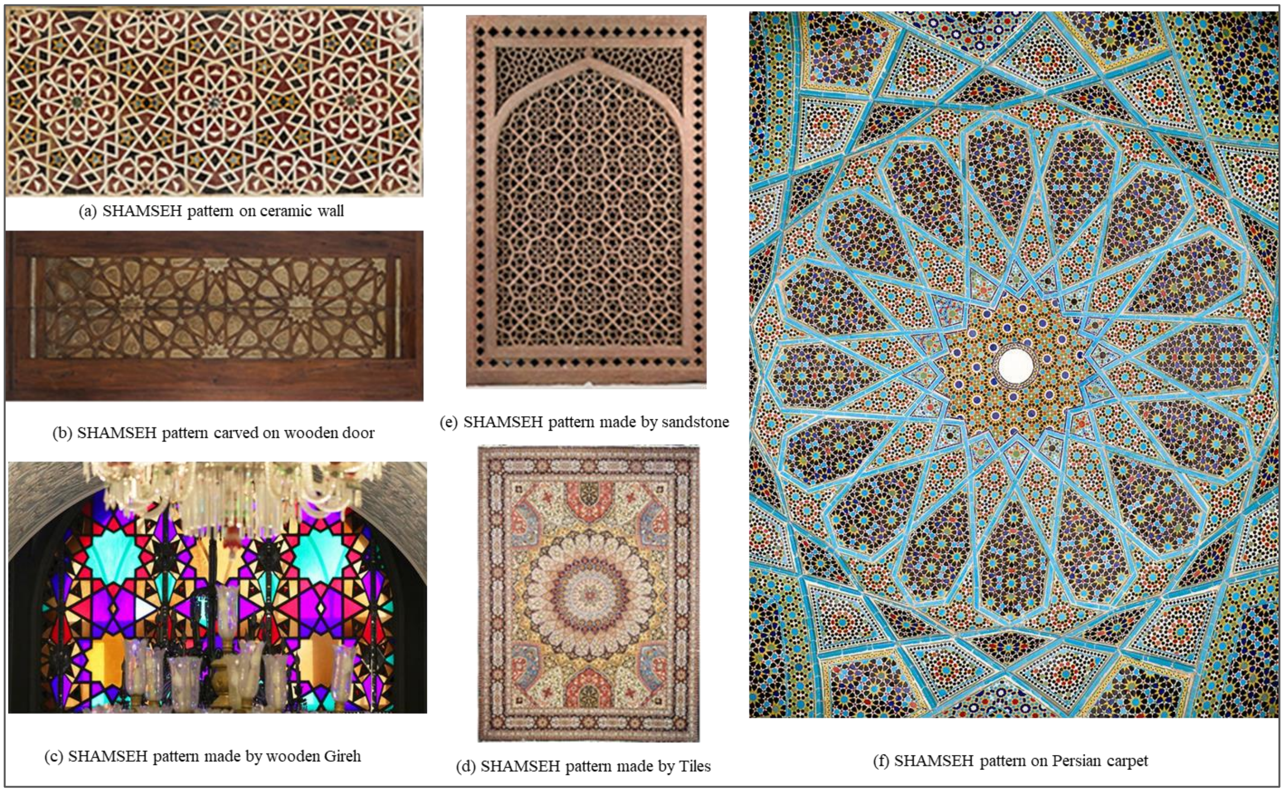
(a) KARBANDI structure

(b) KARBANDI pattern

(c) SHAMSEH pattern

(d) KARBANDI structure used in Doulat-Abad Garden Yazd, Iran

Figure 3: The 2D pattern of *"SHAMSEH"* and the 3D structure of "*KARBANDI*"



(a) SHAMSEH pattern on ceramic wall

(b) SHAMSEH pattern carved on wooden door

(c) SHAMSEH pattern made by wooden Gireh

(d) SHAMSEH pattern made by Tiles

(e) SHAMSEH pattern made by sandstone

(f) SHAMSEH pattern on Persian carpet

Figure 4: Various implementation of "*SHAMSEH*" pattern

The circular geometry of a "*SHAMSEH*" makes it an ideal choice of pattern for learning about mathematical and parametric modeling.

For this activity among different n-fold polygons, a 12-fold polygon is selected. This resembles a 12-fold "*SHAMSEH"* used in the center of "*KARBANDI*" structure at Fin Garden, Kashan, Iran as shown in figure 5. In the next activity, students develop a shading function of the desired facade system based on the selected pattern obtained from activity one.

Activity #2: Shading Function Development

This activity aims to emphasize implementation of patterns as daylight controller especially by focusing on shading screens and facades. Exploring existing shading screens can help students to identify design concepts from socio-cultural values. Also, it assists students in identifying design geometric variables such as angles of penetration, the porosity of facade, the granularity of facade, porosity area, and porosity depth.

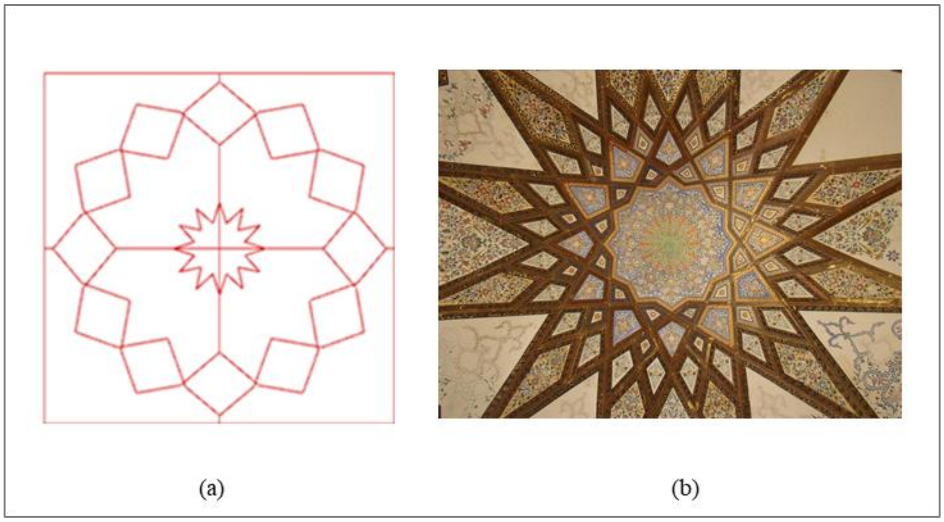


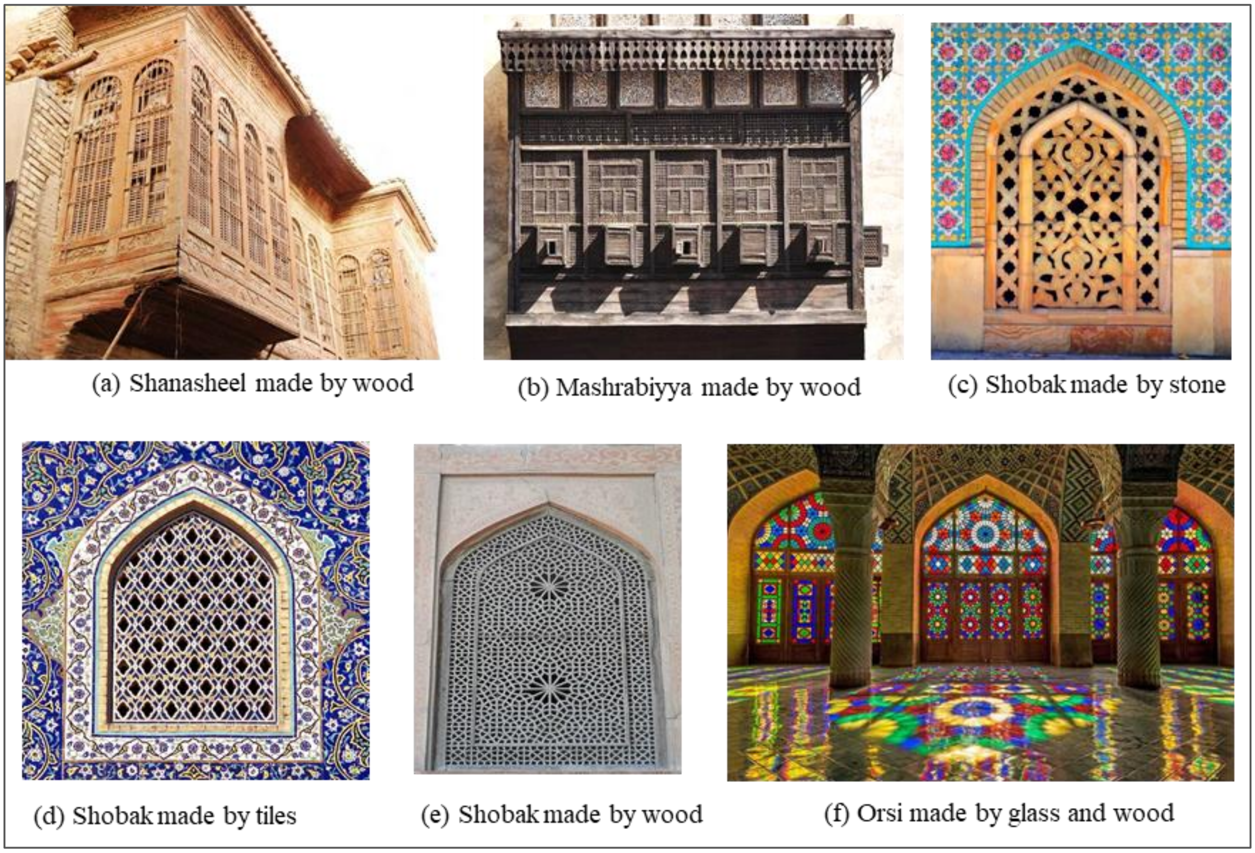
Figure 5: A 12-fold "*SHAMSEH"* used as the selected pattern

Previous studies have shown that the characteristics of geometric patterns such as various porosity percentage and granularity percentage have great potential for application in facade design of contemporary architecture (Emami et al., 2014; Emami & Giles, 2015). Porosity is defined as the ratio of hollowed areas divided by the total area. It is utilized in evaluating daylight performance of shading screens and facades (Emami, et. al., 2016).

Intense sunlight in the Middle East has created the tradition of controlling daylight by shading screens in this region (Sherif, El-zafarany, & Arafa, 2012). These screens were built to follow various purposes such as socio-cultural value, environmental function, and symbolic meanings. Grids of various geometric patterns were designed with different materials across the Middle East as shading screens.

Some of the traditional shading screens are wooden balconies known as Shanasheel (Emami et al., 2015), unglazed latticework known as Shobak made of wood, tile, brick, clay or stone

(Maghsoudi Nia, Hajihasani, Mohd Yunos, & Abdul Rahman, 2015), wooden latticework known as Mashrabiyya (Sherif, El-Zafarany, & Arafa, 2012), and colored glass attached to a wooden grid known as Orsi (Hosseini, Mohammadi, Rosemann, & Schroder, 2018). Figure 6 illustrates different traditional shading screens used in the Middle East.



(a) Shanasheel made by wood (b) Mashrabiyya made by wood (c) Shobak made by stone

(d) Shobak made by tiles (e) Shobak made by wood (f) Orsi made by glass and wood

Figure 6: Different traditional shading screens used in the Middle East

Notable examples of inspiration from traditional shading screens are the terracotta shading screens of Masdar City (Amrousi, 2017), the perforated metal facades of Ali Mohammed T. Al-Ghanim Clinic (Alothman, 2017), the multiple layers of perforated metal facade of Doha Tower (Al-Kodmany, 2014), and the wooden shading screen of Abu Dhabi Central Market (Song & Shim, 2017).

The tradition of using geometric patterns as daylight controllers was used in the concepts of pattern-based responsive facades through the Middle East. Among existing buildings that have used geometric patterns in facade design, are Arab World Institute and Al-Bahr Towers have responsive facade systems (Heidari Matin, Eydgahi, & Shyu, 2017; Karanouh & Kerber, 2015; Heidari Matin, Eydgahi, Shyu, & H. Matin, 2018).

In this activity, students classify existing patterns that have been implemented for shading by various geometric patterns and materials of the past and the contemporary Middle Eastern architecture for each of the screen and facade categories. Then, they identify pattern-based responsive facades among existing projects and buildings. Finally, they develop a shading function for their selected pattern in activity one.

Kaplan (2000) has presented different approaches for visualizing geometric patterns including plain, outline, checkerboard and outline-and-checkerboard approaches. In this activity, these approaches are utilized to develop the three-dimensional shading screen from the two-dimensional geometric pattern.

The shading function of the selected pattern, a 12-fold polygon, is shown in figure 5. Figure 7(a) shows the plain pattern in the x-y plane. The lines that create the pattern are offset on both side of the lines to generate outline. As shown in figure 7 (b), the blue lines indicate the outline of patterns that are used to visualize solids and voids in the screen. Pattern outlines are filled with hatches in figure 7 (c). Hatches with the black color visualize solids while hatches with the white color visualize voids in this screen. These hatches create checkerboard visualization which is utilized for measuring porosity percentage in shading screens as shown in figure 7 (d).

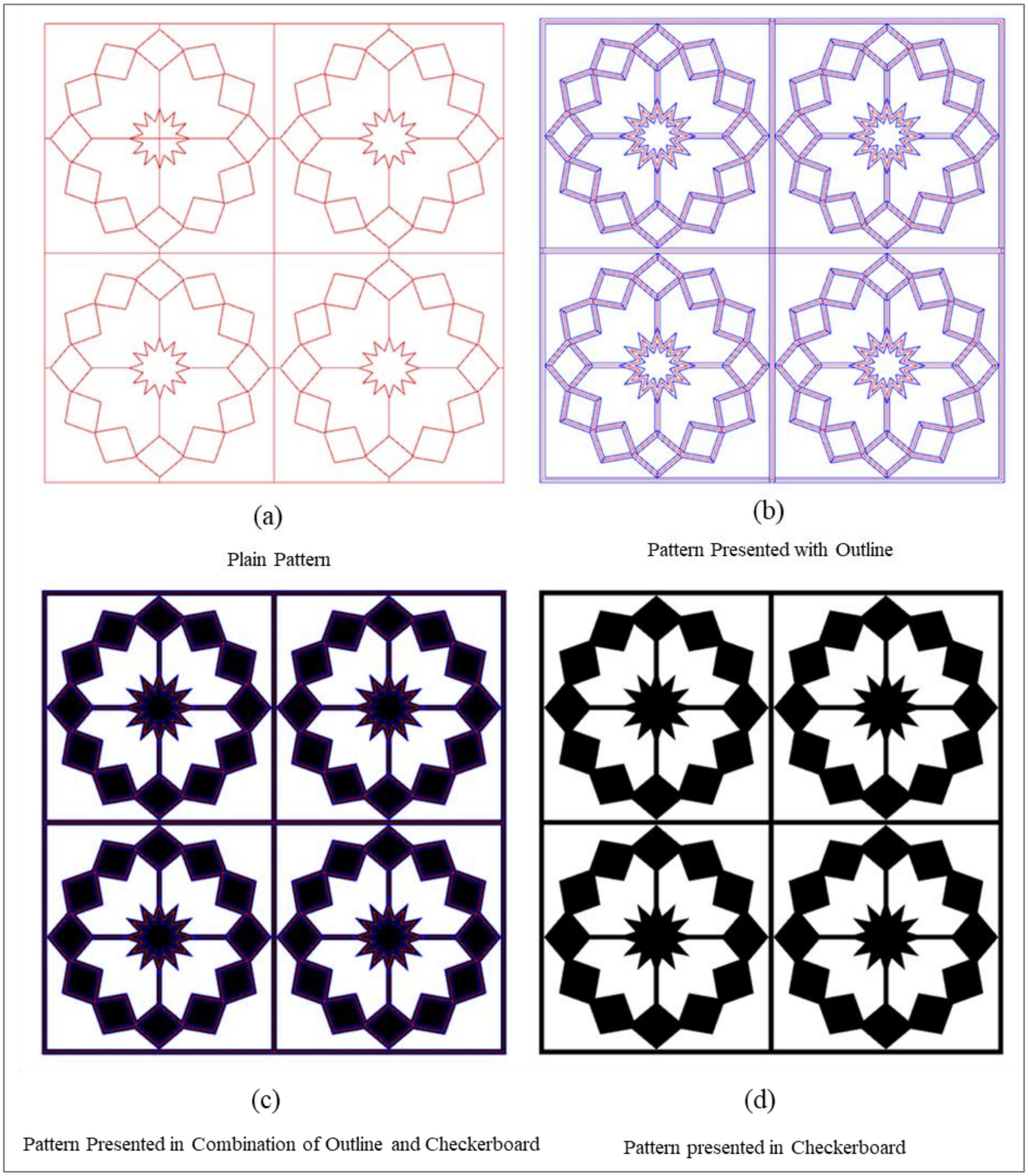


Figure 7: The shading function of a 12-fold polygon

In the next activity, students develop mathematical model of the selected pattern and its corresponding shading function by using geometric parameters.

Activity #3: Developing mathematical modeling

Geometric patterns are deeply mathematical in nature (Kaplan, 2005; Kaplan & Salesian, 2004). Mathematical modeling assists students in analyzing pattern behavior that is utilized in facade design. Pattern generation utilizing compass-straightedge techniques in Persia and the surrounding regions were based on the practical experience of tiling with the real shapes of patterns instead of proving theorems and establishing mathematical facts (Abas & Salman, 1992; Sarhangi, 2012).

This activity consists of three phases. In the first phase, students learn how to visualize a geometric pattern utilizing compass-straight techniques through constructing regular tessellations of 12-fold polygons by repeating the pattern in circles, squares and regular hexagons in order to develop the series of patterns in two-dimensional surfaces (Kaplan & Salesian, 2004). In the second phase, analytic geometry is utilized to develop mathematical formulations for the selected geometric pattern. In the third phase, students use Microsoft Excel to depict geometric variations of the selected pattern while the geometric parameters are changing.

A 12-fold polygon consists of 12 diamonds. A diamond as shown in figure 8 represents one individual representative cell that is repeated throughout the polygon mechanism. This representative cell is repeatedly translated and rotated with a specific manner to generate the polygon mechanism. As shown in figure 8, the cell is represented by four sides of length L, which intersect at four specific corners named 1, 2, 3, and 4. The angle between the sides and the vertical diagonals is θ, and d represents half of the length of the vertical diagonal.

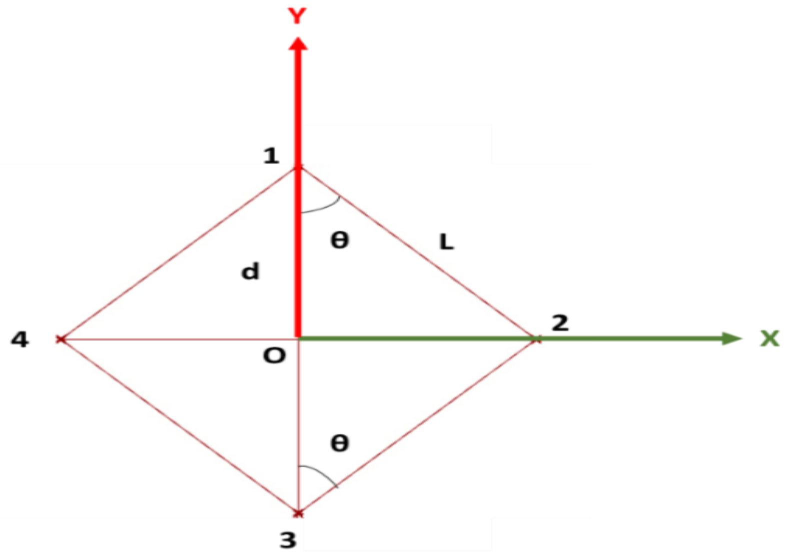


Figure 8: A diamond-shape representative cell

The coordinates of corners 1, 2, 3, and 4 are:

$$X1 = 0$$
$$Y1 = d = L\cos\theta \tag{1}$$

$$X2 = L\sin\theta = \sqrt{L^2 - d^2}$$
$$Y2 = 0 \tag{2}$$

$$X3 = 0$$
$$Y3 = -d = -L\cos\theta \tag{3}$$

$$X4 = -L\sin\theta = -\sqrt{L^2 - d^2}$$
$$Y4 = 0 \tag{4}$$

It should be noted that although the representative cell might be considered in many different locations through the mechanism, the coordinates of its corner always remain the same as above in the local coordinate system of the cell, which is always located at the center of the cell.

The construction of the polygon mechanism is accomplished by repeatedly utilizing a diamond-shaped representative cell at a new location while the bottom corner of the new cell remains attached to the top corner of the old cell as shown in figure 9. This repetition may be characterized first by a translation of the original cell to a new location and then a subsequent rotation. As shown in figure 9, the translation is characterized by the translation vector components X_{offset} and Y_{offset}. The subsequent the rotation is characterized by rotation angle $-\gamma$ about Z axis.

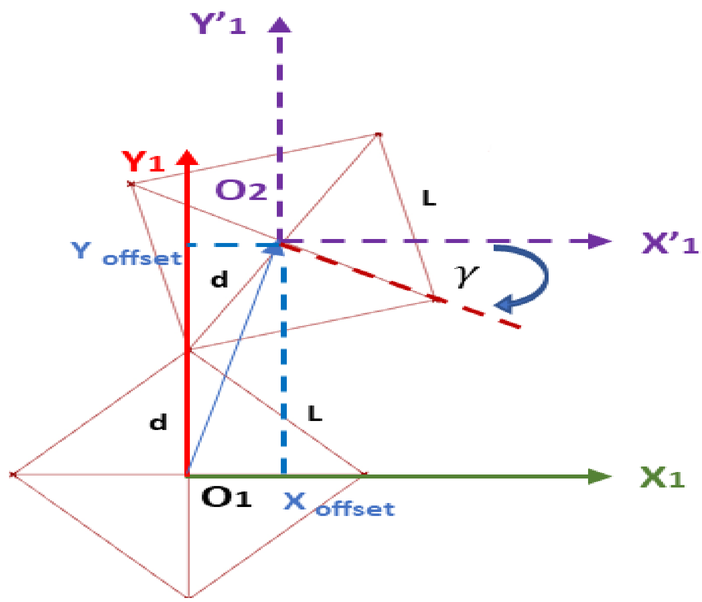


Figure 9: Translation and rotation of representative cell from old to new adjacent location

Considering figure 9, the translation vector components X_{offset} and Y_{offset} are written in terms of the diamond geometric characteristics as:

$$X_{offset} = d \sin \gamma \tag{5}$$

$$Y_{offset} = d + d \cos \gamma \tag{6}$$

$$Z_{offset} = 0 \tag{7}$$

where γ is rotation angle of the representative cell from one cell to its following adjacent cell. Rotation angle γ is dependent on the total number of the diamonds (n) in the mechanism as:

$$\gamma = \frac{2\pi}{n} \tag{8}$$

Now that the translation and rotation of the representative cell are characterized, the local coordinates of each corners in the second cell can be transformed to the local coordinates of the first (original) cell using transformation matrices (Niku, 2010):

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{Bmatrix} = [\, Trans(\text{X offset}, \text{Y offset}, \text{Z offset}\ ,1)\ Rot(z, -\gamma)] \begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{Bmatrix} \tag{9}$$

Or:

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{Bmatrix} = [\, Trans(d \sin \gamma\ , d + d\cos \gamma, 0, 1)\ Rot(z, -\gamma)] \begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{Bmatrix} \tag{10}$$

Equation (10) can be written in the matrix form as:

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{Bmatrix} = \begin{bmatrix} \cos \gamma & \sin \gamma & 0 & X_{offset} \\ -\sin \gamma & \cos \gamma & 0 & Y_{offset} \\ 0 & 0 & 1 & Z_{offset} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{Bmatrix} \tag{11}$$

Equation (11) transforms the local coordinates of the corners in the second cell to the local coordinates of the corners in the first cell. The same concept may be applied in transforming the local coordinates of cell number *n* to the coordinates of cell number *n*-1 as follows:

$$\begin{Bmatrix} X_{n-1} \\ Y_{n-1} \\ Z_{n-1} \\ 1 \end{Bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & X_{offset} \\ -\sin\gamma & \cos\gamma & 0 & Y_{offset} \\ 0 & 0 & 1 & Z_{offset} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} X_n \\ Y_n \\ Z_n \\ 1 \end{Bmatrix} \tag{12}$$

Equation (12) may be utilized to transform the local coordinates of each cell to its prior adjacent cell. If this relationship is used repeatedly from cell number *n* to cell number *n*-1 to cell *n*-2 and extended to cell number 1, the local coordinates of cell number *n* can be transformed to the local coordinates of cell number 1 using the following formula:

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{Bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 & X_{offest} \\ -\sin\gamma & \cos\gamma & 0 & Y_{offest} \\ 0 & 0 & 1 & Z_{offest} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{(n-1)} \begin{Bmatrix} X_n \\ Y_n \\ Z_n \\ 1 \end{Bmatrix} \tag{13}$$

Figure10 depicts the coordinate systems of both cell number *n* and cell number 1. It should be noted that the coordinate system of cell number 1 is considered as the global coordinate system of the entire mechanism.

Equation (13) is utilized to transform the local coordinates of cell number *n* to the global coordinates of the entire mechanism. By varying cell numbers from 1 to *n*, the global coordinates of all four corners of each cell can be calculated for all the representative cells within the range of 1 to *n*. Therefore, the coordinates of all corners of all diamonds in the entire mechanism are calculated in one common global coordinate system. This allows the geometry of the entire mechanism to be mathematically defined. By using equation (13) and while the geometric parameters such as γ, L, and *d* are changed, the mathematical model of 12-fold polygon can be developed.

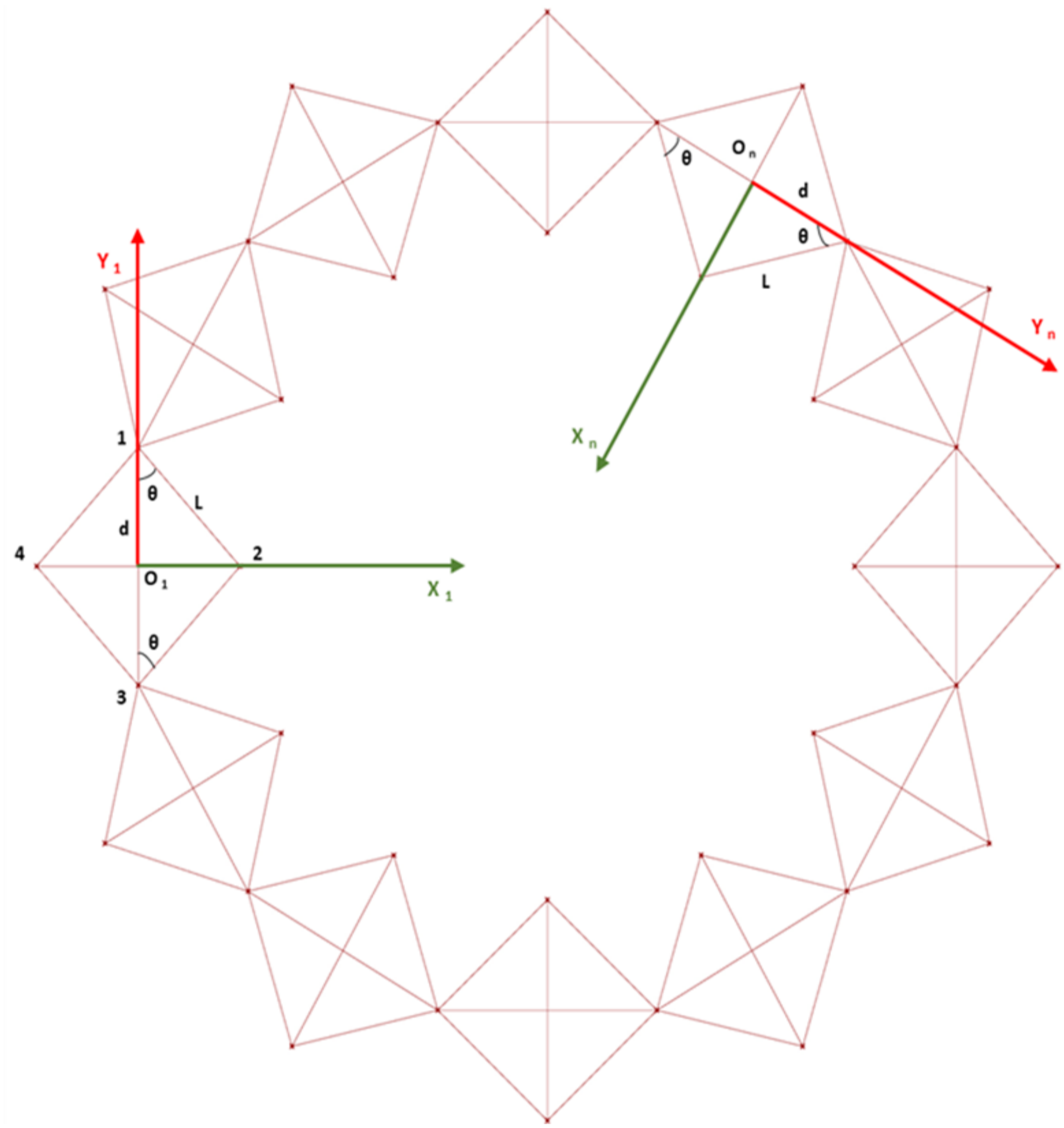


Figure 10: Coordinates transformation between cell n (local coordinate) to cell 1 (global coordinates)

Figure 11 presents the coordinates of the 12-fold polygon's corners for n =12 and for various values of γ, d and L. The corners' coordinates of first diamond is calculated by using equations (1) - (4). The corners' coordinates of the other diamonds are obtained from:

$$\begin{Bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{Bmatrix} = \begin{bmatrix} \cos 30 & \sin 30 & 0 & d \sin 30 \\ -\sin 30 & \cos 30 & 0 & d + d \cos 30 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{(12-1)} \begin{Bmatrix} X_{12} \\ Y_{12} \\ Z_{12} \\ 1 \end{Bmatrix}$$

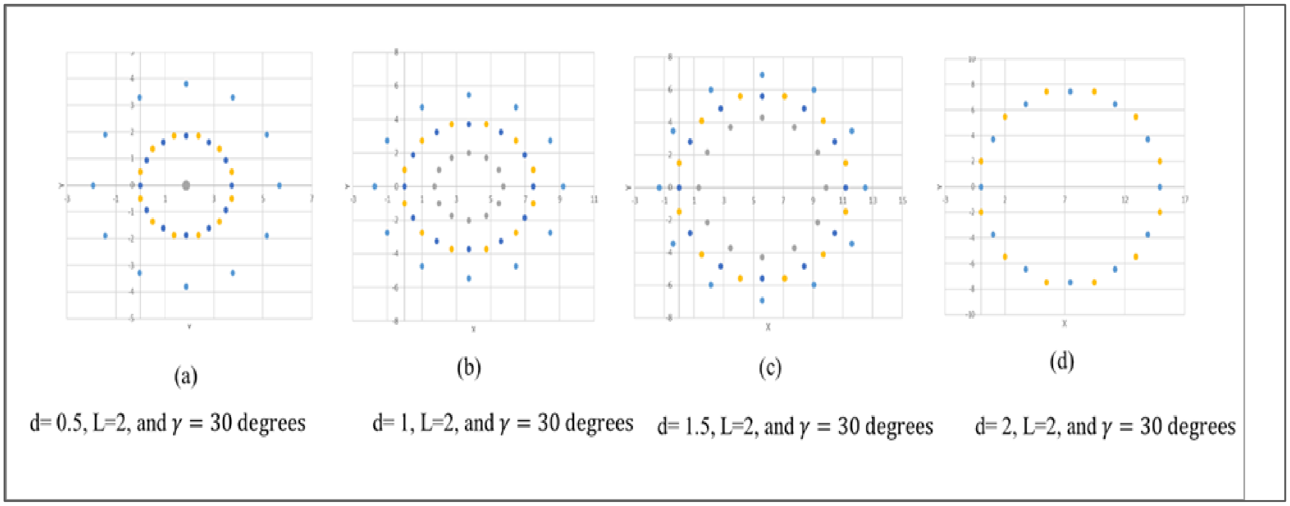


Figure 11: The corners' coordinates of 12-fold polygon modeled with Excel

Joining the corners while *d* is varied from 0.5 to 2 create the pattern shown in figure 12. Based on equation (8), any changes in γ leads to changes in *n*. Figure 13 presents various patterns created by changing γ.

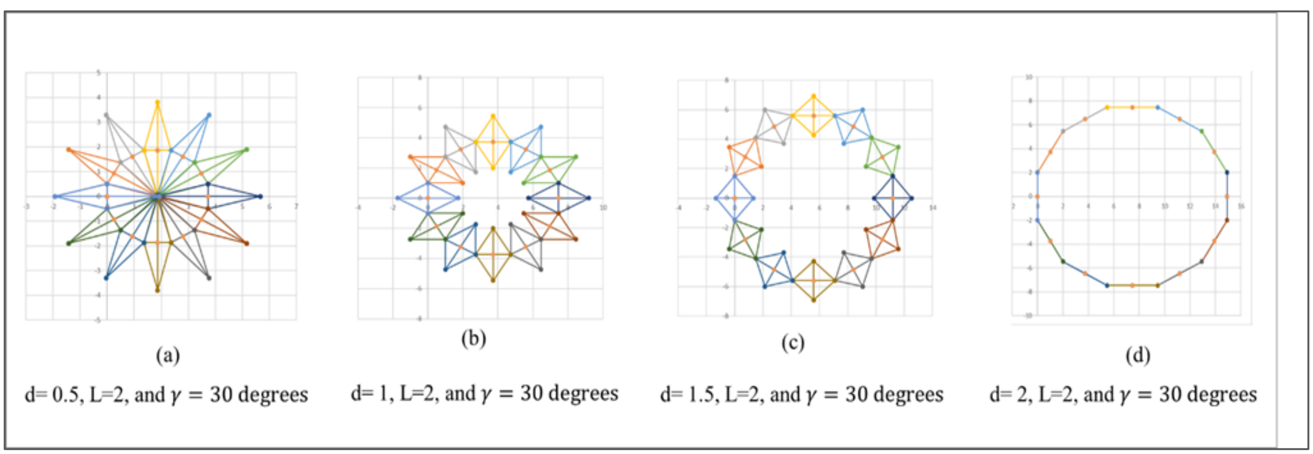


Figure 12: Geometric variations of 12-fold pattern created with Excel when *d* is changing

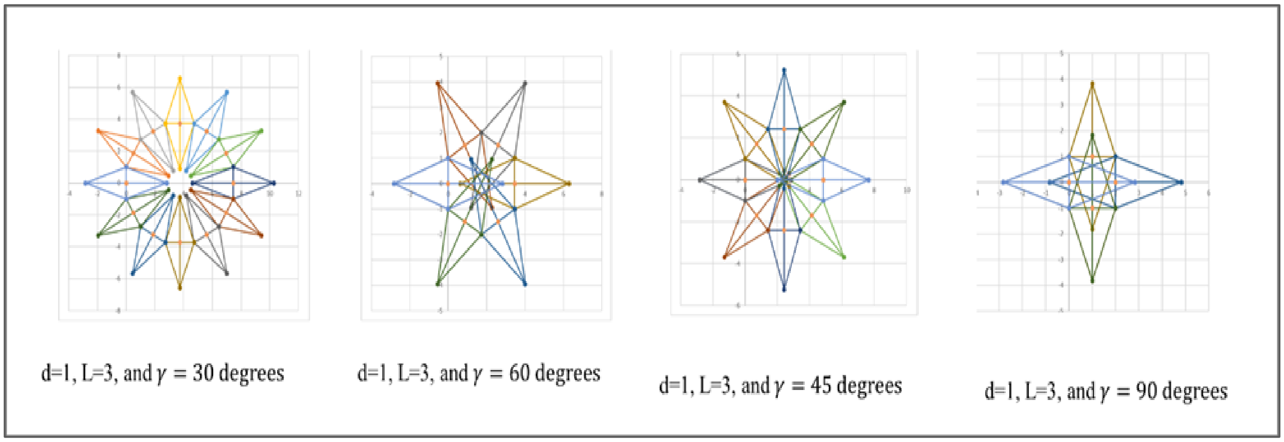


Figure 13: Geometric variations of 12-fold pattern created with Excel when γ is changing

In the next activity, the mathematical model resulting from this activity is used to design the mechanism and the facade system.

Activity #4: Simulating mechanism and the facade system

Various mechanisms can be utilized for creating motion in the facade systems. Advanced parametric or solid modeling tools such as Grasshopper or SolidWorks software can be utilized for designing and applying different mechanisms to a facade system. The Grasshopper modeling tool is a visual programming language that works within Rhinoceros 3D Computer Aided Design software to develop generative algorithms for parametric modeling. In this activity, students utilize the mathematical formulas derived in previous activity to gain experience in developing algorithms for creating a pattern, designing a facade system based on the pattern, designing a mechanism, and utilizing the mechanism to create motions in the designed facade system using the Grasshopper software.

The selected pattern, a 12-fold polygon, is simulated using the geometry of three concentric circles as the coordinates of the diamonds' corners of the 12-fold polygon are located on three levels. The association between these circles is characterized by deploying the relationships among their radiuses. These circles are divided into 12 identical pieces which are indicated with points. The steps involved in creating the 12-fold polygon are presented in figure 14. The Grasshopper components such as circle, rotation, division, list, line, function, and sliders are used to simulate the 12-fold polygon as shown in figure 15.

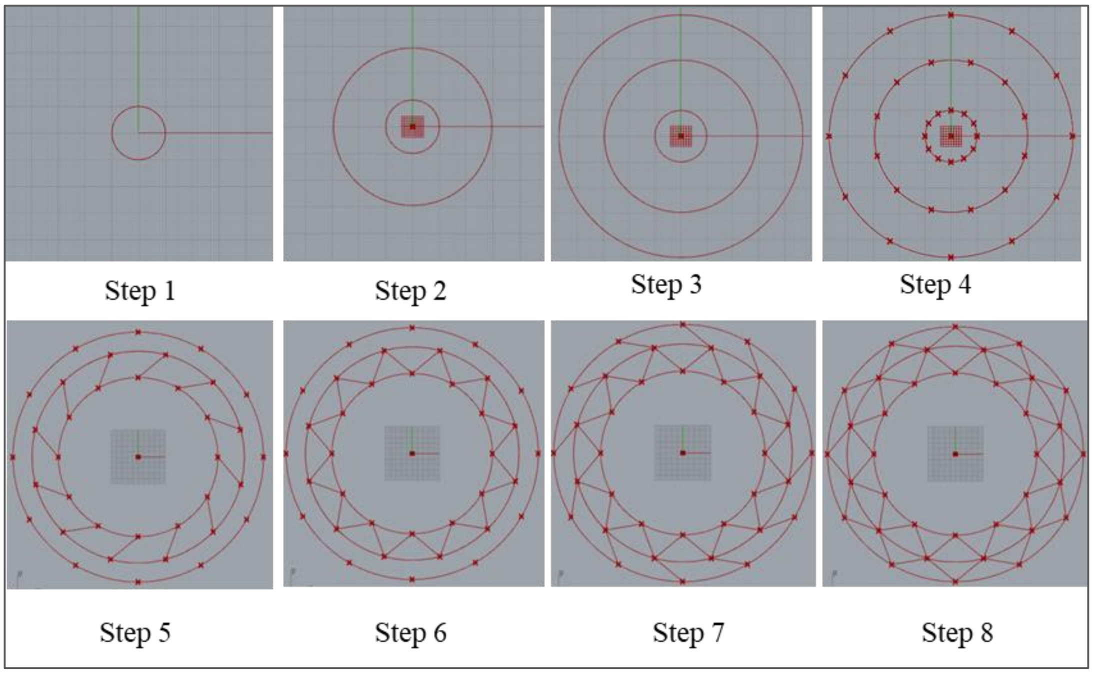


Figure 14: Simulation process of the 12-fold polygon in Grasshopper

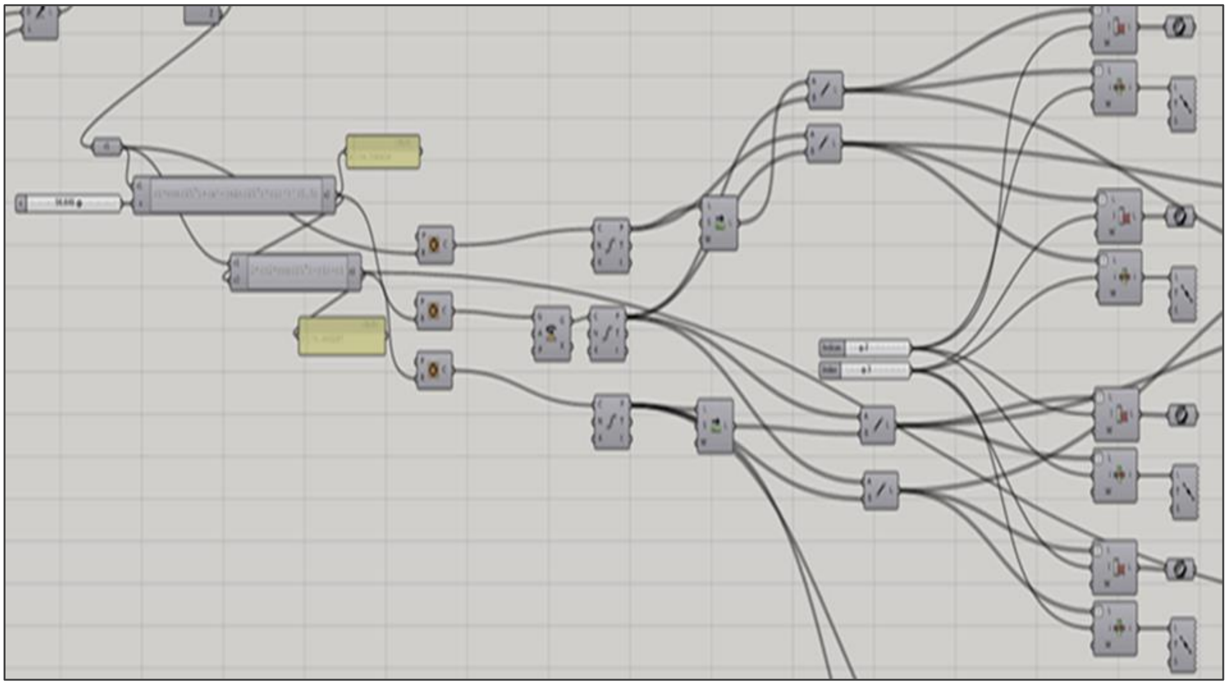


Figure 15: The script for creating 12-fold polygon in Grasshopper

In order to generate motions through this pattern, the linear crank-slider mechanism in Grasshopper is utilized. This mechanism is constructed with a circle and lines as shown in figure 16. The lines present the length of the crank and rod while the circle presents the rotation path of the hinge pin.

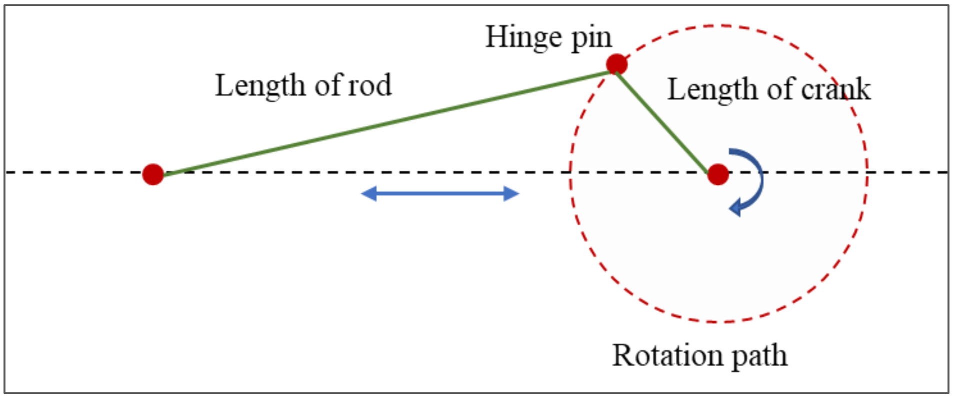


Figure 16: The geometry of linear crank-slider mechanism

The construction of this mechanism in Grasshopper is shown in figure 17. The components used are a plane, rotate plane, rotate, end points, circle, curve, and line SDL, deconstruct point, and line. Some of the motions generated with this mechanism are shown in figure 18. The final simulated facade system is presented in figures 19 and 20.

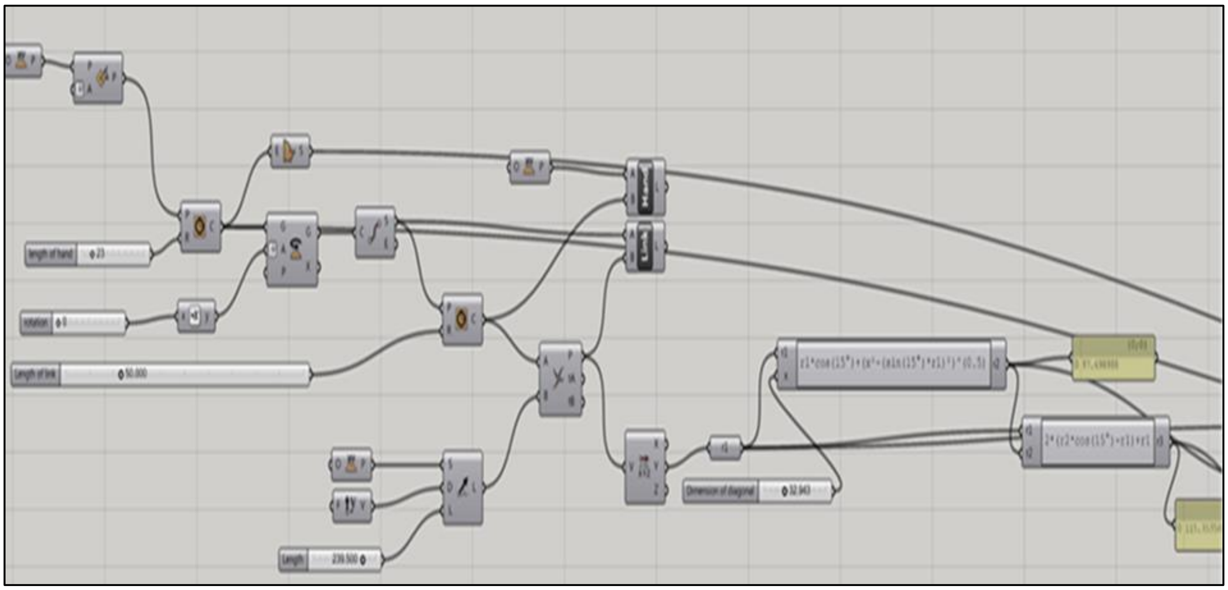


Figure 17: The crank-slider algorithm to create motions in the pattern

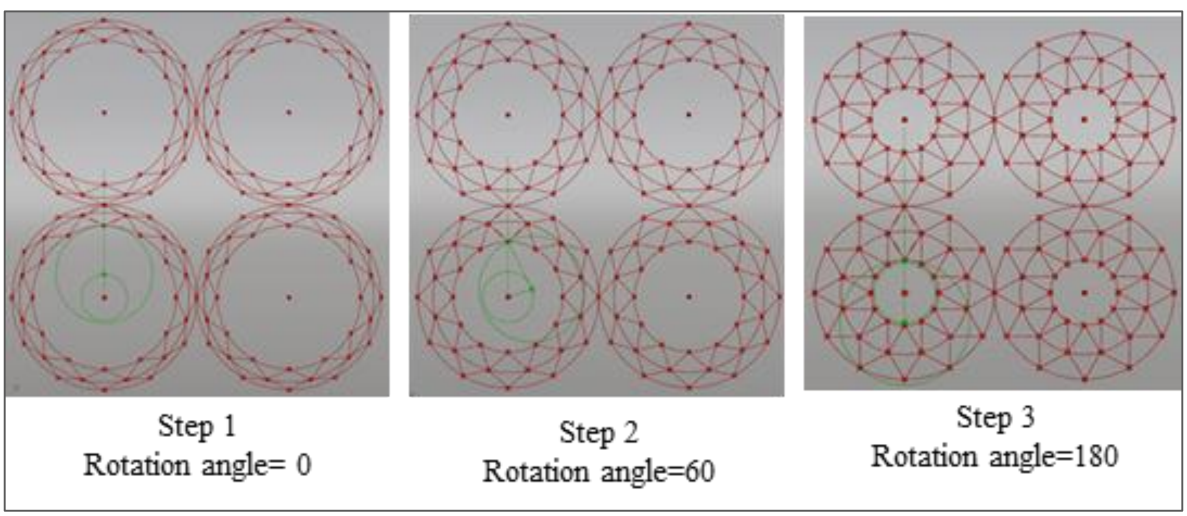


Figure 18: Various configurations of patterns generated by crank-slider algorithm

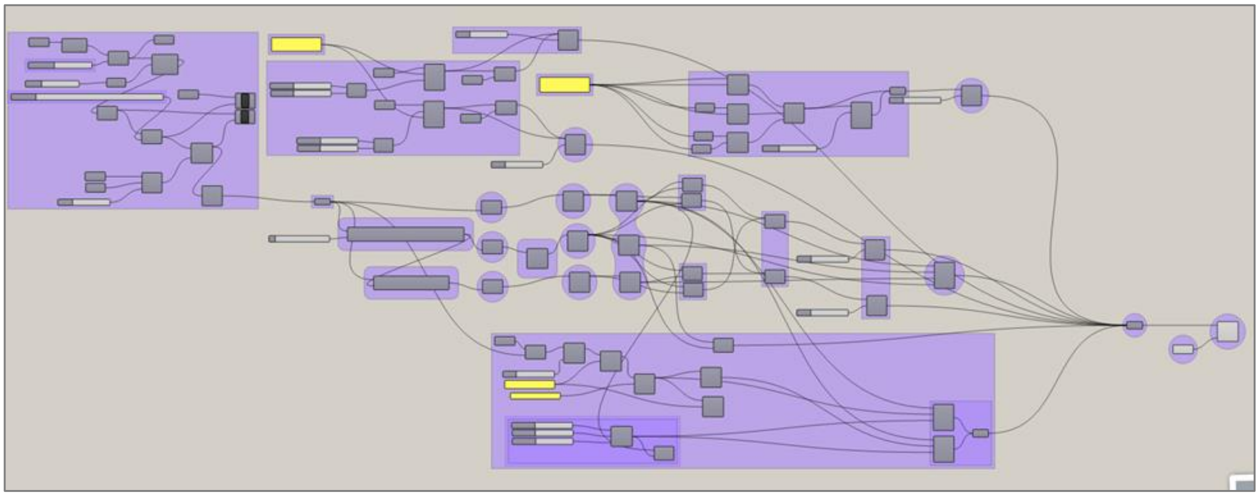


Figure 19: The Grasshopper algorithm of the designed facade system

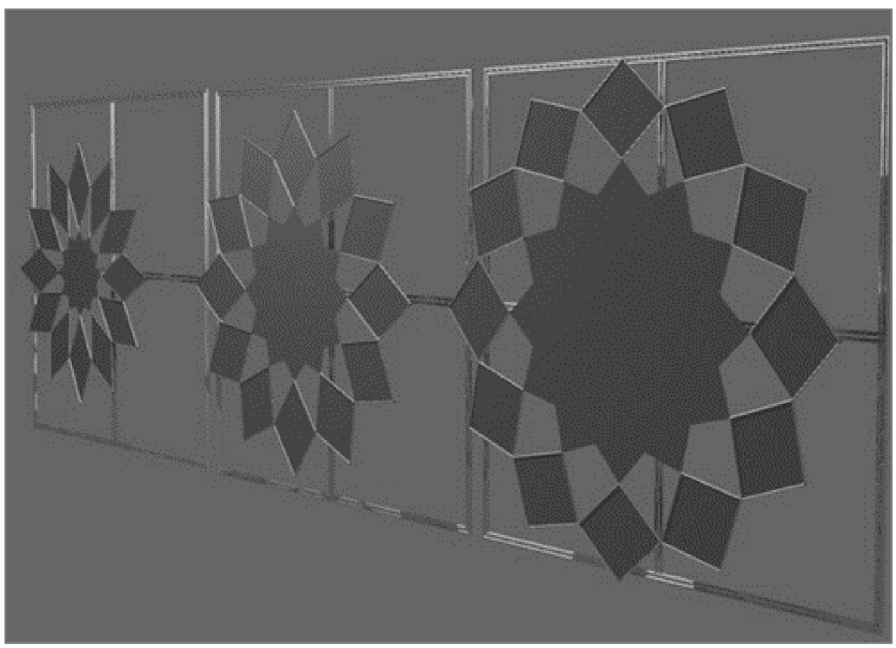


Figure 20: The simulation of the design

The proposed activity modules provide students invaluable hands-on experience and knowledge of various topics such as transformable geometries, pattern modeling, mechanical movement, and parametric simulation and modeling. The nature of these project-based activities allows students to engage in conceptual modeling by applying advanced architectural tools such as Grasshopper along with Microsoft Excel to perform analysis and design of the facade systems.

The use of the modules requires a basic knowledge of college level mathematics, mechanics, Microsoft Excel, and Grasshopper that could be provided by the instructors during the lectures. Also, these modules can be collaboratively used by a team of graduate/undergraduate students as a course project.

Conclusion

In this paper, a number of educational activities have been presented that are suitable for architecture students who want to gain hands-on learning experience in the design process of an interdisciplinary pattern-based course on responsive facades. The presented activities consist of educational modules on pattern identification, mathematical pattern model development, creating a facade shading function, and simulating the designed facade system.

The proposed activities can be complemented with additional modules for studying various facade control systems and their corresponding sensors/actuators utilized for implementing a prototype of the facade system. Another module could be developed for testing the influence of the proposed responsive facade on the visual comfort of occupants and energy efficiency of buildings.

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