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Math Out of the Box: A K-5 Mathematics Curriculum and Teacher Professional Development Program

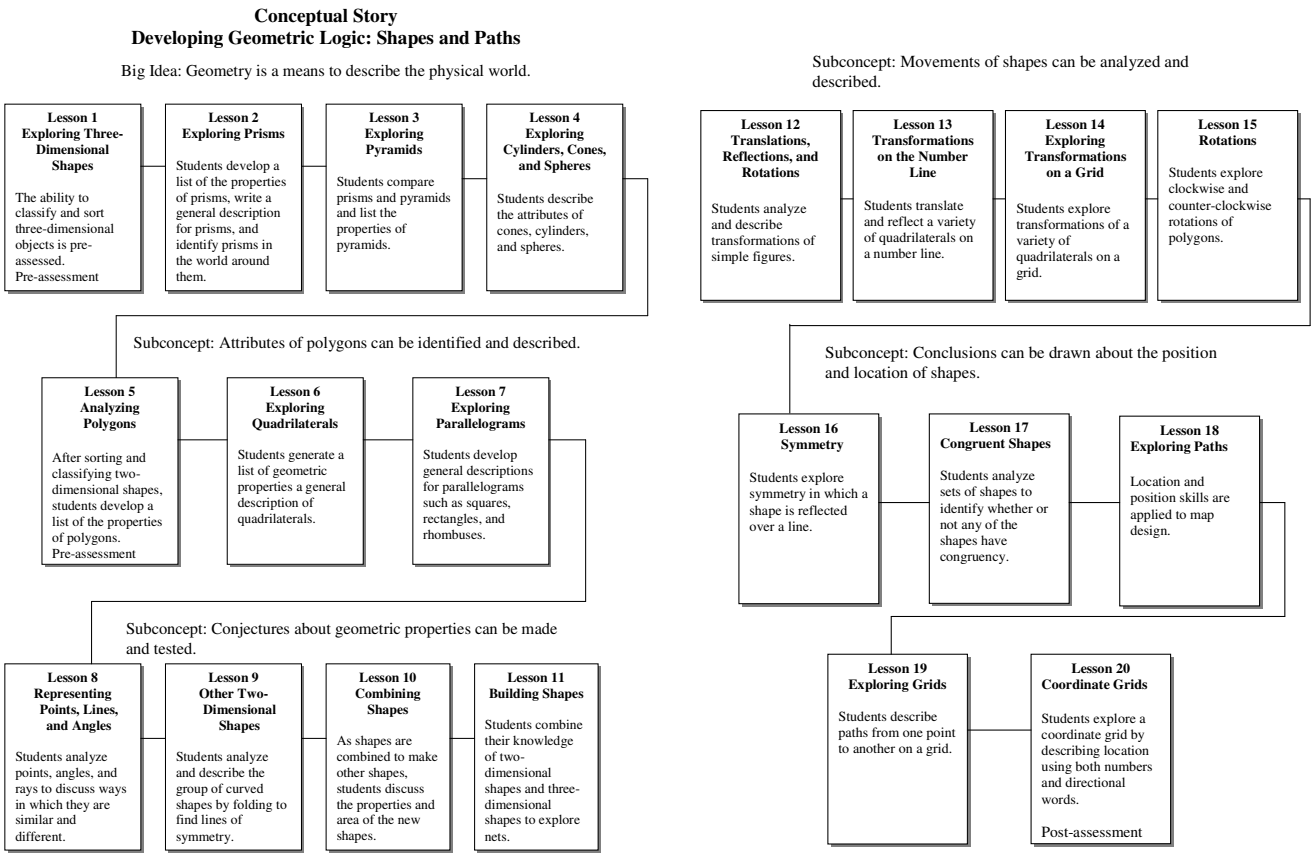
Background information

Math Out of the Box is a K-5 mathematics curriculum and a companion professional development program for teachers under development in the College of Engineering and Science at Clemson University. The program's overarching goal is to fulfill the mathematical promise that exists in every child by providing teachers with innovative materials, a mathematically challenging curriculum, and high quality professional development.

This curriculum is developed to be standards-based, research-based, and inquiry-based. A standards-based curriculum can be recognized by the inclusion of mathematics for all students, the interconnectedness of processes and concepts, the connection to big ideas of mathematics, the continuous building of foundational ideas vertically through the grades, and the thoughtful identification of representations that build "intellectual engagement"[42]. Research-based curricula are those that are based on the body of knowledge that defines how students learn and how teachers teach. In addition, research-based curricula add to this body of knowledge in a continuous cycle of research and revision. Inquiry-based curricula are designed so that students construct their own knowledge under the guided instruction of a teacher who has experienced similar knowledge construction.

The Math Out of the Box curriculum is designed to be released in four strands. The first two strands, *Algebraic Thinking* (which addresses algebra and data analysis standards) and *Geometric Logic* have been field tested and are currently available through the publisher Carolina Biological Supply Company. The last two strands, *Measurement Benchmarks* and *Number Concepts* are under development. The *Measurement Benchmarks* strand will be published early in 2007. The *Number Concepts* strand will be published beginning in 2008. These strands are vertically aligned through the grade levels and provide a comprehensive mathematics curriculum that is designed to support the mathematical development of all students, the professional development of teachers, and the development of the larger school community.

Each grade level manual is designed around a mathematical big idea and arranged in subconcepts. The following conceptual storyline shows the organizational design of the third grade geometry manual:



The interconnectedness between Math Out of the Box and engineering design takes place in the inquiry approach that is used in each lesson of the curricula and in each professional development experience designed for teachers. This paper describes the relationships between engineering design elements and inquiry-based instruction that the developers believe to be essential to successful learning and teaching of elementary mathematics. Early data and promising trends in the learning and teaching of mathematics are shared.

The Math Out of the Box developers have worked at all levels of K-16 STEM education. As a result of their experiences, the developers of this program formed the following beliefs about mathematics teaching and learning through inquiry, which are supported by the research-base of the National Council of Teachers of Mathematics' *Principles and Standards for School Mathematics*[31]:

- All students must have access to a curriculum that connects mathematical ideas.
- All teachers of mathematics need to be confident in their own teaching and learning as well as that of their students.
- Students need to have rich and varied experiences and materials as part of their mathematical learning.
- Assessment guides students in knowing what they have learned, aids teachers in planning instruction, and informs the community.

- Technology supports students and teachers as they engage in rich mathematical experiences.

Literature that describes pedagogy relating to design standards includes many of the same beliefs. In a synthesis of the literature, Burghardt and Hacker described "pedagogically solid design projects" as having the following criteria when focused on the learner:

- Engaging children as active participants, giving them greater control over the learning process.
- Assisting students to integrate learning from language, the arts, mathematics, and science.
- Encouraging pluralistic thinking, avoiding a right/wrong dichotomy and suggesting that multiple solutions are possible.
- Providing children an opportunity to reflect upon, revise, and extend their internal models of the world.
- Encouraging children to put themselves in the minds of others as they think about how their designs will be understood and used[5,36].

The components of this project that make engineering design and inquiry-related investigations possible in typical elementary mathematics classrooms are discussed in the following sections of this paper.

Learning cycle

This curriculum uses a learning cycle to foster inquiry-based learning. The learning cycle used in the lessons gives teachers the structure that is needed to open the mathematics classroom to problems that have multiple solutions, whether the teachers are traditional or inquiry-based in practice, providing teachers with a template that promotes the development of active inquiry and critical thinking. The learning cycle allows students to make connections between past and present learning experiences and is based in the "cognitive principle of assimilation," which implies that understanding cannot be imposed on the learner, but instead is developed progressively by the learner, beginning with concrete and progressing to abstract opportunities. The learning cycle provides the opportunity for students to share ideas with others and to more formerly connect what they have learned with what they already know.

Development of a community of learners

Extensive research corroborates the effectiveness of collaborative groups in K-5 classrooms and their use to build a learning community. After examining the large body of research on cooperative groups, one group of researchers concluded that "Markedly different theoretical perspectives (social interdependence, cognitive-developmental, and behavioral learning) provide a clear rationale as to why cooperative efforts are essential for maximizing learning and ensuring healthy cognitive and social development as well as many other instructional outcomes"[20]. Formal or informal collaborative groups are essential to the design process as students work in teams to solve problems and share their learning before being held accountable for their knowledge of the content and processes of mathematics on an individual level.

A model for verbal and written communication

Communication in the mathematics classroom permits learning to build on the students' informal knowledge, gives students practice in explaining their mathematical thinking to others, and provides students and teachers with evidence that learning has occurred[46,29]. Discussion, questioning, reflection, and writing are communication strategies that ensure that meaningful mathematical thinking has occurred. Mathematics classrooms that use an inquiry approach with more open-ended problem solving must provide a communication model that provides a structure for successful verbal and written experiences so that learning continually takes place as multiple solutions are shared and reflected upon.

Explicit connections that make mathematics meaningful

This curriculum is designed so that students will develop the ability to make meaningful mathematical connections. The ability to recognize relationships among mathematical ideas and to apply those ideas beyond the mathematics classroom has long been recognized as a hallmark of mathematical understanding[4,14]. In recent years, the ability to recognize such relationships is often referred to as "making mathematical connections." The benefits of mathematical connections in developing mathematical understanding is well documented in cognitive psychology, and is recognized as an essential part of learning mathematics by mathematics teachers and educators[41]. In the 2004 TIMSS Video Study, the making of connections among mathematical ideas was cited as the most significant feature distinguishing the higher-achieving countries' mathematics instruction from the other countries in the study[41]. The importance of making mathematical connections in developing mathematical fluency between STEM disciplines cannot be overstated.

Balanced assessment practices

Assessment is an ongoing, essential component of the inquiry-based learning cycle used in this curriculum. Assessments are built around concepts and skills based on mathematics, science inquiry, and technological design standards. The goals for assessment in the Math Out of the Box curriculum are

- to guide students in knowing what they have learned.
- to allow the teacher to understand how students are thinking about mathematics.
- to aid teachers in planning instruction.
- to inform the community.

Two types of assessment are used throughout the lessons. Formative assessments are embedded into the lessons, providing information to the teacher for instructional decisions and information to the students about their own learning. Numerous studies support the practice of formative assessment as a way to increase student success, particularly with low-achieving students[13,45]. Summative assessments provide additional information about student learning and can be evaluative in nature, providing information to a broader community. A variety of assessment strategies are included in each lesson to allow students multiple opportunities to demonstrate their knowledge and skills.

A variety of problem solving experiences

Data from reform curricula of the 1990s indicate that students using curricula, with an emphasis on problem solving, perform as well as students using traditional curricula on basic skills and better on conceptual understanding on standardized tests[38,39]. Research indicates that opportunities to explore new ideas balanced with opportunities to practice skills results in successful problem solving[14].

Researchers continue to find a relationship between the development of students as thinkers and student success in problem solving and conceptual understanding. Studies have examined the issues in classroom application when problem solving is considered as a process rather than another topic in a mathematics curriculum[11,22,23]. The following beliefs of the developers of this project are based on such research:

- A safe environment must be developed as part of the learning community so that mathematical discourse can take place.
- Changes in thinking can occur as errors and misconceptions are reconceptualized.
- Successful problem solving often requires multiple attempts and multiple strategies.
- Problem solving as a community leads to shared understanding of mathematical ideas, individual accountability, and connections to life outside of the mathematics classroom.

A diversity of materials, manipulatives, and models

Researchers advocate an environment of hands-on experiences in mathematics classrooms. In addition to manipulatives, materials needed for this rich environment include charts, graphs, writing models, diagrams, technology, and any tool that aids students in sense-making and problem solving[17,24,40,43].

Including the materials as part of the curriculum and in professional development, ensures that materials are used effectively by students and teachers to demonstrate and develop knowledge, to self-assess learning, and to connect mathematic ideas. Embedding the use of materials throughout the learning cycle of each lesson provides a powerful means of formative assessment for the teacher as students investigate mathematical ideas.

Professional development

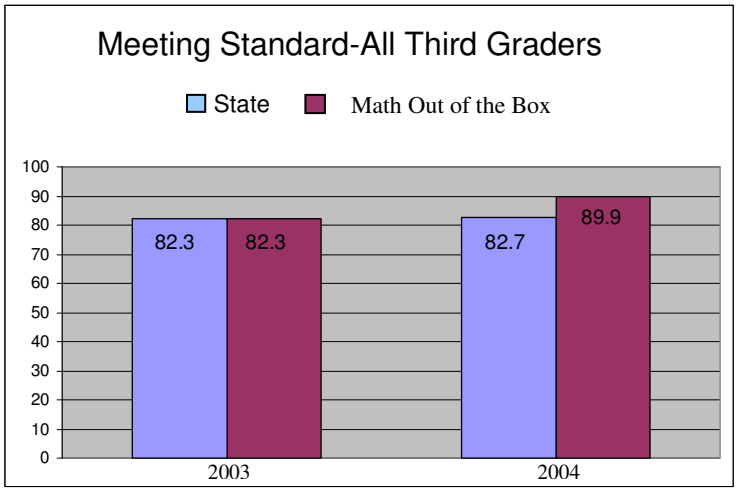
Innovative and meaningful professional development experiences provided in partnership with established organizational structures are needed for successful implementation of any curriculum[9,15,27]. In addition to the inclusion of design strategies in professional development for teachers, this project includes embedded strategies to support and change teachers' knowledge and beliefs about the teaching and learning of mathematics. Throughout the lessons, procedures and processes of effective teaching are modeled for teachers including effective questioning, writing strategies, discussions of open-ended problems, representation as a key to successful

problem solving, and reflective practices. Research shows that teachers' knowledge and belief systems can be affected by such experiences[6,11,12].

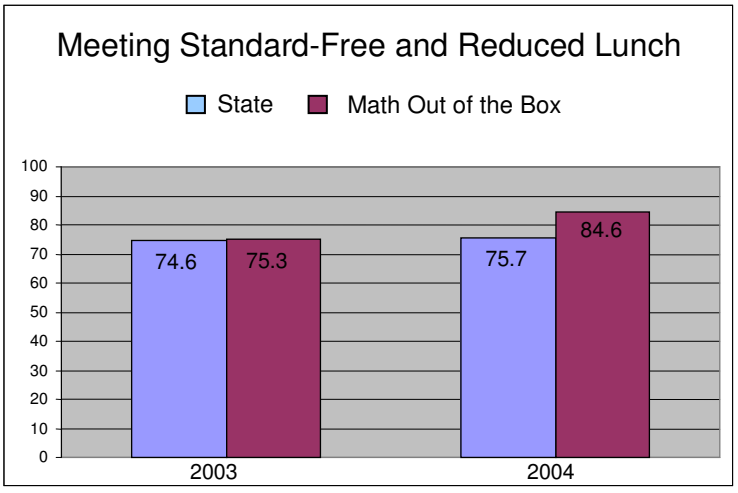
The following portion of this paper provides initial data and developing trends as this project is implemented in a variety of settings. Field tests and pilots have taken place in South Carolina, Pennsylvania, New Jersey, and Illinois in over 500 classrooms with over 11,000 students.

Program assessment: field tests

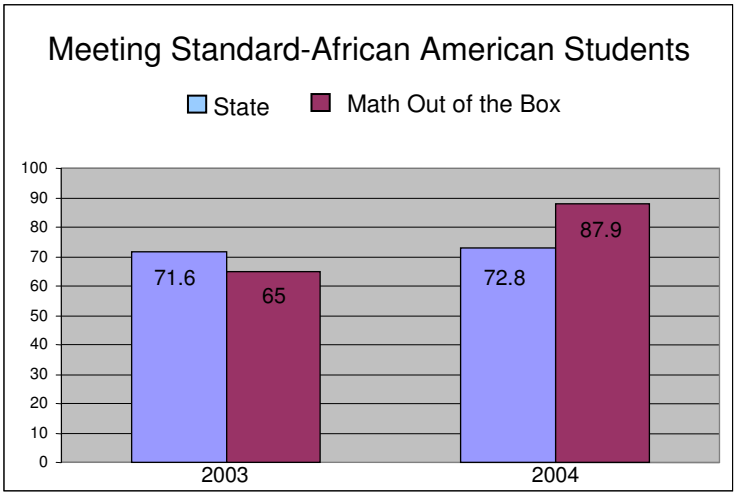
The following three bar graphs display average 2004 Palmetto Achievement Challenge Test (PACT) scores for 250 Math Out of the Box third graders from three field test schools. The PACT is South Carolina's statewide assessment. At the time of testing, these students had completed one of the four Math Out of the Box curriculum strands: Developing Algebraic Thinking.



89.9% of the 250 third graders met standard on the mathematics section of the third grade PACT in 2004 compared to the state rate of 82.7%.



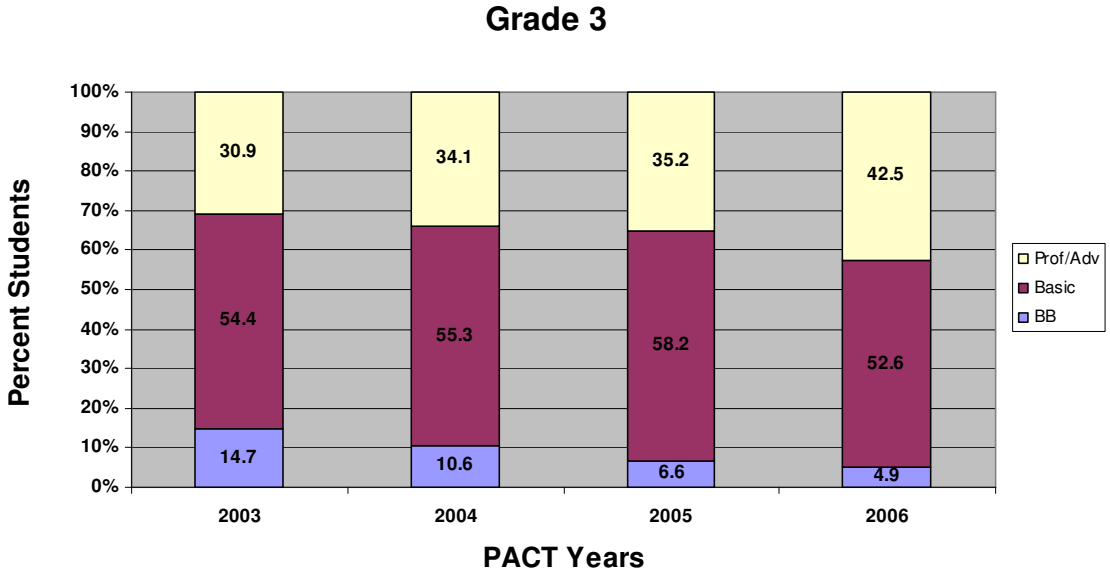
50% of the third graders were eligible for the free and reduced lunch program. Of those students, 84.6% met the standard on the mathematics section of the third grade PACT compared to the state rate of 75.7%.



40% of the third graders were African American students. Of those students, 87.9% met the standard on the mathematics portion of the third grade PACT compared to the state rate of 72.8%.

The following student test data collected over time at a field test site where each strand of MOOTB was implemented along with the companion professional development suggests that as a change in teacher content knowledge and pedagogy takes place, the goal of student proficiency in mathematics is more likely to be met. In 2004, the teachers at this school taught one strand of Math Out of the Box, followed by two strands in 2005, and three strands in 2006. This Title I school has a very stable population. Other programs implemented in the same time frame as Math Out of the Box included a school wide writing program. A mathematics coach was on-site for the first three years of the project.

The top part of each column shows the students who scored proficient and advanced, the middle part shows students who scored basic, and the bottom part, students who scored below basic on the Palmetto Achievement Challenge Test.



Reflections from over 200 teachers participating in field tests were collected electronically or in writing in specially designed reflection sessions. For many elementary teachers, this is a first

experience with reflection on classroom practices. Early reflections often show surprise at something the students know or do not know, although the teacher is sure they have been taught it in the past. The traditional teachers often use the reflections to express frustration about elements of inquiry-based instruction with which they have had little or no experience, such as the following:

- Not knowing all of the possible answers to a problem that was designed to be open-ended.
- Having difficulties with the management of hands-on materials.
- Not having a plan for the development of community through collaborative grouping.
- Not having the time needed to reflect during the scheduled math lesson.
- Not having time or understanding the purpose of writing in math class.

Following are several samples of reflections from the third grade field test for Developing Algebraic Thinking:

> Dear Teachers,
> We hope you have had great experiences with the first two lessons in the third grade manual. Please respond electronically to the following prompts:
>
> Please share something that you learned about your students that had to do with the math in the lessons. Also share something you learned about your students as a result of the kit lessons that wasn't mathematical.
>
> From K. Senger
> I learned that my kids had no idea how to use a tape measure. We have not studied measurement yet and several didn't even know which end to start with. The number 1 was not enough of a hint! My class is also very talkative and I thought the group work would be a challenge for them. So far it has gone okay and since the groups only have 2 in them they seem to work better than in larger groups. I have enjoyed the kit so far and I think the kids have too.
>
> From N. Wolfe
> I learned that my students knew how to measure straight objects using inches and centimeters, but that it was difficult for some of the students to figure out how to measure something round. The students enjoyed working with a partner very much this week. All students had fun in math. For the first time this year my lower level students could participate equally with the faster achieving students. I learned that each of my students would participate in a group if they felt comfortable with the assignment.
>
> From M. Jackson
> My kids have loved using the kit so far. I was very surprised that my kids already knew a lot about data. I even had one who gave me the same definition that is listed in the kit. They were also very aware of how important it is to test things in a fair manner. After planting the bulbs, they were full of even more ideas on why

it is so important to have fair tests. I had one child tell me that it was a good thing that everyone was here today to plant their bulbs, because if someone had been absent, it wouldn't have been fair. I asked her to explain why, and she did it correctly even using the word data in her explanation. In addition, I have learned that I have numerous kids in my class that want to be the "leader." I guess I have always known that, but during these first lessons, it has become more evident. That makes it really hard to work in groups. That is one thing I have been struggling with. I am still trying to match up partners and groups. We are getting there though. Overall, I am pleased with the kit. The kids are having fun and learning at the same time.

Program assessment: pilot projects

Educational Testing Service (ETS) is a private nonprofit organization devoted to educational measurement and research. The ETS Center for Foundational and Validity Research is conducting a three year evaluation of the implementation of Math Out of the Box in Lawrence Township Public Schools (LTPS) in New Jersey. The first year report[16] focuses on the implementation in 2005-2006. The executive summary follows:

- Most of the stakeholders interviewed regarded the pilot of Math Out of the Box as an opportunity to document how teachers changed in their knowledge of mathematics and use of inquiry-based teaching strategies.
    - The most salient dimension mentioned by stakeholders was the enthusiasm of the teachers for Math Out of the Box. This has encouraged more teachers to become involved in the pilot and seems to have had a positive impact on students and parents.
    - One concern expressed primarily by LTPS administrators was uncertainty about what the Math Out of the Box program would look like in final form.
- The professional development sessions met the standards for high quality inquiry-based pedagogical training. Teacher reaction to the training was very positive, although some mentioned a need for additional kit-specific training.
- Classroom observations found teachers to be successful in implementing the Math Out of the Box curriculum.
    - In most lessons, the mathematics was standards-based, appropriate, and challenging. Typically, students had opportunities to communicate their understanding through discussion and/or writing. The lessons allowed students to apply their understanding in activities that went beyond drill and practice.
    - Students were actively engaged in math learning throughout the lesson and were often given opportunity to work together in small groups in a collaborative way.
    - Teachers used a variety of questions – both higher order and factual recall, some of which led to open discussion which pulled the students into the analysis or brainstorming in a way that one-question-one-answer sequences cannot.
    - Since most small group work required students to solve problems together or construct lists of attributes, the conversations appeared to be moving them towards greater understanding or comfort with the mathematical concepts.

- Overall, the mini-assessments developed in the first year of the evaluation were successful in achieving their primary goal of creating items to be used in the pre-/post-assessments to be given to all 3rd through 5th grade students in 2006-2007.
    - A total of 132 multiple-choice items and 36 open-ended items were piloted with 245 students in the 2005-2006 academic year, covering the three Math Out of the Box strands (Algebra: Patterns, Algebra: Data Analysis, and Geometry).
    - These items were found to be measuring a similar construct as that measured by the 2006 NJ ASK mathematics subject test, thereby justifying their future use.
    - Teachers who participated in the scoring of the mini-assessments contributed to the design of the 2006-2007 assessments through feedback on specific items. Additionally, some identified a need for further work with their students on open-ended questions.

Math Out of the Box partners

The business and industry partners who provide funding for the project have an interest in a diverse workforce and recognize that elementary school mathematics is an important piece of the education pipeline. Corporations and foundations including Dupont Office of Education, Michelin North America, Fluor Daniel, American Honda Foundation, John Deere Foundation, Self Family Foundation, Ford Motor Foundation, and General Electric Fund provide financing for pilot programs and research projects. Carolina Biological Supply Company publishes the curriculum, provides financial support to the project, develops the hands-on materials in partnership with the authors, and provides in-kind support for field tests and pilots.

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