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Mathematical Modeling in Preexisting K-12 Engineering Design Challenges (Fundamental)

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Mathematical Modeling in Pre-existing K-12 Engineering Activities (Fundamental)

Introduction

The inclusion of engineering at the K-12 level has increased due in large part to the integrated science framework (i.e., A Framework for K-12 Science Education [1]) and integrated STEM, a pedagogical approach that focuses on making connections across STEM disciplines[2], [3]. Currently, though, there are a limited number of certified K-12 engineering and technology teachers [4] and science and mathematics are the dominant STEM subjects at the K-12 level [4]. As a result, engineering is often taught by science and mathematics teachers [4]. While in the future, engineering may be taught by certified engineering teachers, there are several reported benefits of integrating engineering into a mathematics or science classroom. These benefits include supporting students' understanding of science and/or mathematics content and making what can be seen as abstract content relevant and useful to K-12 students. It is important to be mindful that these purported benefits are not shared equally between science and mathematics at the K-12 level [5], as teachers have experienced challenges, specifically when designing lessons that make explicit connections between mathematics content to the other STEM disciples' content [6].

The framework has been developed to provide some guidance and explanation about how to include engineering content with their science content, (e.g., A Framework for K-12 Science Education [1] and Next Generation Science Standards: For States, By States[7]), the available resources to support the integration of engineering into mathematics have not been the same [8]. At present, there are no integrated K-12 mathematics and engineering educational standard documents. Explorations of existing resources, such as engineering problems and activities that support and promote mathematics objectives, could help to address the lack of integrated K-12 mathematics and engineering documents.

Engineering problems and other engineering activities allow students to use mathematics content while learning engineering content, yet the connections between the content are limited [6]. Advocates of the importance of mathematics for understanding K-12 engineering content lament that when students only use mathematics content in engineering experiences as this positions mathematics in a supporting role instead of as an equal role (e.g., [4], [5], [9], [10]). Additionally, when mathematics takes on a supporting role in a problem or activity, students predominately apply procedural mathematical knowledge (e.g., reproducing mathematical procedures) [11], [12] instead of attempting to understand why mathematics is connected to and needed for specific engineering content. Existing studies of K-12 engineering education efforts point out that mathematics connections in engineering problems have to be made explicit in activities to bring forth the mathematical and engineering concepts to be understood [6], [13].

While the use of procedural mathematics knowledge in engineering problems is an aspect of engineering, it only represents one dimension of mathematics needed for engineering work. Currently, K-12 STEM teachers are often presented with examples of engineering problems that depict procedural mathematical knowledge. For instance, in some professional development opportunities (e.g., [14], [15]) and undergraduate teacher education method courses [16], teachers are introduced to approaches for creating engineering problems with mathematics tailored to the use of procedural mathematical knowledge. While using procedural mathematical knowledge is unavoidable in engineering problems, more attention should be given to engineering mathematics. Engineering mathematics can be defined as “practical mathematical techniques and methods that engineering professionals apply within industry and research settings to better solve problems and complete engineering tasks in a predictive manner” [17, p. 36]. Engineering mathematics differs from the use of procedural mathematical knowledge in engineering problems in that engineering mathematics makes explicit connections between mathematics and engineering content. For example, the engineering problem requiring the redesign of a waste recycling system process to accommodate different methods of collecting and handling materials supports the use of different types of computational tools. The use of computational tools for the mathematics and engineering content in the engineering problem facilitates the development of a system that will meet the needs of the task.

The specific K-12 mathematics topic that aligns with engineering mathematics being advocated for is *mathematical modeling* (e.g.,[9], [17]). Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions, or otherwise provide insight into real-world phenomena [18] and genuine engineering problems. By definition, this form of engineering mathematics can be easily connected to engineering content in the areas of prototyping, evaluating the interactions of parts or systems, and redesign, and as such, can further students’ understanding of a broader scope of engineering work.

Mathematical modeling is an interdisciplinary mathematics topic that is critical and necessary in STEM and non-Stem fields. The use of mathematical modeling in engineering problems has the potential to facilitate mathematics teachers’ understanding of engineering concepts and is advantageous as mathematical modeling is a practice standard in the mathematics educational standards(i.e., Common Core State Standards for Mathematics [19]). In addition, mathematical modeling in engineering problems can help teachers counter the notion at the K-12 level that engineering is more or less a structured process of trial and error [4]. These types of integrated problems would also allow K-12 mathematics teachers to gain a better understanding of the interconnectedness between K-12 mathematics and engineering content. Furthermore, all K-12 STEM educational standards already have mathematical modeling standards; as such, increasing the use of mathematical modeling in engineering problems would likely help all STEM teachers with creating engineering experiences that focus on the importance of mathematics in their disciplines.

Literature Review

Mathematical modeling

There is no standard description or definition of mathematical modeling in STEM disciplines. Still, the general characteristics that scholars agree upon are that the modeling that occurs uses an iterative process to abstract, mathematize, represent, and analyze messy real-world problems, and the designed solutions or predictions are derived from the authentic, real-world situations using mathematics [20], [21]. Accordingly, mathematical modeling is a creative process that requires the mathematizing of the real world that enables one to transition back and forth between the genuine real-world issue and mathematics to understand and come up with viable solutions for the specific real-world problem under study [22], [23]. In simple terms, one can view mathematical modeling as the process of taking open-ended, complex situations from real life, where one must draw on and apply mathematical knowledge to solve the problem. Ferri

[23] insists mathematical modeling problems are different from other modeling problems because the problem context has to be an open-ended and/or real-world situation requiring the use of mathematical modeling practices. These mathematical modeling practices could include making assumptions, defining the problem, defining the variables, analyzing and assessing the model, and validating the model. [23], [24]. The described practices of mathematical modeling support deep learning of mathematical concepts necessary for the mathematical model and underlying concepts of the model, for example, science or engineering. For these reasons and others, mathematical modeling can be classified as an interdisciplinary skill that spans cross-STEM disciplines [25].

Engineering Problems

Engineers solve many kinds of problems related to improving and designing products, systems, and/ or processes [9], [26]. *Design problems*, for example, are ill-structured with underdefined constraints and unknown criteria to assess solutions [26]. In contrast, an *engineering optimization problem* focuses on using data collection and analysis to determine and/ or improve the performance of an existing process, product, and/ or system [9]. A *reverse engineering problem*, on the other hand, encompasses understanding existing processes and/ or systems to document, learn about or from, and/ or redesign it [9]. Although these engineering problems are described as individual problem types, within professional engineering practice, engineers are tasked with working on a combination of the different kinds of problems. As such, engineering problems designed for K-12 settings should reflect the diversity of problems, engineers face in order to develop a range of skills and a breadth of knowledge among these students.

Creating engineering problems for the K-12 setting requires a specific understanding of the previously mentioned types of engineering problems that would require the use of mathematical modeling. The instructor-made design scenario would be open-ended, have unclear constraints and criteria, contain several possible solutions, more than one criteria for evaluating the solutions, and no prescribed skills and/ or procedures would be included in the scenario [26], [27]. Furthermore, to allow K-12 students to comprehend the characteristics of the engineering problem, the context of the situation should be within the student's knowledge of understanding. For this reason, setting the context of the problem in a global or local issue can help students [28], [29].

Mathematical modeling in K-12 engineering problems

Given that mathematical modeling is a critical concept for understanding situations that occur in the real world and in engineering problems, engineering educators and education researchers alike seek to foster this mathematics understanding in K-12 engineering [9], [17], [30]. From this research and practice, findings have demonstrated how the use of mathematical modeling assists with using scientific methods and data to generate and test ideas that are not available using other methods [1], [31]. Consequently, Purzer et al. [9] examined 134 articles published in the National Science Teacher Association journal from 2005 to 2019, spanning elementary through high school, to understand the detailed connections between engineering problems and the integrated science and engineering educational standards [7]. The findings demonstrated how only the secondary (i.e., middle and high school) articles focused on engineering analysis and optimization. Engineering analysis and optimization problems involve using calculations, data analysis, and developing mathematical models to improve performance. Also, these problem types involve determining suboptimal performance, making comparisons, and predictions of designed systems [9]. For example, in one of the middle school engineering problems, the problem focused on developing an earthquake warning system for the general population. The mathematical modeling was facilitated using a web-based simulation platform [32]. One of the high school problems, as another example, involved using a web-based simulation to determine the environmental and economic tradeoffs centered on the issue of how best to manage the growing, fertilization, and selling of biofuels (i.e., corn, switchgrass, or cover crop) related to producing alternatives fossil fuel sources[33]. Overall, Purzer and colleagues' [9] analysis revealed that mathematics-focused engineering problems are not prominent. In general, mathematical modeling-focused problems are underutilized in the development of science-based engineering problems. Increased attention to these types of engineering problems can help to support students' understanding of science, math, and engineering simultaneously.

Even though mathematical modeling is included in K-12 mathematics educational standards (i.e., Common Core State Standards for Mathematics [19]), teachers struggle with integrating mathematics content into engineering problems[14], [15]. As an example, the middle school teachers in Lesseig et al.'s [15] multi-year university and school district partnership used a professional development model to create Science, Engineering, and Mathematics Design Challenges. The mathematics and science teachers perceived the challenges aligned with mathematics problem-solving skills and specific engineering practice. The major problem the mathematics teachers experienced creating the design challenges was that the sequencing for the development of mathematics concepts was being compromised. In other words, the teachers were able to include the prerequisite concepts necessary for the mathematics concept. As such, aligning grade level standards to the design challenge was nearly impossible. Moreover, the teachers conceded it was difficult to design interdisciplinary challenges; in particular, while they could make connections between the engineering and science content, the mathematics connections were perceived as superficial [15].

Conversely, researchers Corum and Garofalo [34] and Mousoulides and English [35] provide evidence that engineering tasks can be tailored to the K-12 mathematical modeling practice standard and include additional grade-level mathematics topics. For example, in one of their tasks, an elementary engineering activity focused on advising the Ministry of Communication and Works, students were asked to advise the Ministry (i.e., their client) whether to build natural gas and oil refinery stations using a data set from worldwide natural gas reserves and annual average consumptions [35]. The results demonstrated that these students developed an understanding of mathematical and engineering assumptions as they used grade-appropriate mathematics (i.e., averages, creating equations, and data gathering/analysis) to develop a linear function (e.g., model). This model was then able to make predictions as to whether the gas refineries and stations would be a good investment based on the rate of consumption and reserves of natural gas [35]. As another example, Corum and Garofalo [34] focused on using mathematical modeling to support middle school students' understanding of engineering technology applications. The engineering technologies included speakers, motors, and generators. Corum and Garofalo [34] designed the engineering task using the model-eliciting activity framework [34], which is a pedagogical approach for teaching mathematics-based concepts that requires one to represent real-world situations mathematically. The engineering-based mathematics task allowed the students to draw on their knowledge of pre-algebra, algebra, and magnetism to develop a multivariable mathematical model that explained why the electrical devices work. Engineering problems with mathematical modeling and engineering content can create suitable interdisciplinary learning experiences.

Teachers' use of instructional resources

State departments of education or local school districts use an established curricular materials process to determine the various STEM and non-STEM instructional materials teachers can use in their classrooms. The materials on the approved lists are based on their alignment to the particular state's adopted educational standards. At this point, not all states have integrated science and engineering educational standards [36], [37], and even for those who have these standards, the state and national standards often establish what teachers should teach but don't provide guidance or support on how to teach it. Given these complexities and, in many cases, the lack of guidance, teachers are using other methods to find K-12 engineering materials. Teachers, for example, rely on their disciplinary teachers' association's (i.e., National Science Teachers Associations or National Council of Teachers of Mathematics) recommendations to meet their needs and/or teachers use of online resources [38]. According to Opfer et al. [39], "94 percent of elementary and 97 percent of secondary teachers report using Google to plan instruction" (p.1) to support their comprehensive standards-aligned curriculum materials in their subject area and grade level. As such, one can assume that K-12 STEM teachers who are integrating engineering into their courses are leveraging online engineering curricula. These curricular materials can influence and improve teachers' "disciplinary content knowledge, pedagogical knowledge, and pedagogical content knowledge" [40, p. 59]. Therefore, online engineering curricula with mathematical modeling have the potential to support K-12 STEM teachers with understanding mathematical modeling used in engineering.

Conceptual Foundations

Model Eliciting Activities (MEA) are widely used to support the development of students' modeling and problem-solving competencies. The model eliciting activities framework was developed through the work of Lesh and colleagues [41], [42]. This framework specifically focuses on using mathematical processes to develop a model. The model is a mathematical description of genuine situations that are rooted within specific systems of practice, and the modeling is the representation of the process that requires numerous testing and revision cycles [42]. The MEA framework has been used in engineering classrooms to design engineering tasks at the K-12 level [43] and university level [44], [45] as the framework supports various engineering practices and problem-solving practices [46].

For this study, we make the claim that engineering activities with the K-12 mathematical modeling standard should have some alignment to the MEA mathematical modeling perspective. Mousoulides and English [35] described these types of engineering problems as Engineering Model Eliciting Activities (EMEA) that are based on genuine real-world problems, are mathematics content-focused, and are student-centered. Additionally, the problems are open-ended, client-driven, and use a test and revise cycle to come up with viable mathematical modeling solutions. The six principles used to create EMEA are *Model Construction, Reality, Self-Assessment, Model Documentation, Model Shareability and Reusability,* and *Effective Prototype* (see Table 1) [41]. In the design of EMEA, the *Reality* principle is the context for the engineering problem where the mathematical model (*Model Construction* principle) is designed for the needs of a specific client or group of people (*Shareability and Reusability* principle) [47]. Additionally, student teams use the data in the problem's context and gather additional data to evaluate their constructed model ideas (*Self-Assessment* principle) [47]. The designed solutions must be tailored and presented to the client (*Model Documentation* principle) [47]. Furthermore, the resulting mathematical model must be transferable to similar engineering problems or situations (*Effective Prototype* principle) [47]. For the study described in this paper, this framework enabled our team to examine online engineering curricula created with the K-12 mathematics mathematical modeling practice standard to determine the extent to which they align with engineering-based mathematical modeling problems.

Table 1. Six principles of Engineering Eliciting Activities and Model Eliciting Activities (from[41], [48])

	Diefes-Dux et al.'s [48]	Lesh et al.'s[41]
Reality	Requires the activity to be posed in a realistic engineering context so that students can interpret the activity meaningfully at different levels of mathematical ability and general knowledge.	The mathematical model is used to solve a realistic problem.
Model Construction	The activity requires the construction of an explicit description, explanation, or procedure for a mathematically significant situation.	A mathematical model is created to address the needs and purpose of a given client.
Self-Assessment	The activity contains criteria the students can identify and use to test and revise their current ways of thinking about the problem.	Students evaluate their progress as they work on the problem.
Model Documentation	Students are required to create some form of documentation that explicitly reveals how they are thinking about the problem situation.	The documentation shows explicitly how the problem solver is thinking about the problem and the model.
Model Shareability and Reusability	Students are required to produce solutions that are shareable with others and modifiable for other engineering situations.	The model is transferable, shareable, easily modified and/ or reusable in the situation.
Effective Prototype	The model produced will be as simple as possible yet still mathematically appropriate for engineering purposes.	The created model can be used as a prototype for other similar situations in the future.

The literature on mathematical modeling in engineering activities at the K-12 level strengthens the argument that mathematics needs to be made explicit in engineering tasks to support the understanding of engineering concepts. Yet, there are a limited number of engineering problem examples to support STEM teachers in their specific disciplines. Additionally, the literature points out that properly designed engineering problems can develop mathematical concepts and mathematical practices by using mathematical methods. The mathematical practices and methods allow students to learn and use engineering concepts and skills.

Research Question

We sought to answer the following research question: *To what extent do K-12 engineering activities with the K-12 mathematical modeling standard reflect the characteristics of engineering-based mathematical modeling problems?*

Research Methodology

This qualitative document analysis study used direct content analysis to determine the alignment of pre-existing online K-12 engineering design challenges with mathematical modeling to engineering-based mathematical modeling activities. Document analysis is an analytical method used in qualitative research to systematically review or evaluate either print or online material [49]. The method requires the text examined is organized into themes, categories, and/ or examples to understand and gain insights into the phenomena under study [49]. The use of document analysis is appropriate to better understand the potential of online engineering curricula to support the teaching and learning of engineering mathematics, specifically, mathematical modeling, which is a mathematics technique and method used to solve societal engineering problems and complete engineering tasks in a predictive manner [17].

Data Collection

We used purposeful sampling [50] to identify specific entities with online engineering curricula that may have mathematical modeling. The initial list (see Table 2) of possible online engineering curricula came from the American Society for Engineering Education Pre-College Engineering Education division, the Institute of Electrical and Electronics Engineers Pre-University Education list, and the International Technology and Engineering Educators Association. The resources were reviewed, and those that did not list the K-12 mathematics mathematical modeling practice standard, mathematical models, or mathematical modeling were eliminated. This process resulted in the use of the TeachEngineering Digital Library as our sample. The other resources reviewed did not meet the criteria.

Table 2. List of online engineering curricula

Source	Resources
American Society for Engineering Education Pre-College Engineering Education division	Engineering Go For It (eGFI) Link Engineering The TeachEngineering Digital Library
The International Technology and Engineering Educators Association	TechChallenge Carnegie STEM Girls Discovery Education Design Squad Design and Technology Association
Institute of Electrical and Electronics Engineers Pre-University Education	TryEngineering.org

To determine the most appropriate grade level for our analysis of the engineering problems with mathematical modeling on the TeachEngineering Digital Library, we used the work of Opfer et al.[39] addressing teachers' implementation and use of K-12 standards for mathematics and English language arts. According to Opfer et al.[39], secondary teachers' understanding of what it means to model with mathematics aligns more with mathematical modeling, which is to create solutions for real-world complex mathematical situations that occur in the real world. Therefore, we opted to analyze activities at the beginning of the secondary level. Using the search by standards feature on TeachEngineering, we selected the grade 6 band as the grade level is the beginning of the secondary level. The grade 6 level on TeachEngineering contained (n=9) activities.

To evaluate the presence of the six principles of the framework in the nine activities, a rubric (Table 3) was designed. The first author and second author (an expert in engineering education and mathematical modeling) created the evaluation rubric using the six principles in Table 1. The second author's expertise in engineering education and mathematical modeling helped ensure the rubric's face and content validity. Additional reliability and validity testing was not feasible during this study and should be the focus of future work. The first and second authors scored four activities together from a different grade band: the elementary grade band. The authors discussed the similarities and differences in coding. The differences in coding led to the refinement of the rubric.

The rubric (Table 3) was used to explore the activities' content sections and student worksheets using direct content analysis. The content sections included (1) activity summary, (2) introduction/motivation, (3) educational standards, and (4) engineering connection. Each of the six principles was scored on a scale of three points. The scale was 1= Does Not Meet Principle, 2= Partially Meets Principle, and 3= Meets Principle. The rubric sections were summed, and the total possible score is 21. The list of activities reviewed and their scores are presented in Table 4. The activities (see Table 4) that met the score threshold of greater than 14 indicated the activities contained satisfactory evidence of meeting the principles. The activities that met the score criteria of greater than 14 were further analyzed using direct content to understand how mathematical modeling was incorporated with the engineering content in the activities.

Table 3. Evaluation rubric

Principle	3-Meets principle	2-Partially meets principle	1-Does not meet the principle
Reality	There is an engineering problem posed in a realistic engineering context.	There is somewhat of an engineering problem posed in a realistic engineering context.	There is no engineering problem posed in a realistic engineering context.
Grade level appropriateness	The context allows students to interpret the activity at their grade level using grade-level mathematical knowledge.	The context somewhat allows students to interpret the activity at their grade level using grade-level mathematical knowledge.	The context does not allow the students to interpret the activity at their grade level using grade-level mathematical knowledge.

Model Construction	There is an explicit description, explanation, or procedure for a mathematically significant situation that is derived from the engineering context and the created mathematical model meets the needs and purpose of a given client.	There is an explicit description, explanation, or procedure for a mathematically significant situation that is derived from the engineering context. or The created mathematical model meets the needs and purpose of a given client.	No explicit description, explanation, or procedure for a mathematically significant situation that is derived from the engineering context. No mathematical model is created to meet a given client's needs and purpose.
Self-Assessment	The engineering context contains criteria for students to assess the successfulness of the mathematical model through the use of a test/revise cycle. and Students evaluate their progress as they work on the problem.	The engineering context contains criteria for students to assess the successfulness of the mathematical model through the use of a test/revise cycle. or Students evaluate their progress as they work on the problem.	There are no criteria for students to assess the successfulness of the mathematical model through the use of a test/revise cycle in the engineering context. Students do not evaluate their progress as they work on the problem.
Model Documentation	Students are required to create some form of documentation that reveals how they are thinking about the problem. and Mathematical model	Students are required to create some form of documentation that reveals how they are thinking about the problem. or Mathematical model	Students are not required to create some form of documentation that reveals how they are thinking about the problem and the mathematical model
Model Shareability and Reusability	Requires students to produce mathematical solutions that are shared with the client and modifiable for other engineering situations	Requires students to produce mathematical solutions that are shared with the client and not modifiable for other engineering situations	The produced solutions are not shared with the client or modifiable for other engineering situations
Effective Prototype	The mathematical model can be used as a prototype for similar engineering contexts.	The mathematical model might be able to be used as a prototype for a similar engineering context.	The mathematical model cannot be used as a prototype for similar engineering contexts.

The first author read the content sections and students' worksheets for each of the grade 6 activities multiple times, analyzed the content and worksheets using the rubric, and assigned a score. If an activity's content sections or student worksheets were questionable for a principle, the second author was consulted. The concerns about an activity's score were resolved through discussion. One activity was excluded from the analysis as the activity was no longer recommended for use on the TeachEngineering website. The first author analyzed across the data for each principle to make sense of the connections between mathematical modeling and the engineering content in the activities at the completion of assigning scores to each of the activities.

Results

This study aimed to understand the alignment of existing engineering activities with mathematical modeling to engineering-based mathematical modeling problems. The section below presents the overall evaluation score for each activity (see Table 4) against the principles in the framework. Following the presentation of the evaluation score is the description of the principles that were present across the activities.

Table 4. Grade 6 Engineering Activities Evaluation Score

Activity title	Equal to or less than a score of 14	Greater than a score of 14
Composting Competition	10	
Digest Your Food!	9	
Designing and Packaging a Distance-Sensing Product	12	
Making Moon Craters	7	
Mmm Cupcakes: What's Their Life Cycle Impact?	14	
Sliding Textbooks	7	
Strum Along with Shoebox Stringed Instruments: Sound or Music?	7	
The Great Algae Race	10	
Ultrasound Imaging	No score	

No score: The curriculum is no longer supported because the materials are no longer available or are outdated.

Reality

The two activities that met the reality principle were the Designing and Packaging of a Distance-Sensing Product and the Mmm Cupcakes: What's Their Life Cycle Impact? The scenario for the Designing and Packaging a Distance-Sensing Product was an open-ended design problem that required the creation of a product that used a microcontroller sensor to assist others in their local community or to provide a service to a specific user group and determine how to address the three listed constraints. The required engineering decisions included determining the client groups for the intended product and how best to address the constraints. The Designing and Packaging a Distance-Sensing Product problem did require grade-level mathematical knowledge. The Mmm Cupcakes: What's Their Life Cycle Impact activity contained an engineering problem that required the use of the life-cycle assessment tool to determine the environmental impact of a common snack item. Again, the activity's engineering context required six-grade-appropriate mathematical knowledge. These engineering problem scenarios can be interpreted as engineering problems.

The activities that did not meet this principle alluded to engineering and did not provide a real engineering problem. Instead, these activities did not meet the principle had an implied engineering context. For instance, in the Great Algae Race activity, there was a context of a regional power plant wanting engineers to use their carbon dioxide waste gas for the biofuel farm. The challenge required the students to create an experiment to compare algae growth with and without adding carbon dioxide gas, which helps biofuel production. The work in the activity was hands-on science tasks.

Model Construction

Three activities had some description, explanation, or procedure for a mathematically significant situation that could be derived from the engineering context. The activities that partially met this principle included: (1) The Great Algae Race, (2) Designing and Packaging a Distance-Sensing Product, and (3) Mmm Cupcakes: What's Their Life Cycle Impact? Additionally, the activities did not require a mathematical model that met the needs and purpose of a given client. In the review of the activities, when there was a model in the activity, it was a physical object where mathematics was used to determine a relationship between variables. The Strum Along with Shoebox Stringed Instruments: Sound of Music activity, for example, no mathematical model was required to determine the frequency of a physical waveform and pitch by making an instrument.

Self-assessment

None of the activities' engineering contexts contained criteria to assess the successfulness of a mathematical model that would require the use of a test/revise cycle. There were no requirements in the activities that enabled students to evaluate their progress as they worked on the problem.

Model Documentation

One of the eight activities required students to create some form of documentation to reveal how they are thinking about the problem but not the mathematical model. The Mmm Cupcakes: What's Their Life Cycle Impact activity partially met this principle. The activity had a requirement that documents the form of the mathematical model. The designed mathematical model in the activity were graphs that helped to show the difference between the various disposal methods in the problem. The activity did not have a requirement of how the mathematical model could be applied to the problem.

Model Shareability and Reusability

None of the activities required the production of mathematical solutions that are shared with a client. Also, there were no requirements in the activities to create mathematical solutions that could be modified for other engineering situations.

Effective Prototype

A requirement in the Mmm Cupcakes: What's Their Life Cycle Impact activity was that the mathematical model could be used with different sets of data for the problem. The purpose of the mathematical model was to design an optimal life-cycle plan with the least amount of environmental impact. There was no requirement in the activity that the mathematical model be used in a similar engineering context. For this reason, the activity partially met the principle.

Discussion and implications

This qualitative study illustrates that these grade 6 online engineering curricula with the K-12 mathematical modeling practice standard do not reflect the characteristics of engineering-based mathematical modeling problems. Five of the grade 6 engineering activities did not have an engineering problem. Two of the engineering activities made hints to engineering. The hints to engineering included engineers using models to study systems or using composting technologies to design waste management systems. Given that engineering is more than a context for teaching subject matter, the unique concepts and skills of the engineering discipline should be depicted accurately, as described in the reality principle [17]. An implication for K-12 STEM teachers is that to replicate open-ended engineering problem scenarios specific to their discipline. To assist in the replication, the work of Grubbs and Strimel [28] and Strimel [29] provides guidance for meeting the reality principle. The reality principle allows one to interpret the engineering problem meaningfully and facilitates the use of engineering concepts and skills to meet the needs of users or clients [51].

Concerning the principles of Model Construction, Model Documentation, Model Shareability and Reusability, there may be confusion about how to set up an engineering problem that requires mathematical modeling, as the evidence for these principles was rarely present in the activities. Five activities did not require the use of the model construction principle, and three activities contained evidence of partially meeting the principle. Seven activities did not require the use of the model documentation principle, and one activity contained evidence of partially meeting the principle. None of the activities contained evidence of meeting the model shareability and reusability principle. The minimal evidence of the modeling principles that appeared in the activities is a troubling issue since K-12 STEM teachers are searching out online engineering activities that align with their specific content area standards. The use of these grade 6 activities could cause misconceptions about how mathematical modeling is used and needed in engineering problems. Based on our analysis, the activities reviewed do not accurately reflect engineering-based mathematical modeling problems. Consequently, K-12 STEM teachers should be supported with learning how to evaluate online engineering activities with mathematical modeling and how to revise misaligned sections of activities to reflect the modeling represented in the framework. Since the modeling principles in the framework is critical to connecting engineering and mathematics concepts and skills, assisting K-12 STEM teachers with evaluating and revising engineering activities could be an approach further enabling the integration of mathematical modeling with engineering.

Since the framework closely aligns with open-ended engineering problems, it can be used to create the kinds of engineering problems that focus on mathematical models. As reported by Huffman and Mentzer [45], the structure of the framework enables course activities to further students' understanding of the use of mathematical concepts necessary for testing and optimization in engineering problems. Furthermore, Huffman and Mentzer [43] assert that the practice cycle of testing, evaluating, and revising contained in the framework can aid students in attending to the testing and evaluation procedures essential in engineering problems. The use of the framework to create engineering problems or revise existing online engineering problems has the potential to leverage the natural connections between engineering and mathematics analysis and modeling.

Given that the framework requires communication via the Model Shareability and Reusability principle, this particular concept is similar to the Framework for P-12 Engineering Learning[17] description of a reason for models in engineering. A purpose of models in engineering, in addition to determining "how well a design will perform" p.72 is "to communicate design ideas to others" p.72. Hence, the use of the framework could contribute to students' understanding of how to design mathematical models that are comprehensible and useable by others. The use of the framework could support students in understanding how to create mathematical models appropriate for a current engineering problem but also transferable to a similar problem, aligning with processes that occur in professional engineering practice. We recommend that K-12 STEM teachers and those K-12 STEM education scholars consider using the framework to align engineering activities and problems with engineering content focused on mathematical models.

Limitations

There are several important limitations to this study. Only part of the secondary (e.g., middle school) engineering activities were evaluated using the framework. Evaluating the activities in grades 7th and 8th would offer a more comprehensive view of the mathematical modeling in the middle school pre-existing engineering problems. Although the coding of the activities involved consultation from an expert in engineering education and mathematical modeling, incorporating interrater reliability would help to strengthen the data analysis process.

Conclusions

This qualitative study sought to gather evidence of the extent to which K-12 engineering activities with the K-12 mathematical modeling practice standard reflect the characteristics of engineering-based mathematical modeling problems. The results suggested that most of the reviewed grade 6 engineering activities with mathematical modeling do not reflect engineering-based mathematical modeling problems. The results and interpretations in this study adds to the body of research investigating how to integrate engineering and mathematics content in K-12 engineering activities at the pre-college level. K-12 STEM teachers are seeking out online engineering resources that align with their educational standards and that will work with their curricula; therefore, it is vital that teachers can efficiently evaluate the activities. The framework is a valuable tool to meet this purpose.

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