Mathematics for Scientists and Engineers II
at the Colorado School of Mines

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Colorado School of Mines (CSM) is in the process of a major curriculum revision as a response to the changing world for today's science, mathematics, and engineering graduates. An integral part of that revision is the development of new mathematics courses to better prepare students for their studies in science and engineering. Mathematics for Science and Engineering II is a four hour course which addresses the concerns of faculty from other disciplines who comment that students finish their calculus and differential equations courses unprepared or under prepared to do the mathematics in their courses. These same faculty claim that students are unable to translate the calculus they know into new concepts where problems appear to be different. This input led to the idea of developing a set of problems based on the concepts from these various disciplines. By having students work through these problems, students begin to understand the necessity of their mathematics studies, the connections of the mathematics to other subjects, the richness of the other disciplines, and the underlying mathematics beneath the disguises imposed by the variations in terminology in the disciplines.

This course's development was in response to input from faculty on the CSM campus but it draws from similar projects on other campuses. The National Science Foundation has funded several of these projects such as the ones at Dartmouth, Rensselaer Polytechnic University, and the University of Pennsylvania.[i] In developing this set of problems, these projects were consulted, but the mission of the Colorado School of Mines, with its emphasis on energy and the environment, has driven the characteristics of this set.   Further reasons for developing the course are based on recent calculus reform efforts where an emphasis is put on problem solving with less emphasis of the strict lecture technique which students often find dull and unwelcoming.[ii]

Course Content   The course replaces the traditional second semester of calculus. Instead of the applications of integration and integration techniques which usually inhabit this course, the course starts with vectors, then moves into parametric representations of functions, then lines, curves, and planes in space with an emphasis on the linear algebra requisites, then into multivariate calculus, both differential and integral. The rationale is that these students are concurrently enrolled in physics and they need to understand vectors and multivariate concepts.   Topics such as techniques of integration will be pulled in as needed.

Course Design The course is designed to have two classes per week in which the students learn new concepts. Some of these classes are lecture and others are discovery. In the third class period, the students spend two hours working "Real Problems" from the

various disciplines. In the problem sessions, the students work in teams to solve problems and then turn in a team solution at the end of the two hour period. The problems were collected through personal visits with professors or through email.. Students are not expected to "learn the science," but only to be able to work through the mathematics. It is not the intent of these problems to teach material in other subject areas, but to use that material to help students make use of their mathematical knowledge. Often, engineering and science texts introduce a theory and jump to the formula without developing the mathematics. In fact, rigorous mathematics is avoided. Then, those engineering and science faculty who teach these courses claim that the students cannot "do math." The approach of this course is to let the students see the formulas in their disguises and strip away the mystique of the terminology to get at the heart of the mathematics.

Performance objectives, syllabus, and problems were all posted on the web page for this course. Use of email for asking questions was encouraged. Each student was asked to send an email message to the professor at the beginning of the semester. These were used to form a mailing list for each class. The initial messages also served to introduce the students to the professor.

Problem Development To get a more concise understanding for what needed to be done in constructing these problems, a faculty survey was developed and sent out to faculty members in each of the science and engineering options. The first question asked for a specific list of mathematical topics or skills which pose difficulty for students. The response included topics such as complex variables, matrix multiplication, definite integrals, differentiation, simple first-order differential equations, algebra, trigonometry, interpretation and construction of graphical relationships, application of symmetry information to simplify integrals in two or three dimensions, vector differences, differentials, and the chain rule. The second questions asked if each respondee felt that students have forgotten the math, never learned the math or are confused about the change in vocabulary or symbols. Responses indicated that a change in vocabulary or symbols creates the most problems, followed by students not learning the math in the first place. The third questions asked how much time is spent in reteaching math. Most responded that little reteaching is done, but students are expected to review on their own. Each faculty member was asked to provide sample problems. They submitted problems by email and in person to the graduate student compiling the problems. The response was excellent and those problems form the basis for the problems used in the course and also in a thesis for a master's degree.

Evaluation This course was taught as a pilot course in Fall, 1996. The students were entering freshmen who scored a 4 or 5 on the Advanced Placement AB calculus exam. There were 33 students in one section and 48 in the other. The same students continued into the third semester course in the Spring. The students had a variety of backgrounds, but they did have their AP test scores in common. The students were asked to complete a written evaluation of the course at the end of the semester. When asked what element of the course helped them learn calculus the most, a majority of the students listed the

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problem solving sessions as being very helpful in learning the calculus concepts. Only one said that they were no help at all. The biggest complaint about the problems was that they had to sit and think. When asked to list connections of calculus to chemistry, engineering, physics, and economics, nearly all students were able to list at least two and many listed three or four. The same students are being tracked through the first semester of physics in the Spring 1997 semester to determine what effect the problem solving had. Early reports indicate the physics faculty feel they are able to deliver their course with a greater mathematical content. All of the physics students were given a preliminary test of basic mathematics skills with the problems posed as they would be in a physics setting. The students from the pilot mathematics class did significantly better on this examination than the students from the regular calculus classes. Several older CSM students asked to visit the class as they had heard how excited the students were about the content and methods. They commented that they felt that the students were seeing problems which they would be exposed to later and this was the best way to learn mathematics.

This model will be tested on students who did not have advanced placement credit in the Fall, 1997. In fact, four classes of students who did not pass the first semester of calculus in the Fall, 1996, will be enrolled in this course.

Evaluation of the problems, as opposed to the course itself, needs to cover three areas. First, do all of the problems work well, i.e., are the students able to follow the directions and come up with a reasonable solution? Second, do the problems enhance the learning of the calculus concepts by extending student knowledge and fostering student interest? Third, do the problems help the students learn to translate their knowledge of calculus as they encounter it in other courses. It will take several years to answer these questions.

In changing curriculum in higher education, it is often difficult to get the support of alumnae. A letter received from a CSM '49 graduate, in response to an article in the Mines magazine, reads as follows: "My years at Mines were in the forties and fifties and throughout calculus and differential equations we frequently asked our professors for applications of what was being taught to our options. Unfortunately, especially in differential equations, the teacher knew little about engineering and even less about the practical side of math. As you realize, math is a tool to be used to solve the problems of industry. Today's students must be shown how to apply the tools to the actual problems. …Please try to convince your colleagues in the math department that this is the right track." Hopefully, all alumnae feel the same way.

One of the unexpected benefits of the problem development is that the faculty involvement in providing problems has improved the interaction between the various departments and the Mathematical and Computer Science Department and has improved the image of MCS.

Example problems follow. Others can be found on the web page for MAGN 142 listed on the CSM home page under On-line courses.

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Example Problems

Search and Rescue

This problem uses properties of three dimensional vector for finding a downed aircraft. Using consistent vector notation is important to the solution of the problem. The calculus and physics topic is vectors.

A ranger stationed in a remote section of the Rocky Mountains spots a small Cessna aircraft with smoke pouring from the engine. In order to give the search and rescue team as accurate information as possible, he immediately contacts a second ranger who can also see the plane. At the same time they both take bearings on the plane in order to calculuate its exact position. The first ranger is located at (0,0,6800) and sights the plane at 42° south of east and 11.5^o above the horizon. The second ranger is located at (17321, 10000, 7600) and sight the plane at 58° south of west and 3.4^o above the horizon. Three minutes later, they both take a second bearing on the plane. The first ranger sights the plane at 41° east of north and 2.3^o above the horizon. The second ranger sights the plane at 83° west of north and 1.6^o below the horizon. The coordinates are in feet.

1. Draw a reasonable accurate sketch of the problem. Although you have three dimensions in the problem, it may be easier to work with a two dimensional sketch with the elevations listed next to the points.
2. Calculate the coordinates of the plane at the two sightings.
3. Find both the displacement vector and the velocity vector (in feet per second) for the plane between the two points.
4. If the plane is heading toward a meadow averaging 6300 feet in elevation and assuming the plane maintains its current direction and velocity, calculate the coordinates where it will crash and find the amount of time elapsed between the second sighting and the crash.
5. A rescue helicopter leaves an airfield at the same time the second sighting is taken by the rangers. It flies on a velocity vector of <170,248,7>. The coordinates of the airfield are (150000, -200000,4000). When will it reach the crash sight?

Students really seemed to enjoy this problem. Some had programmed triangle solution programs in their calculators which helped them. Others had to work very hard to draw the diagram and complete the problem.

Ball Mill (Mining Engineering)

A ball mill is used to bread large chunks of ore into smaller pieces. It is a large barrel into which is put a quantity of ore and a quantity of steel spheres. As the mill turns about its axis, the ore and the spheres tumble around together. The spheres break the ore chunks by impact. The mill must be rotated fast enough that the material tumbles well, but not so fast that centripetal force keeps the material plastered up against the wall. The engineers who design ball mills need to know, among other things, how the mass moment of inertia of the steel spheres changes as they wear down (their radii decrease). The mass moment of inertia is calculated by :

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$$I = \int_m r^2 \, dm = \int_V r^2 \, \rho dV = \rho \int_V r^2 \, dV$$

where I = mass moment of inertia, r = the perpendicular distance from the axis of rotation to the element dm, m = mass, V = volume of the object and ρ = density of the object (assumed constant in this case).

Plot how the mass moment of inertia of a steel sphere changes as its radius decreases with wear. The initial radius is 3.00 inches, the final radius is 0.75 inches and the steel density is 7.88 g per square centimeter. Consider 0.25 increments.

Acceleration (from Dynamics course)

The acceleration of a particle is $a = k \sin(\pi t/T)$. Knowing that both the velocity and the position coordinates of the particle are zero when $t = 0$ and that T is a constant, determine

1. the equations of motion, i.e. v in terms of t and x in terms of t
2. the maximum velocity
3. the position at time $t = 2T$.

The interesting thing about this problem was the difficulty which students had in integrating the sine function with the added constants. The other factor was the interpretation of how you would find the maximum velocity.

Conclusion: The introduction of problem sessions to the calculus courses has had a number of effects. Among them are perceptions by the students that they can tackle any problem by carefully analyzing the underlying mathematics and stripping away the terminology difficulties. Another effect has been the improvement of the image of the mathematics department by the requests for input from other disciplines. One of the drawbacks of this approach is the constant need for more problems and the lack of a suitable textbook. The "Real Problems" are intended as a supplement to the regular calculus curriculum. Although the excitement of the students is contagious, faculty input is substantial.

The pilot course was taught with minimal use of computers. The students used graphing calculators extensively. In the past, Mathematica was used in our calculus sequence, but the departmental laboratory was in disrepair in Fall, 1996. The addition of a new computer laboratory in the department has added to the follow-on course and will probably be integrated into Mathematics for Scientists and Engineers II as the three dimensional graphing capabilities will enhance students' visualization.

The curriculum reform effort at CSM includes assessment and evaluation throughout. This should lead to continual improvement of courses, thus this course will be constantly updated and

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improved.   Some of the problems and their solutions for MSEII will be published as part of Douglas Poole's thesis.   Others will be available upon request.

[ii] Block, I.E., NSF Initiative Promotes Sea Change in Math Education, Sept., 1996 Siam News

[ii] Shaping the Future, New Expectations for Undergraduate Education in Science, Mathematics, Engineering, and Technology, National Science Foundation

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DOUGLAS POOLE is currently working on his master's degree at the Colorado School of Mines.   His master's project included gathering and writing the problems for the new mathematics courses.   He holds a BS in Geological Engineering from CSM.   He taught high school mathematics and science in overseas schools for six years prior to coming to CSM and will be teaching mathematics next year at Leysin School in Switzerland.