2024 ASEE Midwest Section Conference

Measurement of Out-of-Plane Thermal Conductivity Using Steady-State Heat Conduction

Clancy Milam [a], Chinmaya Joshi [a], Stephen Pierson [a], Han Hu [a], Ying Sun [b]

[a]Department of Mechanical Engineering, University of Arkansas, Fayetteville, AR 72701

[b]Department of Mechanical and Materials Engineering, University of Cincinnati, Cincinnati, OH 45221

Abstract:

Most engineering students only have a basic understanding of thermal properties, such as insulators having low thermal conductivity, but never look any further into them. This lack of understanding results in them not knowing how to measure these properties, or what they physically represent. Knowing what data one will need to calculate thermal conductivity, and how this data is analyzed can greatly improve one's understanding of thermal conductivity, thermal systems, and data analysis. The analysis that will be discussed in this paper is straightforward and could be used to set up experiments for students or engineers in industry. The experimental setup to gather data includes a cold and hot end made from a bar of aluminum 6061 T6, thermocouples placed throughout the cold and hot ends, a sample placed between both bars, and Polyether Ether Ketone (PEEK) insulating the entire setup. The only data that will need to be gathered for this analysis are temperature throughout the cold and hot sides, the spacing of the thermocouples, the sample's thickness, and the sample's cross-sectional area. This data allows one to find the temperature distribution and heat flux through the cold and hot ends, which allows one to calculate the thermal resistance and out-of-plane thermal conductivity of the sample.

Keywords:

Undergraduate Student Poster, Thermal Conductivity, Measurement, Data Analysis

Introduction:

When dealing with thermal systems, there are many thermal properties one must understand well and have accurate data of. The thermal conductivity of a material is one of the most important and sought-after thermal properties in engineering because of how it affects cooling heat-sensitive components [1]. Electronics are one example where research is done on thermal conductivity, because of trying to cool heat-sensitive components [2], [3]. Because of this, there are many methods to measure thermal conductivity, such as the laser flash, or guarded bar method [4], [5]. For the analysis below, the only data needed are various temperatures throughout the cold and hot ends of the experiment, the spacing of thermal couples, the sample thickness, and the sample cross-sectional area. The steady-state temperatures will be used for this analysis. To achieve this the thermocouple data is plotted against time and only stopped after temperatures stay consistent for a significant amount of time. With these temperatures, linear regression will be used to find the temperature distribution across the cold and hot ends. The heat flux through the bars, the sample's thermal resistance, and the sample's thermal conductivity will be calculated using the temperature distribution. By going through this full analysis, one will greatly improve their understanding of thermal systems, and analyzing data.

Data Analysis Process:

Throughout this analysis, a setup like the one shown in Figure 1 will be referenced. This setup has a cold and hot side, with a sample placed in between, allowing heat transfer to occur. For this setup to match the model that is being used to analyze the data, there will need to be insulation to prevent significant convective heat transfer, making the linear model invalid.

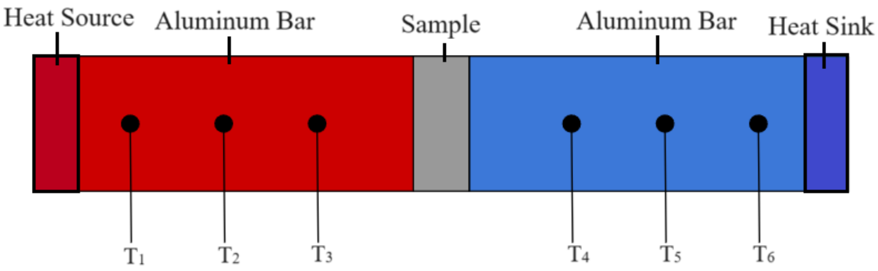


Figure 1. Simplified example of the experimental setup used to gather necessary data for analysis. All temperatures are gathered with thermocouples and plotted over time.

To see why a linear equation can be used, the heat equation shown in Equation 1 will need to be analyzed.

$$\mathrm{k}\nabla^2 T + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \tag{1}$$

First, it is assumed the setup has reached a steady state, which leads to the right-hand side being set to zero (Assumption 1). The heat input is also constant, meaning there is no heat generation (Assumption 2). This means the heat generation term can be set to zero. The temperature is also assumed to change only in the y-direction (Assumption 3). This is because the x-direction and z-direction are heavily insulated, preventing large changes in temperature in those directions. This will lead to Equation 2, which when solved will lead to Equation 3. Equation 3 shows that temperature is a linear function, allowing us to assume temperature varies linearly.

$$\frac{d^2 T}{dy^2} = 0 \tag{2}$$

$$T(y) = C_1 y + C_2 \tag{3}$$

If all assumptions are valid for the setup, then start by plotting temperature throughout the aluminum bars against time. Figure 2 shows such a plot, with three temperatures being taken for both the cold and hot sides. Initially, the temperatures rapidly change with time, and eventually, level out. Once the temperatures remain the same out for a significant amount of time, it can be assumed that the setup has reached a steady state. There are multiple ways one could then get the steady-state temperatures, such as selecting six temperatures within the steady-state period or averaging the temperatures for each thermal couple within the steady-state period. Whichever method is chosen, only temperatures during the steady state period should be considered.

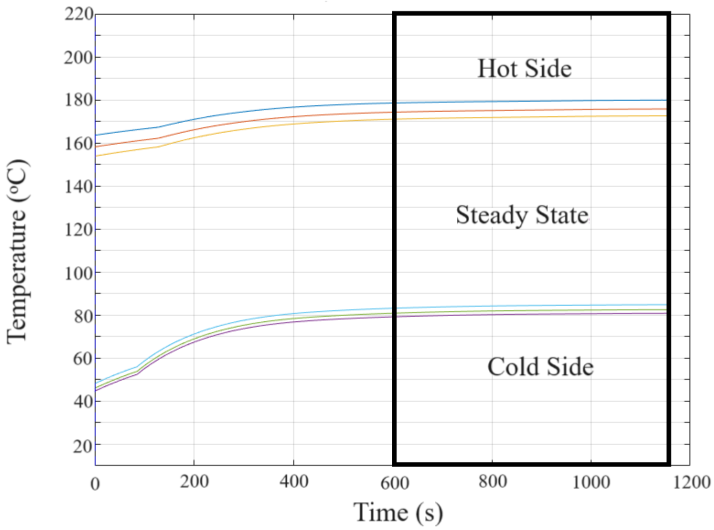


Figure 2. Temperature history through cold and hot sides of thermal resistance tester during representative testing. Only temperatures within the steady state area should be considered for analysis.

The temperature distribution throughout the aluminum bars can be found using linear regression using the steady-state temperatures. The temperature profiles across the cold and hot sides will then be plotted in the form $T(y) = my + b$ where y is the distance from the cold plate. The surface temperature of the sample on the cold and hot sides can then be interpolated by using the equations for the temperature profiles. With the two surface temperatures, a linear temperature profile across the sample can be obtained. Figure 3 shows the temperature distribution throughout the aluminum bars and the sample.

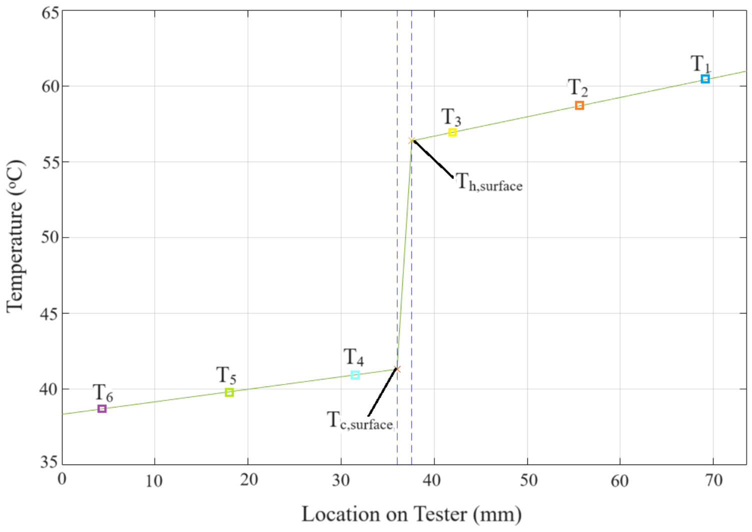


Figure 3. Steady-state temperatures and temperature distribution through cold and hot sides of thermal resistance tester during a representative test.

With the temperature distribution and well-known materials for the cold and hot sides, it is possible to calculate the heat flux through the aluminum bars. Fourier's law of heat conduction can be seen below in Equation 4. By using the temperature distribution of the cold and hot ends, as well as their thermal conductivity, the heat flux through both ends can be calculated.

$$q" = -k\frac{dT}{dy} \tag{4}$$

The heat flux through the cold and hot sides should be the same, but since there will be some losses in the system, they may be off by a small amount. The heat flux through the sample can then be found by averaging the cold and hot side heat flux. With the heat flux through the sample, both the thermal resistance and out-of-plane thermal conductivity can be calculated. The thermal resistance can be found by the change in temperature through the sample divided by the heat flux through the sample. The out-of-plane thermal conductivity can be found by dividing the average heat flux by the temperature gradient across the sample. The temperature gradient will represent the slope of the line found using linear regression with the two surface temperatures. To see what these properties are with samples varying in thickness, the thermal resistance term will need to be expanded. This can be seen in Equation 5 below, where R_{th} is thermal resistance, R_c is contact resistance, y_s is the thickness of the sample, and k_s is the out-of-plane thermal conductivity of the sample.

$$R_{th} = 2R_c + \frac{y_s}{k_s} \tag{5}$$

With Equation 5 the thermal resistance of this system can be represented by a simple linear plot of thermal resistance against sample thickness. A linear trendline of the plot can be found by plotting the thermal resistance of multiple samples with varying thicknesses. By taking the inverse of the slope of the trendline, one will calculate the effective thermal conductivity. To further improve the qualitative data from this plot, one can repeat the same set of samples multiple times, and using the mean and standard deviation, add error bars to the plot as shown in Figure 4.

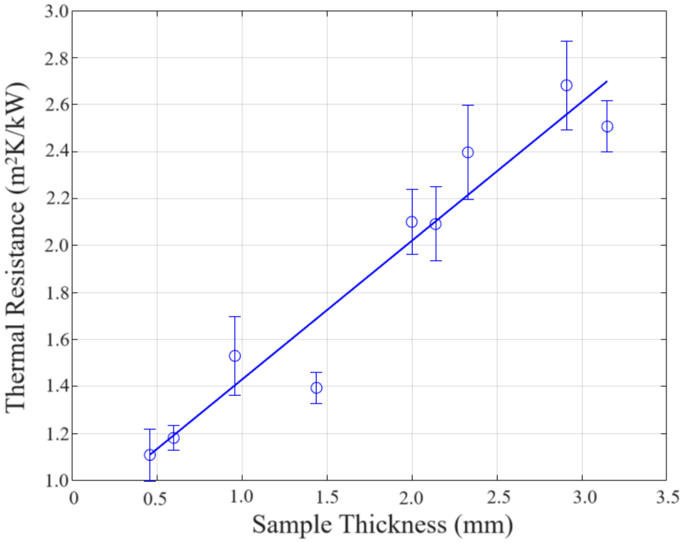
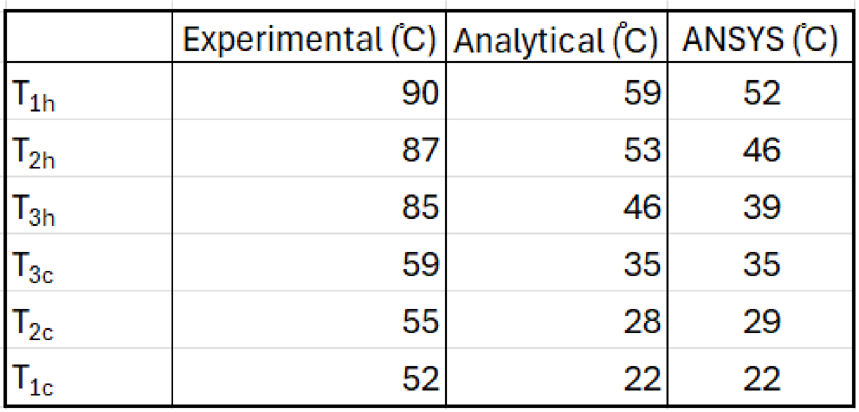


Figure 4. Plot of multiple thermal resistance tests with error bars. The inverse of the trendline slope is equal to the out-of-plane thermal conductivity.

In Figure 4 the results of two thermal properties from the experimental data, and any major problems with their data, can be seen. The error bars will help show how repeatable the test is, if certain samples are more inconsistent than others, and how off the experimental trendline could be from the actual trendline. If it is noticed that samples above a certain thickness had a large increase in error compared to the thinner samples, this would be a great indication that the setup has some issues with larger samples.

Results:

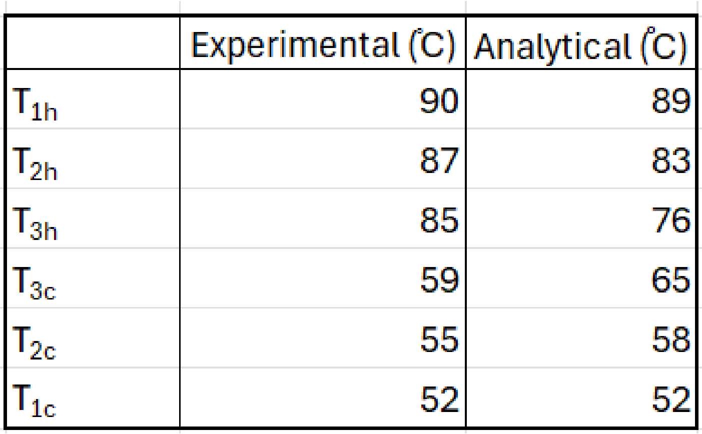
To see the results, experimental data will be compared to the results from analytical and simulation data. For the experiment, a 20W heat load with a grade 2 titanium sample was chosen. The data collected can be seen in Table 1 below.



	Experimental (°C)	Analytical (°C)	ANSYS (°C)
T_{1h}	90	59	52
T_{2h}	87	53	46
T_{3h}	85	46	39
T_{3c}	59	35	35
T_{2c}	55	28	29
T_{1c}	52	22	22

Table 1. Comparison of temperature at thermocouple locations.

Table 1 shows the large difference between the experimental, analytical, and simulation data. With the analytical and simulation data, it was assumed that the bottom of the cold end bar was reaching the temperature of the cold plate. However, the experimental data shows this is not true. This means there must be significant contact resistance between these two. If the boundary condition is closer to what the experimental data shows, the analytical data more closely resembles the experimental data. This can be seen in Table 2.



	Experimental (°C)	Analytical (°C)
T_{1h}	90	89
T_{2h}	87	83
T_{3h}	85	76
T_{3c}	59	65
T_{2c}	55	58
T_{1c}	52	52

Table 2. Comparison of temperatures at thermocouple locations with the boundary condition for analytical data the same as the experimental data.

With the boundary condition changed, the two end temperatures are closer to what the experimental data shows, however, the temperature distribution is very different. The analytical data shows that the temperature should drop more rapidly through the aluminum bars. This inconsistency could be caused by a high resistance between the aluminum and the sample. The resistance between the aluminum and the cold plate might also contribute to this. By reducing these losses, better results could be obtained. Thermal interface materials (TIMs) would be one option to reduce these resistances, and there is research on making TIMs cheaper, which would make them a great option for student research [2]. However, to know what is exactly causing the data to be inconsistent, calibration and testing of the setup will have to be done.

Acknowledgements:

This work was supported by NASA under Grant No. 80NSSC23K023. C.M. acknowledges support from the Arkansas Space Grant Consortium Workforce Development program.

References:

[1] "Thermal Properties of Materials." Infinitia Industrial Consulting, July 28, 2022. https://www.infinitiaresearch.com/en/sin-categoria-en/thermal-properties-of-materials/.

[2] Dunn, Alexandra, Josh Henderson, Zachary Kubas, and Chandan Roy. "Capstone Engineering Project to Design an Apparatus for Testing the Thermal Impedance and Apparent Thermal Conductivity of Different Thermal Interface Materials." ASEE, 2021. https://sites.asee.org/se/wp-content/uploads/sites/56/2021/04/2021ASEESE84.pdf.

[3] Guo, Xiaoxiao, Shujian Cheng, Weiwei Cai, Yufeng Zhang, and Xue-ao Zhang. "A Review of Carbon-Based Thermal Interface Materials: Mechanism, Thermal Measurements and Thermal Properties." Elsevier Ltd., July 6, 2021. https://www.sciencedirect.com/science/article/pii/S0264127521004901.

[4] "Thermal Conductivity Measurement." CALNESIS, September 6, 2023. https://www.calnesis.com/en/measurements/thermal-conductivity/#:~:text=The%20flowmeter%20method%20involves%20determining,thermal%20conductivity%20using%20Fourier's%20law.

[5] Clausen, Edgar, Roy Penney, Dave Marrs, Megan Park, Anthony Scalia, and Nathaniel Weston. "Laboratory/Demonstration Experiments in Heat Transfer: Thermal Conductivity and Absorptivity Measurement." ASEE, 2005. http://myweb.astate.edu/sharan/PMC/Labs/ASEE%202005%20Thermal%20Conductivity%20Absorbtivity%20Meas.pdf.

Clancy Milam

Clancy Milam is an ungraduated at the University of Arkansas majoring in mechanical engineering with a concentration in aerospace. Clancy's interests include thermal and fluid system analysis, as well as the design and application of propulsion systems.

Chinmaya Joshi

Chinmaya Joshi is an undergraduate honors student from Uttarakhand, India. He is double majoring in Mechanical Engineering with Aerospace Concentration and Applied Mathematics with a minor in Computer Science at the University of Arkansas. He has worked on unmanned aerial vehicles in the past and is interested in material science, for aircraft. He intends to use his degree and research to create a new and efficient aerospace propulsion system.

Stephne Pierson

Stephen Pierson is a Ph.D. student in the Department of Mechanical Engineering at the University of Arkansas. Stephen obtained his BSME (*summa cum laude*) from the University of Arkansas in 2023. He is a 2023 Goldwater Scholar, a 2023 Arkansas SURF fellow, a 2024 Razorback Classics, and a 2024 Distiguinshed Doctoral Fellow. His current research interests lie in the two-phase cooling, acoustic sensing, and advanced manufacturing.

Han Hu

Han Hu is an Assistant Professor in the Department of Mechanical Engineering at the University of Arkansas. He received his B.S. from the University of Science and Technology of China in 2011 and Ph.D. from Drexel University in 2016. His research is focused on electronics cooling, thermal diagnostics, advanced manufacturing, signal processing, and engineering education.

Ying Sun

Ying Sun is currently Herman Schneider Professor and Head of the Department of Mechanical and Materials Engineering at the University of Cincinnati. Prior to joining UC, she was Hess Family Endowed Chair Professor in Mechanical Engineering at Drexel University. In 2019-2022, Dr. Sun served as Program Director of the Thermal Transport Processes Program at the National Science Foundation. Dr. Sun is an ASME Fellow and a recipient of the NSF CAREER Award, AFOSR Summer Faculty Fellowship, French CNRS Visiting Professorship, and Drexel College of Engineering Research Achievement Award.