AC 2011-216: MEASUREMENT UNCERTAINTY IN UNDERGRADUATE PHYSICS STUDENT MISCONCEPTIONS AND POINTS OF DIFFICULTY

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Measurement Uncertainty in Undergraduate Physics – Student Misconceptions and Points of Difficulty

Abstract

One key concept in physics is that a measurement always has an associated uncertainty. This paper examines several observed student misconceptions about this concept, discusses the difficulties encountered in overcoming these misconceptions, and suggests some possible alternative solutions. Prior work in this area by Saalih Allie and Andy Buffler of the University of Capetown and Fred Lubbin of the University of York has shown that students often enter college with the notion that scientific measurements are exact and that "measurement error" is due to a fault on the part of the experimenter. Students also often believe that uncertainty is a concept that arises in physics only in the context of quantum mechanics and have misunderstandings of the Heisenberg uncertainty principle that can be difficult to overcome.

These problems are often exacerbated by misconceptions regarding statistics. Even when students in introductory physics classes are able to perform basic statistical calculations, they frequently have weak conceptual understanding of probability and statistics. In particular, they struggle to apply statistics to the interpretation of experimental results.

In this paper, we survey solutions to these problems that have been proposed by authors in the past and suggest a possible approach that combines these solutions with ideas on teaching statistics and best practices from physics education research.

Introduction

Uncertainty is a concept that appears in physics courses in many forms. These range from the basic recognition that measurements include a reported uncertainty along with a value, to the analysis of fundamental noise sources (such as shot noise and fluctuation-dissipation noise) and the derivation of the probability distributions associated with various noise sources, to the sources of uncertainty associated with quantum mechanics. Unfortunately, these many aspects of uncertainty sometimes blur together in our student's minds and a first step along the path of reducing our student's confusion is to first clearly differentiate these ideas in our own minds and curricula.

These are clearly not all topics for our introductory courses. While there is general agreement that measurement uncertainty is an important topic in introductory courses[1] (both in courses for engineers and scientists and also in survey courses), there is not agreement at present on what topics should be covered or to what depth[2]. Many of the topics mentioned above typically appear in later courses and because they are not always explicitly linked back to the introductory concepts our students frequently form incorrect links between them.

International Standards

Although measurement is essential to physics, most physicists do not specialize in the field of metrology. As a result, many are unaware of the existence of two important international standards in this field. The International Bureau of Weights and Measures (BIPM),the same organization responsible for maintaining and promulgating the SI units of measure, collaborated with the International Electrotechnical Commision (IEC), International Federation of Clinical Chemistry (IFCC), International Organization for Standardization (ISO) International Union of Pure and Applied Chemistry (IUPAC), International Union of Pure and Applied Physics (IUPAP) and the International Organization for Legal Metrology (IOML) to produce two standards relating to measurement terminology and uncertainty. These two documents are the *International Vocabulary of Basic and General Terms in Metrology* (abbreviated VIM and originally published in 1984) and *Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement* (abbreviated GUM and originally published in 1993). In 1997, the BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML formed the Joint Committee for Guides in Metrology (JCGM) to update and maintain these documents as well as to create further documents aiding the further standardization of metrology[3].

The VIM provides standardized definitions for terms such as error, precision, accuracy, repeatability and reproducibility. These definitions reflect the shift that has occurred over the last 40 years from a focus on error as a difference from an unknown true value of the measurand to a focus on uncertainty as a measure of the likely range of values of the measurand[4]. For example, the VIM defines *Measurement error* as "measured quantity value minus a reference quantity value"[5] where the phrase "reference value" is used because in reality the true value of the measurand is rarely if ever known. *Measurement uncertainty* is defined as a "non-negative parameter characterizing the dispersion of the quantity values associated with the measurand, based on the information used" leaving a wide range of possible choices for the parameter, such as a standard deviation, a confidence interval half-width, or the total range of possible values. The VIM then goes on to define *standard measurement uncertainty* as "measurement uncertainty expressed as a standard deviation".

The GUM provides guidance on how to narrow these options to produce useful uncertainty statements. The GUM's approach is to report either the standard measurement uncertainty or an expanded uncertainty which is simply the standard deviation multiplied by a coverage factor to produce a confidence interval. Uncertainties due to unknown systematic errors are given as the standard deviation of the Bayesian probability distribution describing the experimenter's knowledge of the systematic error. For example, if an unknown systematic error is bounded by ±a and no other information is available, the maximum entropy principle suggests a uniform probability distribution with a standard deviation $\sigma = a/\sqrt{3}$. Uncertainties are propagated in the same manner as in traditional error analysis[6]. For a measurement result *y* based on a set of n direct measurements x_i through a measurement equation $y=f(x_1..x_n)$, the expanded uncertainty in y is

$$U_y = k_p \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}{}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} \sigma_{x_i x_j}}$$

Where k_p is the coverage factor and the second term under the radical accounts for covariances, $\sigma_{x_i x_j}$, between the x_i and is frequently ignored. This is very similar to the approaches presented in popular undergraduate texts in error analysis[7,8] but the terminology used is different. Some of these differences can help us reduce confusion in our introductory courses.

Error vs. Uncertainty

The VIM makes a clear differentiation between measurement error and measurement uncertainty and the GUM provides advice on the reporting of uncertainty rather than error. Yet in our introductory courses we often discuss error propagation rather than uncertainty analysis. This difference in terminology can reinforce a common confusion that our students bring with them to their college courses. When asked to report sources of experimental error, students often want to list "human error" as a predominant source. It appears that they have been conditioned in their high school classes to believe that ideally their experimental results should match theoretical predications and that error is a measure of the deviation between the expected result and the actual experimental result. They commonly believe that any deviations are an indication of insufficient care or mistakes in measurement or calculation.

Several researchers in this area have examined the effect of using the GUM approach to uncertainty in improving student's understanding of uncertainty in introductory classes[9,10,11,12]. All have reported that GUM compliant courses have resulted in improvements in students' conceptual understanding of measurements producing an interval estimate for the result rather than a point estimate. Unfortunately, many programs don't have room in their introductory courses for a GUM compliant unit on uncertainty and some authors have written that such a level of detail in an introductory course can mask key concepts and that the use of statistics is inappropriate for systematic errors[13] (this argument is echoed in the metrology community by researchers who have proposed alternatives to the GUM framework).

A middle ground would be to use the VIM's terminology in our introductory classes without incorporating the entire GUM framework. We can certainly justify a preference for uncertainty rather than error since we don't know the true value of the measurand. This focus on uncertainty rather than error helps focus on the interval nature of measurement results. Although it does not appear in the VIM, the term "blunder" commonly used in the metrology community to denote procedural mistakes on the part of the experimenter that are not valid sources of uncertainty, but which must be avoided. The use of this term uniformly as a replacement for "human error" should encourage students to consider more carefully the sources of real uncertainty in their work.

Uncertainty in Introductory Courses

Regarding uncertainty, the primary goal in an introductory course is simply to ensure that students appreciate that measurements are interval estimates and that we always need to provide a measure of uncertainty, not the difference between the measured and textbook result[14]. Research performed by Duane L. Deardorff found that students often focused on the mechanics of calculating uncertainties rather than the importance of uncertainty itself as part of the measurement process[15]. A similar problem often occurs with significant figures. When students do pay attention to significant figures they often fixate on a set of rules for determining the number of significant figures required in an answer rather than the fact that we limit the number of figures presented in order to avoid misleading others about the uncertainty of our results.

Given these concerns, it seems most reasonable in an introductory course to focus on the idea of measurement uncertainty, clearly differentiating between valid sources of uncertainty and blunders. Students should be able to estimate the range of possible systematic errors, and unless repeated measurements are made, this is really all they need. Instead of incorporating the full GUM framework in an introductory course, we can use range rather than standard deviation as our measure of uncertainty and combine these via the equation:

$$\Delta y = \left|\frac{\partial y}{\partial x_1}\right| \cdot \left|\Delta x_1\right| + \left|\frac{\partial y}{\partial x_2}\right| \cdot \left|\Delta x_2\right| + \cdots \left|\frac{\partial y}{\partial x_n}\right| \cdot \left|\Delta x_n\right|$$

which becomes the familiar addition of uncertainties for addition and subtraction and addition of relative uncertainties for multiplication and division. This simple propagation rule can be combined with the rule of expressing uncertainty to 1 significant figure (or 2 if the first digit is 1) and the rule that the result should not include digits beyond the least significant digit of the uncertainty. While not compliant with the GUM framework, this elementary uncertainty framework has the advantage of focusing on the importance of thinking about and reporting uncertainty, makes the use of significant figures a consequence of uncertainty rather than a set of rules to be followed slavishly, and avoids the need for statistical concepts which are frequently misunderstood by students at this level[16].

It should certainly be pointed out to students that this approach provides a very conservative number because it assumes worst case addition of inaccuracies and that more sophisticated techniques will be introduced later. If students are familiar with basic statistical techniques we can differentiate between random and systematic errors and show that random errors can be reduced by averaging the results of repeated measurement. In this case, for random errors, the range can be replaced with $\pm 2s/\sqrt{n}$, where s is the experimental standard deviation and n is the number of samples averaged. This gives a 95% coverage interval for normally distributed data and, by Chebyshev's inequality at least 75% coverage for any possible distribution[17].

By using simple rules for uncertainty propagation and significant figures, we can focus student attention on the more important issue of quantifying the uncertainty in a result. By using the terminology of uncertainty rather than error, we can encourage a differentiation between the uncertainty inherent in a measurement and blunders resulting from carelessness.

Noise sources and the Unavoidability of Measurement Uncertainty

In subsequent courses taken by engineering students, the full statistical treatment of uncertainty should be introduced and used. In these courses, students encounter a number of physical phenomena that result in unavoidable sources of measurement uncertainty. For example, the discrete nature of electric charge results in shot noise that places a lower limit on our ability to measure electric current. Similarly, the fluctuation-dissipation theorem[18] results in unavoidable noise sources ranging from Johnston-Nyquist noise in electrical circuits[19] to photorefractive and photoelastic noise in interferometers[20]. It is important that we make this connection between physical phenomena and their effect on measurement uncertainty. Without this connection, students tend to see classical measurement as a potentially ideal process with uncertainty introduced only by imperfect equipment and assume that this uncertainty could eventually be reduced to zero.

A related issue is that measuring instruments do not simply passively observe the measurand. For information to be exchanged between the system measured and the measuring system there must be interactions between the two systems and thereby the system under test is modified. Engineering students often misunderstand this as a quantum mechanical effect while it is easily observed in classical systems. This misconception has been observed among engineering juniors and seniors who had completed all of their required physics courses and their required engineering courses dealing with instrumentation issues. It appears that this concept that the interaction between measuring equipment and the system is present in all measurements is a fundamental concept that we should ensure students have grasped by graduation and is probably best addressed in physics courses.

Uncertainty in Quantum Mechanics

Related to the engineering student's misconception that the interaction between measuring system and system under measurement is a quantum phenomenon is that this interaction is the explanation of Heisenberg's uncertainty principle in quantum mechanics. While these students have typically not taken a physics course in quantum mechanics, this misconception is quite sad. The actual description of quantum mechanical uncertainty is a consequence of momentum and position being related through a Fourier Transform. This is something that engineering students can easily understand since they have studied Fourier transforms in their circuit analysis courses and should understand that localization of a function in one domain requires delocalization in the Fourier domain. This connection of ideas in different contexts helps students build a more robust concept of Fourier Transforms and we have observed that dual majors in physics and electrical engineering generally have a firmer grasp of Fourier Transforms than either pure physics or pure electrical engineering students.

Daniel Styer included this as one of the 15 common misconceptions students hold about quantum mechanics and suggests that the use of the term indeterminacy in the place of uncertainty in quantum mechanics both more accurately describes the phenomenon and avoids the confusion of uncertainty and indeterminacy[21].

Conclusions

We have looked at several student misconceptions about uncertainty. Some of these can be addressed very simply in introductory courses, while others are more appropriately addressed later in the curriculum. Very small changes in nomenclature, such as referring to measurement uncertainty as uncertainty rather than error and quantum indeterminacy as indeterminacy rather than uncertainty can go a long way to eliminating these misconceptions. Focusing student attention on key concepts rather than computational details can also help.

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