Session 1465

Measuring true strain - an application of the logarithm

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Capitalizing on their increasing control of the material world, design engineers will soon incorporate high-strain elastomers and biology-mimicking materials into critically stressed structural components [1]. 'High-strain' is described by a simple and essential mathematical concept to which engineering students are uniformly exposed. Nevertheless, in this writer's view, the theory is seldom internalized.

Engineering educators who seek to solidify the connection between the mathematics and their students' eventual workplace toolbox may, therefore, wish to re-visit a nice little hands-on exercise [cf. 2, 3] which illustrates the concept of one-dimensional strain and its elegant mathematical analog, the logarithm.

Following a sufficient number of lectures on the topic of deformations and stress, engineering students memorize the definition of *engineering strain*, the change in length divided by the initial length, $\varepsilon = \Delta l/l_i$. In later courses, students use this *engineering strain* formulation in virtually all analysis problems that call for a strain calculation. Understandably focused on the problem-solving aspects of the current topic, rather than on the mathematical foundation, most students remain unmindful of the fact that *engineering strain* is a pure approximation, a useful calculation for only small values of deformation. This circumstance presents a classroom opportunity to deepen the understanding of mechanical deformations by applying the logarithm in a hands-on exercise.

Mechanical engineering students respond readily to hands-on experiments; this is especially true when the attendant instrumentation requires no explaining. Such is the case when two points, defining the extremes of a (albeit long gage length) strain gage, are laid out lengthwise on a standard rubber band. Using the simplest of tools, namely a piece of cardboard backing, two thumb tacks, and a finely graduated scale, the results of the *true strain* formulation may be directly compared with those of *engineering stain*. For large strains, greater than say, 10%, these two values disagree significantly, which leads to a quick, physically based explanation of the logarithm.

Beginning with a counter example, students are asked to lay out several small ink dots along the length of a slightly stretched rubber band and then log the distances between the dot pairs. As is soon discovered, the dots define a series of strain gages laid end to end. Stretching the band

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slightly more, say 25% or so, the students are again asked to carefully measure and log the new distances separating the dot pairs and then use these data to calculate the value of engineering strain along the rubber band's length.

Repeating once this procedure for a single gage (a single dot pair) yields, for example, the following table.

initial length l_i (mm)	final length l_f (mm)	change in length Δl (mm)	engineering strain $\varepsilon = \Delta l/l_i$
40	50	10	25%
50	60	10	20%

As the rubber band is stretched, it is seen that the distance separating any given dot pair ranges from 40 to 60 mm. When this operation is done in *two* steps, as shown above, the total engineering strain clearly sums to 45%.

Students are then asked to repeat the experiment by stretching the rubber band in a *single* step during which time the dot separation again ranges from 40 to 60 mm. But, for a single stretch, the engineering strain is clearly 50%, and the contradiction is compelling.

Reinforcing the point, students may be directed to repeat the exercise a few more times while using more, and smaller, steps. In any event, converging results suggest a differential formulation which is abridged in the next few lines.

True strain, ε_t, is defined by the infinite sum

$$\varepsilon_t = \Delta l_i/l_i + \Delta l_2/l_2 + \Delta l_3/l_3 \ldots + \Delta l_f/l_f = \sum \Delta l/l.$$

Here, the summation $\sum$ extends between the limits of l_i and l_f, the initial and final gage lengths, respectively.

As $\Delta l \rightarrow 0$ then $\varepsilon_t = \int dl/l$, with limits again given by l_i and l_f. Engineering students usually recognize this form, which leads to

$$\varepsilon_t = \ln(l_f - l_i) = \ln(l_f/l_i).$$

Also, since $l_f = l_i + \Delta l$, we have $\varepsilon_t = \ln[(l_i + \Delta l)/l_i]$.

To finish the derivation, since $\Delta l/l_i$ is defined as engineering strain, ε, we get

$$\varepsilon_t = \ln(1 + \varepsilon).$$

A quick check verifies that true strain is indeed true. Using the original tabulated two-step values,

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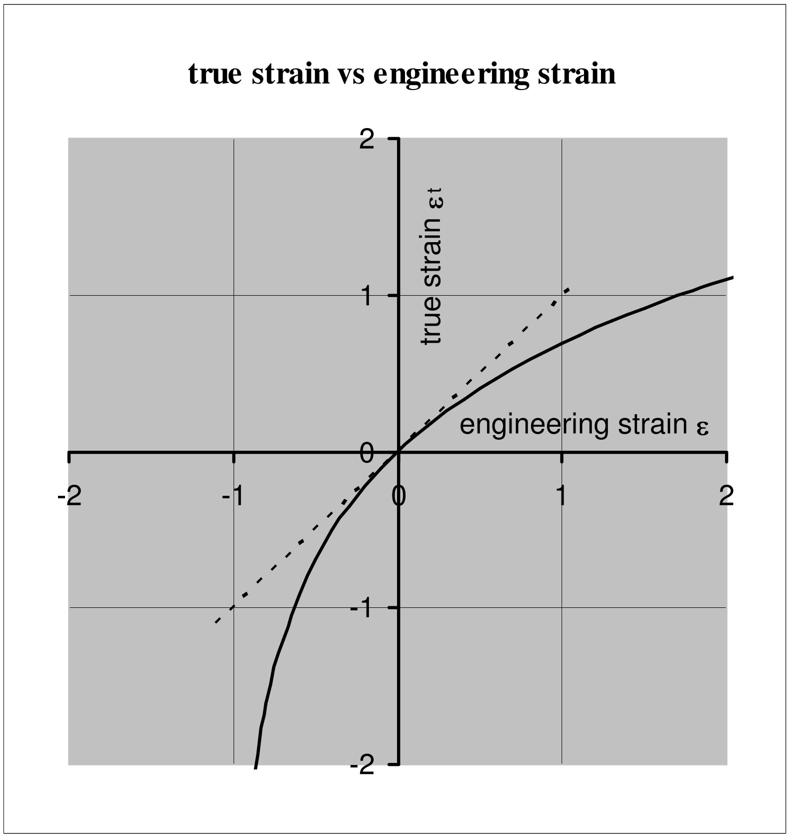
$$\varepsilon_t = \ln(1 + \varepsilon) = \ln(1 + .25) = .223$$
$$\varepsilon_t = \ln(1 + \varepsilon) = \ln(1 + .20) = .182,$$

it is seen that the two-step true strain sum is precisely equal to the one-step true strain value:

$$\varepsilon_t = \ln(1 + .5) = .405.$$

Graphical representations are almost always illustrative, and so students are directed to plot, on a sheet of graph paper, the values of ε_t vs ε, shown (solid line) in the figure below.



The slope at the origin (dashed line) is seen equal to unity, leading to the conclusion that one-to-one correspondence between ε_t and ε exits only when ε is vanishingly small. Further, students may also note that the value of true strain is always less than the value of engineering strain when the loaded structure is in tension, and conversely, always greater when the structure is compressed. The perceptive student may also infer the physical implication of the negative asymptote – that a compressed structure cannot have negative length, no matter how large is the load.

If students make use of sufficiently translucent graph paper, the page may be flipped over, exchanging dependent and independent variables to view the function $y = e^x$ (displaced by one on the abscissa). This simple transformation illustrates an interesting property of the exponential

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function – that the y-value is always equal to the slope – further emphasizing the term 'natural' logarithm.

Lacking experience, engineering designers shall always resort to the proverbial 'fundamentals'. This will certainly be the case as succeeding generations of high-strain materials increasingly find their way into crucially loaded machine components. The few classroom minutes expended in this exercise can bring to life one of these fundamentals.

References

1. 'The Machinery of Life,' Mechanical Engineering, v126, n2, p30-4, February 2004.

2. 'Classroom Demonstrations and Laboratory Experiments,' Chapter 7, Wiley Higher Education, http://wiley.com/college/msci/callister39551X/demonstrations/ch07.html

3. 'Strain in One Dimension,' ABAQUS Theory Manual Version 6.3, http://gong.snu.ac.kr/down/on-line-documents/abaqus/V6.3_HTMLdocs/books/stm/ch01s04ath05.html

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