Mechanical Vibrations Modal Analysis Project with Arduinos

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Abstract

This paper details a new laboratory project in a senior-level Mechanical Engineering Vibrations course. Students are to determine the first four natural frequencies of a 6061 Aluminum free-free beam in a laboratory using three methods. First, they use the idealized theoretical continuous beam model. Second, they use Finite Element Analysis (FEA). Finally, they determine the frequencies experimentally. Using student survey data, it is shown that the project bolstered the following skills: (1) use of measurement equipment to acquire and transmit real-world data, (2) performing a Discrete Fourier Transform (DFT) and creating the Power Spectral Density (PSD) plot of empirical data, (3) creating and modifying FEA code in MATLAB to find natural frequencies and test for convergence of results and (4) connecting the distinct topics of the course together.

Introduction

Our Mechanical Engineering program’s Mechanical Vibrations course has been completely based in theory and simulation. Students did not have hands-on interaction with a real-life spring, mass or damper nor a beam in oscillation. Students may take a dynamics lab as a laboratory elective but can graduate without practical application of the vibrations topics. Experiential and real world application of theory acts to reinforce what is learned in the classroom [1, 2]. This hands-on method of learning is beneficial to students with preferred learning styles other than lecture [3, 4].

Students typically take a MATLAB programming course their first or second year but do not continually apply it in their later courses. Furthermore, most of their programming experience consists of solving problems from a MATLAB textbook and they do not have a chance to analyze their own experimental data.

Our students have the option to take a microcontrollers class as an elective, but again, can graduate without interfacing a computer to the real world through sensors. This interfacing has been shown to increase students’ ability at programming [7]. A microcontroller gaining popularity because of its low cost and flexibility is the Arduino. Students are able to quickly create the hardware and software for their Arduino project [5]. Additionally, the community support and tutorials help the students with their project [6]. Previous work has used Arduinos in Vibrations courses for collecting experimental data [8]. For demonstrating and educational purposes, the results of the data collection were comparable to analytic results.

For an engineer, the ability to interface computer programming with their engineering knowledge is an important skill for their future work [10, 11]. The ability to analyze real data has been seen as an important ABET outcome by industry [10] and technical skills [11].

The intention of this project was to act as a comprehensive assignment that combined what students learned in this Mechanical Vibrations course with what they have learned previously in their Instrumentation, MATLAB programming, and technical writing courses. Students were given the chance to collect real data on a physical object and created a program to analyze the empirical
data. They then compared their actual results to what the theory predicted. This addition was made in the Fall 2016 version of the course.

**Project Description**

Students were asked to determine the first four natural frequencies (in transverse vibration) of a 6061 Aluminum free-free bar (Figure 1) utilizing three methods. First, they were to calculate the values based on the theoretical 4th-order partial differential equation (PDE). Next, they were to create a finite element code and determine the frequencies numerically. Finally, they were to determine them experimentally by recording and analyzing acceleration data. Using the natural frequencies found from the three methods, they then compared the results.

![Figure 1: Aluminum Beam Suspended from Ladder](image)

Students first measured the dimensions of the beam and found the appropriate material properties.

For the analytic method, students were expected to derive the frequencies by beginning with the 4th-order PDE for a beam in transverse vibration. Applying the four boundary conditions (zero shear force and bending moment at the ends), they found the eigenvalues. The first four eigenvalues were used with the material properties and dimensions to calculate the first four natural frequencies. This was done for vibrations along both principal transverse axes.
Students were provided a basic FEA code for modal analyses of beams in axial or transverse deflection. They needed to modify the code for use with the specific beam and apply a mesh. Then, the local and global stiffness and mass matrices were created (it was assumed to have negligible damping). Then, they applied the proper boundary conditions for these supports. For free-free, no modification to the matrices was needed. The matrices to find the eigenvalues of the system. Finally, the natural frequencies were calculated. Using the FEA code, they needed to demonstrate convergence of their solutions by solving with a finer and finer mesh. Again, this process was done for vibrations along both principal transverse axes.

In the experimental section, students utilized an ADXL-335 analog accelerometer affixed to the bottom end of the beam (Figure 2). The accelerometer was connected to the inputs of student-provided Arduino microcontrollers. Students were provided a template Arduino code for reading the sensor and logging it to the serial terminal. They needed to modify the code to poll at an appropriate sample rate and read from the correct analog pins on the microcontroller.

Students began the data logging to record acceleration along a single principal axis (transverse to the beam). Then, they used they sharp side of a welding hammer to strike the beam along that axis orthogonal to the surface (treated as an impulse) and allowed it to vibrate for at least 90 seconds. After stopping the data logging, the time and acceleration data were copied out of the terminal and into a spreadsheet for later analysis. This data collection was repeated several times along the same
surface, striking at different locations. Then, the experiment was repeated along the other principal transverse axis.

The raw data was imported into Matlab one trial at a time. The Arduino did not sample uniformly so data first had to be resampled, using linear interpolation, to a uniform time step. This was typically around 1800 Hz. Students plotted the data and decided where to trim to include only those data after the initial strike and before the vibrations become “too small.” A Discrete Fourier Transform (DFT) was performed on each truncated data set. The coefficients were divided by the square of their associated frequency (effectively integrating) to find the coefficients for the position of the end of the beam. These coefficients were used to create a Power Spectral Density (PSD) plot. Students manually identified the first four peaks in the PSD and recorded these as the natural frequencies, again assuming no damping in the system. The values from the multiple trials were averaged together to find the experimental values.

**Sample Student Work**

Using the theory, the dimensions of the bar and the material properties, students found the natural frequencies for principal axes designated as $x$ and $z$ in Table 1.

<table>
<thead>
<tr>
<th>$\omega_n$</th>
<th>x-axis (rad/s)</th>
<th>z-axis (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>617</td>
<td>494</td>
</tr>
<tr>
<td>2</td>
<td>1702</td>
<td>1361</td>
</tr>
<tr>
<td>3</td>
<td>3336</td>
<td>2669</td>
</tr>
<tr>
<td>4</td>
<td>5514</td>
<td>4412</td>
</tr>
</tbody>
</table>

By modifying the FEA code, the numerically-determined natural frequencies differed by less than 0.1% with those found by the analytic method. A typical plot of the convergence of the natural frequency values is shown in Figure 3. For this simple beam, convergence for the first four frequencies was reached by five elements.

![Figure 3: Convergence of Natural Frequency Values as the Number of Elements Increased](image-url)
Acceleration values were sent in bits (0-511), rather than g’s, from the Arduino to the computer to increase the sample rate. A typical plot of the trimmed acceleration data is shown in Figure 4. The acceleration is clearly attenuated over the time span of recording. For the purpose of this project, this damping is ignored as it was in the analytical and numerical sections.

![Acceleration Raw Data X Axis 1](image)

*Figure 4: Typical Plot of the Raw Acceleration Data from the Arduino. The data has been trimmed to include acceleration only after the hammer strike and before it becomes “too small.”*

After a DFT of the acceleration data and double integration, the position PSD was found. Typical PSD plots of the results are shown in Figure 5. Students manually determined the first four peaks of the PSD as marked in the figures.

![Power Spectral Density Analysis X Axis 1](image) ![Power Spectral Density Analysis Z Axis 1](image)

*Figure 5: Typical PSD of Position. The first four peaks are marked and taken to be the natural frequencies.*

Comparisons of the first four calculated natural frequencies for the x-axis and z-axis in this sample are shown in Table 2 and Table 3 respectively. Frequencies determined by the Arduino and PSD analysis were generally “close” to the analytic and numeric values. In most cases, all four frequencies were lower for the experimental values. These typical results from the experimental
portion are consistent with those found in previous classroom experiments using an Arduino to measure natural frequencies [6].

**Table 2: Typical Comparison of Natural Frequencies for x-axis**

<table>
<thead>
<tr>
<th>$\omega_n$</th>
<th>Analytical (rad/s)</th>
<th>FEA (rad/s)</th>
<th>Experimental (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>617</td>
<td>617</td>
<td>555.4</td>
</tr>
<tr>
<td>2</td>
<td>1702</td>
<td>1702</td>
<td>1532</td>
</tr>
<tr>
<td>3</td>
<td>3336</td>
<td>3336</td>
<td>2946</td>
</tr>
<tr>
<td>4</td>
<td>5514</td>
<td>5514</td>
<td>4033</td>
</tr>
</tbody>
</table>

**Table 3: Typical Comparison of Natural Frequencies for z-axis**

<table>
<thead>
<tr>
<th>$\omega_n$</th>
<th>Analytical (rad/s)</th>
<th>FEA (rad/s)</th>
<th>Experimental (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>494</td>
<td>494</td>
<td>442.1</td>
</tr>
<tr>
<td>2</td>
<td>1361</td>
<td>1361</td>
<td>1237</td>
</tr>
<tr>
<td>3</td>
<td>2669</td>
<td>2669</td>
<td>2437</td>
</tr>
<tr>
<td>4</td>
<td>4412</td>
<td>4412</td>
<td>4033</td>
</tr>
</tbody>
</table>

Students concluded that the assumption of zero damping was not completely accurate. The attenuation in the acceleration plot and the difference in natural frequencies demonstrated the limitation of this assumption on real world problems. The observed differences demonstrated the difficulties of modeling an ideal system versus how it behaves in reality [2].

**Student Outcome Measurements and Discussion**

After submitting their final project, students were given a survey about the project. They were asked to rate their ability in several areas on a 0-100 continuous scale before starting and after finishing the project. Statements were anchored at 0, 33, 66 and 100 for each of the prompts but students could select any value. The prompts and anchor statements are shown in Table 4.

**Table 4: Prompts and Anchors for Student Survey. The code in italics under each prompt corresponds to the responses in Fig. 6 and Fig. 7.**

<table>
<thead>
<tr>
<th>Prompt</th>
<th>0</th>
<th>33</th>
<th>66</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate your proficiency level of <strong>wiring and programming an Arduino (or PIC) for data collection...</strong> (Ard)</td>
<td>I have never attempted this</td>
<td>I know how to read and create very basic code but cannot attach a sensor</td>
<td>I am able to attach the sensor to the Arduino but not read data from it properly</td>
<td>I can read data taken from a sensor I have attached to the Arduino in a format ready to be used in a subsequent step</td>
</tr>
<tr>
<td>Prompt</td>
<td>0</td>
<td>33</td>
<td>66</td>
<td>100</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------</td>
</tr>
<tr>
<td>Rate your proficiency level of performing a Discrete Fourier Transforms (DFT) and creating and interpreting Power Spectral Density (PSD)...</td>
<td>I do not know what either of these is</td>
<td>I can find the Fourier coefficients but am not sure what to do with them</td>
<td>I can make a PSD plot using my coefficients, but I am not sure what it shows me</td>
<td>I am able to find the Fourier coefficients, plot them on the PSD and then identify peaks on the PSD</td>
</tr>
<tr>
<td>Rate your proficiency level of finding natural frequencies of continuous beams using analytic methods...</td>
<td>I do not even know where to start</td>
<td>I can identify the boundary conditions correctly</td>
<td>I can set up the equation, but not fully solve for the natural frequencies</td>
<td>I can set up the equations and solve for the natural frequencies numerically</td>
</tr>
<tr>
<td>Rate your proficiency level of creating (or modifying) Finite Element Analysis (FEA) code to find natural frequencies of continuous beams...</td>
<td>I do not even know where to start</td>
<td>I can set up some things (e.g. parameters of the system and boundary conditions) but the code will not run</td>
<td>I can run the code, but the results do not look correct</td>
<td>I can size my elements correctly, create my mass and stiffness matrices, apply proper boundary conditions and then find the eigenvalues</td>
</tr>
<tr>
<td>Rate your level of overall understanding of the course content and how it fits together...</td>
<td>I do not understand anything in this course</td>
<td>I can generally identify the type of system I have but I am not always sure what equations are appropriate to use</td>
<td>I am able to identify the type of system I have and find the appropriate equations to solve for unknowns, but I am not sure how the units all fit together</td>
<td>I am able to identify the type of system I have and find the appropriate equations to solve for unknowns. I also understand how all the mathematics review, MDOF systems and continuous systems fit together.</td>
</tr>
</tbody>
</table>
The survey was distributed via Qualtrics at the end of the semester and was completed by \( n = 11 \) students. The results of the numerical responses are displayed in Figure 6 and Figure 7. The prompts are in the same order as presented in Table 4. The “A” response is perceived ability before starting the project while “B” is perceived ability after finishing the project.

![Boxplot of Survey Results](image)

*Figure 6: Boxplot of Survey Results (1). There is a significant difference between the median pre-project ability (A) and the post-project ability (B) for all prompts.*
To compare the pre- and post-ability for each prompt, a one-sided Wilcoxin sign rank test was employed. The distributions were not normally or symmetrically distributed around the median. All median post-measurements (B) are significantly larger than the pre-measurements (A) with 95% confidence, i.e. $p < 0.05$.

One limitation to the following inferences of the survey results is that students completed these projects as a team so not every member may have been as involved in every aspect of the project. Additionally, all ability assessments were self-reported and an objective concept quiz was not administered before or after the project.

A small gain in the median ability to use microcontrollers was observed. In the pre-ability, we see a large spread in experience with Arduinos. Many of the students were concurrently taking a microcontrollers class using PICs and already were familiar with using sensors and controllers. We observe the spread decrease in the post-ability, but the median value is not impacted.

The students’ ability to create and interpret PSDs was greatly increased after completing the project. At the beginning of the course, we discuss and create PSDs as part of learning about Fourier Transforms and DFTs. However, at that point in the course, the discussion is limited to 1-DOF of systems and using premade empirical data sets.

Students’ ability to both analytically and numerically determine natural frequencies was improved. Because we do not address continuous systems until the last third of the class, they had the least previous experience with these topics. They completed homework assignments on the topics, but this was the first time they had a chance to use them on a larger scale.
Finally, there is an increase in the students’ understanding of how the course topics in the course fit together. The topics are initially presented in silos: mathematics, multi-DOF systems, and continuous systems. This project was the first assignment to synthesize course topics from the entire semester and have students apply them to solve a real-world problem.

Anecdotally, students reported in the survey that the hands-on project helped their understanding of the theory presented in class. This report agrees with previous literature on the benefits of laboratory activity [3]. Additionally, students appreciated working with teammates with different strengths. The team members were able to learn from each other while working on the project.

The positive results from the survey data demonstrate that the project was worthwhile and met the intended objectives of improving student ability in several areas. Students were asked in the survey to list what they found most frustrating in completing the project and suggestions to improve it. These responses will be taken into account to improve the project for next year’s class. Additionally, an objective measurement will be made to gauge the improvement of student ability.

**Future Project Improvements**

Based on student feedback from the survey and grading the submitted projects, several changes are planned for the next version of the project. One issue was that the project was not assigned until the end of the course when all the topics and been covered. This did not allow enough time to thoroughly complete the project and complete a report. In the fall, the project will be assigned earlier as the data collection and FEA can be done before covering the final topic of continuous systems.

Students appreciated using the Arduino and the sensor but felt the code template gave them too much at the start of the project. They would have preferred coding the Arduino from a blank template and understanding how it worked better. In the next version, students will be given coding resources and references but will complete the hardware and software interfacing themselves. With the experimental portion beginning much earlier in the semester, they will have more time to do this.

**Acknowledgements**

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References


