Medieval Engines of Siege Warfare and Modern Engineering Tools

Ron Goulet
College of Engineering and Computer Science
University of Tennessee at Chattanooga

Abstract

The College of Engineering and Computer Science at UT Chattanooga offers second year engineering students a three credit hour lecture course in Engineering Dynamics. Seeking to supplement the traditional lecture approach, *experiential problem-based learning* projects are inserted. *EPBL* is the outcome of a 'learner centered' classroom process that uses real work problems to motivate students to 'perform' or 'act out' the discovery and application of concepts. By doing so, students develop *familiar* as well as *formal* understanding of course content. The EPBL insertions compel students to *'perform'* the tasks of analysis, design, prototype construction and proof testing of the gravity powered catapult known as the trebuchet. Individuals complete kinematical and kinetic analysis of planar rigid body motion requiring applications of modern engineering tools including Maple®, Excel® and Visual Basic for Applications®. Team activities include building and proof testing prototypes. The paper describes the development and delivery of the EPBL insertion, results and feedback and instructors' reflections and recommendations for future improvement. Access to this course BlackBoard® website is also provided.

Background

Engineering Dynamics is offered to all second year mechanical, civil and industrial engineering students of the College of Engineering and Computer Science at UT Chattanooga. The learning objectives of the three credit hour course include knowing and applying the dynamical principles of impulse & momentum, work & energy, kinematics and kinetics for particle and planar rigid bodies. The course also seeks to add depth to student proficiencies in the application of modern engineering tools, particularly Maple®, Excel® and Visual Basic for Applications® (VBA)

The method of course delivery generally follows the traditional teacher-centered lecture/exam paradigm requiring pre-lecture student preparations and post-lecture reinforcement. Pre-lecture student preparation is promoted through graded classroom recitations where students are randomly selected to solve pre-assigned problems at the board. Post-lecture reinforcement is advanced through the daily assignment of homework and through a mandatory correction policy. This policy, based on the premise *"one learns from one's mistake"*, compels students to correct the mistakes contained in all graded work by rewarding perfect corrections with a 50% recovery of any loss points. To manage the correction policy, students are required to submit an end-of-semester portfolio containing all work and corrections.

To maximize the likelihood that the course learning objectives are achieved, a strategy to supplement the otherwise traditional lecture course is employed through *insertions* of experiential problem-based learning (EPBL) projects. EPBL is the outcome of a learner-centered process that uses real work problems or scenarios to motivate students to perform or act out the discovery and application of concepts and information. By doing so, students develop *familiar* as well as *formal* understanding of course content. [1,2]

Evolvement of the EPBL Project

The original EPBL activity was developed and launched in 1999 then evolved to its current state through two full cycles of the well-known process of *closing* the assessment feedback loop. A recap of the earlier versions is provided below.

EPBL Version 1.0 Spring 1999

The first Dynamics EPBL insertions, in the spring of 1999 through spring 2000, were catapult projects similar in some respects to those reported by others[3,4]. These out-of-class projects required applying the principles of projectile motion, particle kinetics, work & energy and impulse & momentum in the design, construction and proof testing of spring-loaded catapults known as tennis ball launchers. The team-based projects were delivered to the class in the form of a contest or team challenge to obtain the longest launch distance. At the midterm, five teams of 3 to 4 students were formed and contest rules were distributed including design requirements and constraints, schedule of deliverables and grading expectations. The launcher design requirements called for a spring-loaded "striker", to impact a stationary tennis ball upon release of a trigger. To pit design ingenuity rather than the teams' material resources, each was limited to use one 8 ft. length of *official* 1" x 3" pine board and one 12 in. length of *official* surgical tubing. Team deliverables included one written design report with launcher predictions, supporting analysis and sketches, one functional launcher and a proof test to compare actual launch distance to the predicted. The predicted performance of the launcher and supporting design analysis was required prior to construction of the catapult. The analysis included: 1) Estimate striker velocity as a function of spring displacement by applying the principle of work & energy; 2) Estimate the initial velocity of projectile as a function of striker momentum by applying the principle of impulse & momentum and; 3) Estimate the projectile launch distance as a function of launch angle and initial velocity by applying particle kinematics. Little emphasis was placed on grading in this first version of the EPBL insertion. An *A* was assigned to members of the winning team and a *B* to all others except for "no shows" who received a *D*.

Three of the teams submitted substantially complete and correct design reports on their first submittal. The other two required varying degrees of coaching and rework before receiving a release for construction. All teams delivered their catapults at the specified time and place and actively engaged in the shootout contest. The performance of roughly 75% of the devices was surprisingly consistent with predictions. While no quantitative assessment of the impact on learning outcomes was attempted, student feedback indicated that the EPBL insertion increased the interest in and perceived relevance of the subject matter. Students praised the project as a welcome change of routine and roughly half of those expressed an appreciation for the hands-on aspect of building and testing. On the other hand, a few students complained about freeloaders and about the logistical difficulties inherent in executing team tasks outside of the classroom. A few students also expressed concern about the hands-on aspect because of their inexperience with tools, the shop and hand building.

EPBL Version 2.0 Spring 2001

In the spirit of continuous improvement and in response to two cycles of instructor observations and student feedback, the EPBL insertion was revised from a team project to an individual project, the contest *problem* was modified and the use of modern engineering tools was incorporated.

The change from team to individual deliverables was due primarily to the author's desire to hold all students individually accountable for the dynamical analysis. This followed from the observed freeloading noted earlier. The shift away from the team-based project was also in response to student complaints about scheduling out of class meetings, poor attendance and unequal distribution of workload among teammates. With this change, version 2.0 shifted to an entirely individual project where each learner was responsible for the design, construction and proof testing of his or her launcher.

Knowing that the occasional student seeks the shortcut offered by re-using earlier work, the contest *problem* was changed from a competition of longest distance to greatest accuracy. This revision challenged students to launch a tennis ball on a *prescribed trajectory*. Specifically, the launcher would be placed on the floor of the basketball court at the free-throw line 15 ft. from the backboard. The ball would strike the backboard 11.5 feet above the floor then rebound to the floor 7.5 ft from the free throw line. The remaining launcher specifications remained unchanged.

While the UTC engineering program embraced the EC2000 outcome related to developing competencies with modern engineering tools, this particular engineering Dynamics course did not compel students to apply any of the available p.c. based tools including Maple®, Matlab®, ANSYSR®, Excel® or TK Solver®. Therefore the project was modified to require students to use Maple® in the dynamical analysis to derive and solve the system of equations. While no students in the class used Maple® in any of their engineering course work, most had used Maple in prior calculus labs. Leveraging this prior experience, example worksheets were posted to the course website as supplemental instructional support.

Roughly 75% of the students submitted perfect or substantially complete and correct design reports with supporting Maple® worksheet files on their first submittal. The remaining students required varying degrees of coaching in the dynamical principles or the use of Maple®. Of these all but one eventually received his or her release for construction. All students produced their launchers on time and again actively engaged in the shootout contest. The performance of the devices could best be described as erratic, however, as only one launcher roughly followed the prescribed trajectory. This unexpected outcome proved to be both entertaining and instructive as the class posited theories underlying the misfires. Student feedback again indicated that the EPBL activity captured their interest in the application of the subject matter. Students again welcomed the change of routine and again several, roughly half, enjoyed the opportunity to show off their handiwork. The other half, however, expressed frustration with the construction aspect due to limited knowledge, skill and shop experience. Roughly half complained about the time demanded to learn or relearn how to use Maple®.

EPBL Version 3.0 Spring 2002

In light of concerns that flowed from instructor observations and student input, the assessment feedback loop was closed again by another revision of the EPBL project. The concerns included the absence of a team-based activity, construction frustration and poor (yet entertaining) device performance. Another concern arose after review by the instructor: the problem was not very challenging. This conclusion was based on the observation that the design analysis was limited to basic particle kinematics and kinetics and contained none of the thornier applications of rigid body dynamics. Furthermore, the application of Maple® to solve the system of linear equations was a trivial task easily accomplished by hand without the use of a solver. The EPBL project was therefore again revised in Spring 2002. A description of the current EPBL activity follows.

The Floating Axel Trebuchet

The trebuchet is a medieval gravity powered engine of siege warfare. The idea of inserting a trebuchet into the dynamics course originated with a student who completed the Spring 2001 dynamics course. Knowing nothing about medieval siege engines, a web search found a plethora of medieval siege engine sites and trebuchet enthusiasts, both academic and hobbyists[5,6]. Searches of the educational literature revealed a few reports of trebuchet learning projects used in an engineering course[7,8,9]. The trebuchet appeared to be a good candidate for an EPBL insertion into the engineering dynamics course because it would require knowledge and application of general planar rigid body kinematics and kinetics and because it could be constructed with readily available materials. On the other hand, the classical solution to the trebuchet *and sling* applied the method of Lagrange and its extension to holonomic constraints[10], approaches beyond the scope of this second year dynamics course. By excluding the sling, the motion of the throwing arm and its projectiles could be estimated without advance tools of analysis.

The Specifications

The EPBL project was delivered to the class as an individual challenge where each student developed a design with supporting analysis to predict trebuchet performance and as a team contest where each team selected, optimized and constructed a FAT with the longest throwing distance. The project schedule was divided into two phases that corresponded to individual and team-based activities and deliverables. In Phase1 all students proposed his or her design solution supported by a substantially complete dynamical analysis. The dynamical analysis included concept sketches, properly labeled free body diagrams, derivations of governing equations of rigid body kinematics and kinetics, solutions to the system of dynamical equations using Maple®, and a functional Excel design tool that could be used to predict and optimize throwing distance. In Phase 2, students were assigned to teams of 3 to 4 members who evaluated their design proposals then selected one as the team's preliminary design. Teams then enhanced their design with sufficient detail for construction and reported the predicted performance. On acceptance of the report by the instructor, the teams constructed the prototype then delivered the FAT for demonstration and proof testing.

The project specifications called for a gravity powered floating axel trebuchet with a counterpoise, a throwing arm or lever, trigger release mechanism and a freestanding wood support structure. The throwing arm articulated on two slider-mounted axels where one pair of sliders was constrained to translate vertically and the other translated horizontally. Referring to the free body diagram in Figure 1, points B and C indicate the vertically and horizontally constrained axels, respectively; the counterpoise hangs from point A and the projectile is seated at point D.

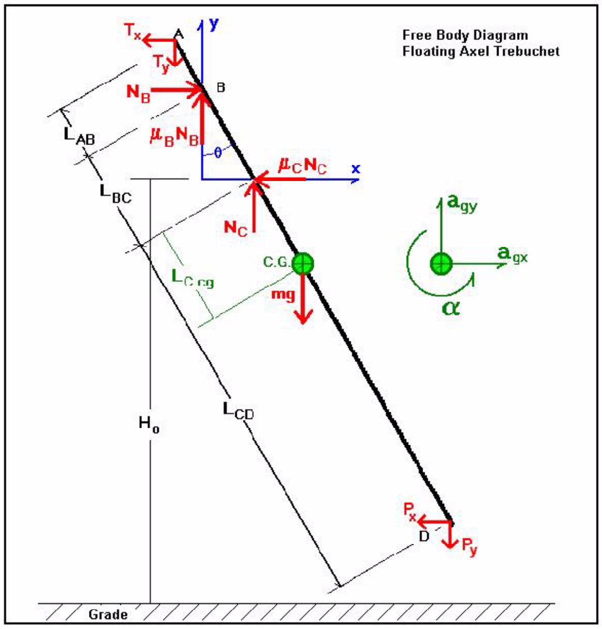

Figure 1 FAT free body diagram

Students were directed to limit their analysis and predictions to the *sling-less* trebuchet but all were invited to build *sling-capability* into their launchers. The specifications for Phase 1 design proposal deliverable required each student to submit:

- Concept sketch that described all elements, centers of mass, supports, loads and dimensions and labeled with appropriate symbols.
- A set of complete and properly labeled free body diagrams with the corresponding set of derived kinetic and kinematic relations
- Maple® worksheet containing the solution of the system of equations: kinetic equations of motion and kinematic equations of general plane motion
- Excel® worksheet containing plots of:
  - Angular acceleration vs. angular position
  - Angular velocity vs. angular position
  - Projectile Range vs. angular release position

Applications of Modern Engineering Tools

The project specifications called for the use of Maple® to solve the system of equations in order to estimate the angular velocity as a function of angular position. The closed form solution for angular acceleration is readily obtained in Maple, however that for angular velocity is not. Therefore a numerical approximation of velocity was obtained using Excel. The worksheet was constructed to estimate the angular velocity ω as a function of acceleration and position by minimizing the difference in the following expression, a variation of the well known kinematic relation: $\omega d\omega = \alpha d\theta$:

$$diff = \frac{\omega_{i+1} + \omega_i}{2} \cdot (\omega_{i+1} - \omega_i) - \frac{\alpha_{i+1} + \alpha_i}{2} \cdot (\theta_{i+1} - \theta_i) \qquad \text{eq.1}$$

The angular velocity was estimated by calling the Excel tool *Goal Seek* to find the value ω that minimized the equation 1. Because the Goal Seek command was repeated for every row in the spreadsheet, a macro was created in Visual Basic for Applications as follows:

```
Sub Solve_w()
count = 0
Do While count < 180
   count = count + 3
   cell = "G" & count
   cell2 = "E" & count
   Range(cell).GoalSeek Goal:=0, ChangingCell:=Range(cell2)
   count = count - 2
Loop
End Sub
```

Maple's symbolic solution for angular acceleration was not trivial as it contained over 5000 characters and spaces, well over the size limit (1024) for an Excel formula. Therefore a custom function was created in VBA by copying then pasting the expression from Maple to the VBA editor where each line held the same 10-bit size limitation. An example of a custom function (for equation 1) follows:

```
Function diff(a1 As Double, a2 As Double, w1 As Double, w2 As Double, theta1 As_
```

```
        Double, theta2 As Double) As Double
        diff = (a2 + a1) /2* (theta2 - theta1) - (w2 + w1) /2* (w2 - w1)
    End Function
```

Excel worksheets with custom functions and macros were designed by students to predict projectile range as a function of user-defined values for counterpoise, lever and projectile masses, critical lever dimensions, coefficients of friction, and initial angular position.

Assessment of the Design Proposals

Roughly half of the students submitted satisfactory individual design proposals on their first submittal. All but one of the remaining students satisfied the content requirements on their second submittal. Unexpectedly, the instructor did not have sufficient time to properly review, debug and grade the individual analyses and Excel worksheets. The *plan* was to assess the goodness of each student’s design by comparing it directly to the instructor’s design template. However, suffering a total lack of imagination, this instructor did not anticipate the numerous ways a free body diagram could be culled together. Thus the design template poorly accommodated the multitude of differences in FBD labels, dimensions and coordinate systems of the students’ work. Therefore, the feedback to individual students was limited to: “Good: Submittal is complete. Results are generally consistent with instructor’s”; “Satisfactory: Submittal is complete. Results are not consistent with instructor’s”; and “Unsatisfactory: Incomplete submittal”.

The FAT “Shootout”

In Phase 2 teams were assigned and designs were finalized. Due to time limitations, the team design report requirement was waived and construction was fast-tracked. Two teams build their trebuchets on campus in the engineering shop. The third catapult was constructed off campus. The specified time and date for delivery of the functioning trebuchets for proof testing coincided with the scheduled date and time of the final exam. The class met at an outdoor playing field. In spite of the overcast damp weather, many curious onlookers gathered including dozens of students, faculty, staff as well as the local radio, television and print media outlets. For the shootout, teams test fired the trebuchets by hurling the official projectile, a baseball. Volunteers shagged fly balls, measured and recorded distances. Although one team encountered numerous mechanical problems eventually corrected with a substantial quantity of duct tape, the other catapults operated reliably. Once all the requisite test measurements were made and recorded, the students remained on the practice field for well over an hour, to show off their catapults, to hurl more objects, to chat with curious onlookers, to take pictures, to play, to enjoy the moment.

How closely did the actual performance match the predicted? Not closely at all. The maximum observed range for “*sling-less*” catapults measured roughly 15 m while the predicted range was 42 m. It was reasoned, at the time, that the deviation was due the absence of a means to “arrest” trebuchet motion at the optimum angular position, a shortcoming observed in all three team designs. The instructor determined at a later time, however, the deviation was also related to a common deficiency in the dynamical analysis.

Student Feedback

Student feedback was acquired informally through group and one-on-one interactions. Again, most students agreed that the EPBL insertions increased their interest in and their perceived relevance of the course materials. The most recurring dissent pertained to the amount of out-of-class time consumed completing the analysis. Several others expressed frustrations using Maple® and programming VBA and Excel®. One student opined that trebuchet construction should be an individual deliverable. Several, after complaining about the time consumed developing, debugging and refining their analyses, pointed to the inadvertent learning results: they could readily derive the equations of motion and kinematics; they could solve the system of equations in Maple®; and they were able to create Excel® macros in VBA! Finally, most expressed an appreciation for the opportunity to demonstrate their handiwork and expertise on the campus practice field to a curious and impressed audience.

The formal student evaluations of the spring 2001 course were unusually high with median scores of 6.0 on a scale of 6.0 in three of four categories: instructional quality, course content and interest in subject matter. The accompanying student comments reiterated the positive and negative feedback cited above.

Conclusions, Reflections and the Future

The floating axel trebuchet EPBL project was successful. With regard to the maximizing the likelihood of achieving learning objectives, the trebuchet challenge turned out to be a good EPBL project choice because it reinforced the *formal classroom knowledge* with the *familiar knowledge* obtained when immersed in the solution of a real, hands-on problem. With regard to enhancing competencies in the application of modern engineering tools, the EPBL project is also a good choice in two ways. First, it exposed students to a sufficiently complex problem where the use of a solver such as Maple® was warranted; Second, it presented an opportunity to obtain a numerical approximation in Excel® using VBA.

While the above conclusion asserts that the EPBL project was a success, student feedback and instructor observation suggest there is much room for improvement in both content and delivery. Regarding content, the sling will be added to the prediction of FAT performance without relying on advanced methods of analysis. Slightly overestimated, yet reasonable, predictions of system behavior would be obtained by superimposing the motion of sling (as a rigid pendulum) onto the kinematics of the throwing arm independent of and uncoupled to sling kinetics. Regarding delivery, student complaints of too much time could be addressed by spreading the project over a longer time period. Frustrations with Maple® and Excel® would be addressed by inserting supporting preliminary exercises. Therefore, the EPBL experience would be modified to be a series of preliminary and preparatory projects. The first project, SP1, would require students to apply particle kinematics to derive expressions for projectile motion, to apply the calculus of extrema in Maple®, then to plot functions and trajectories in Excel. The second project, SP2, would introduce the students to the problem type whose solution defies the *closed form*, specifically, where the expression for ω cannot be derived from $\omega d\omega=\alpha d\theta$. In the third project, SP3, students derive the kinematic equations of motion for two trebuchet types. The fourth project, SP4, students, given α, utilize Excel® and VBA to find an iterative numerical solution to the angular velocity where ω cannot be expressed in closed form. The fifth project, SP5, would be the Trebuchet challenge where students develop a design tool in Excel® that predicts the performance of a floating axel trebuchet with sling based on a set of user-defined parameters and to complete an assigned parametric study. The sixth project, SP6, would be the construction and proof test of the FAT. To minimize freeloading that short-circuits the learning process, the five preliminary projects would have individual deliverables. The last project, construction and proof testing would be team-based. For additional information regarding special projects SP1-SP5 go to: http://utconline.utc.edu. Visitors may log on with user name: test and password: test.

Bibliography

[1] The Power of Problem-Based Learning *A Practical "How To" for Teaching Undergraduate Courses in Any Discipline* Edited by Barbara J Duch, Susan E Groh, Deborah E Allen, Stylus Publishing, LLC, 2001.

[2] R. Goulet, J. Owino, *Experiential problem-based learning in the Mechanics of Materials Laboratory*, Proceedings of the 2002 ASEE Annual Conference, Montreal, June 2002.

[3] Yilmaz, E., *Winning the World Punkin' Chunkin' Competition with a Student Design Project*, Paper 3548, Proceedings of the 1999 American Society for Engineering Education Annual Conference, Charlotte, NC, June 1999.

[4] Huang, M, *The Catapult Experience –A Learning Project on Taguchi Method for Design Optimization*, Paper 2525, Proceedings of the 2001 American Society for Engineering Education Annual Conference, Albuquerque, June 2001.

[5] P.V.Hanson, *Experimental Reconstruction of a Medieval Trebuchet*, Acta Archaeologica, vol. 63, pp. 198-268, 1992.

[6] URL: http://members.iinet.net.au/~rmine/gctrebs.html; The Grey Company Trebuchet Page.

[7] Cartwright, L, *Humorous Engineering*, Paper 2081, Proceedings of the 2001 American Society for Engineering Education Annual Conference, Albuquerque, June 2001.

[8] Matlock, M., Osborn, S. LePori, W. and Engler, C., *Development and Implementation of a Common Investigative Methods Course for Undergraduate Engineering Students*, paper 2326, Proceedings of the 2001 American Society for Engineering Education Annual Conference, Albuquerque, June 2001.

[9] URL: http://www.tbullock.com/trebuchet.html; Tom Bollock, Some experiments with trebuchets.

[10] URL: http://www.algobeautytreb.com/trebmath35.pdf; Siano, D.B., Trebuchet Mechanics, 2001.

RON GOULET
Ron Goulet is an associate professor of mechanical engineering in the College of Engineering and Computer Science at UT Chattanooga where he teaches dynamics, mechanics of materials, material science and design. He received a Ph.D. in Engineering from University of New Hampshire in 1997. His research interests include experimental mechanics and orthopaedic biomechanics.