AC 2010-1037: MODE OF FAILURE ANALYSIS OF STUDENT RESPONSES TO PRE-REQUISITE KNOWLEDGE ASSESSMENTS IN FLUID MECHANICS

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Mode of Failure Analysis of Student Responses to Pre-Requisite Knowledge Assessments in Fluid Mechanics

Abstract

In Mechanical Engineering, and in particular in Fluid Mechanics, advanced concepts build extensively on a student's understanding of both Mathematics and their core Mechanical Engineering courses (Statics, Dynamics, Solid Mechanics). Within these core courses are a number of central concepts and skills which form threads that connect one content area to another within a discipline. An incomplete understanding in any of one of these concepts at an early stage in a student's education can lead to a cascade of failures or difficulties that resonate throughout their education. The current research has identified and mapped a number of central content and skill trajectories that are present in engineering education, focusing on science and mathematics content and skills. Student competency in these content and skills has been assessed along these trajectories for selected classes. Common student errors within these assessments have been identified and classified to generate a profile of the error modes for each topic. Validation of the error modes has been conducted through inter-rater reliability studies and student interviews.

Trends and insight in to student difficulties with pre-requisite knowledge and an early curricular profile of issues with pre-requisite knowledge in Mechanical Engineering will be presented. Knowledge about the modes of failure (error) and the overall success or failure of content and skill trajectories will permit focused attention on teaching practices and the development and assessment of activities and learning materials aimed at developing long-term improvement of the student knowledge base. Through this research we are beginning to gain an understanding of student performance at various stages of a content or skill trajectory and we are able to examine the structure of the curriculum and determine where learning and transfer breaks down.

Introduction: Content and Skill Trajectories

In engineering education there are a number of central concepts and skills that form threads which connect one content area to another within a discipline. These threads generally consist of basic or simple concepts and are central to a student's engineering education because they form the scaffold upon which higher-order knowledge constructed. The recurrence of these threads throughout a curriculum or program of study is referred to as a trajectory.

One aspect of these recurring concepts and skills is that they may enter a given class or subject at a variety of different levels ranging from "central to the development of concepts" to "tool oriented". In addition, when these trajectories are used to support advanced content their development need not follow a logical progression within an advanced course since they are considered pre-requisite knowledge: a student may use the concept in a sophisticated manner in one class and then simply as a tool in a later class. An incomplete understanding in any of one of these concepts at an early stage in a student's education can lead to a cascade of failures or difficulties that resonate throughout their education. Since construction of new knowledge is built using these concepts, the inability to immediately and accurately apply these central principles means that students will find that the impact of these knowledge gaps is recurring. While coping skills may enable a student to pass a class or "get through" a topic, if they choose to repair their knowledge gaps they may find themselves re-learning these concepts/skills while at the same time learning advanced concepts that depend upon these skills.

A simplified example of a trajectory essential to mechanical engineering is the concept of the dot product, also known as the scalar inner product. Figure 1 shows a representative program of study in Mechanical Engineering at Kettering University with the intersections of the dot product trajectory with various classes highlighted. Mechanical Engineering students (generally in the United States and, specifically, at Kettering University) typically receive their first full instruction on this topic in their intermediate-level Calculus classes. In this course the theory behind the dot product is developed from the perspective of vectors and vector multiplication. In their math class tools are developed to facilitate its implementation and, while the focus here is often applied, discussions and investigations also develop the mathematical properties of the concept. Students are often next exposed to the dot product in their introductory Physics classes, primarily as a method for determining the component of a vector but they can also, depending on the level of instruction, be exposed to the dot product within the concept of flux if Gauss' Law is discussed in the second semester of their introductory Physics class. In some circumstances, the instruction on the dot product in the Physics class precedes its instruction in the Calculus class.

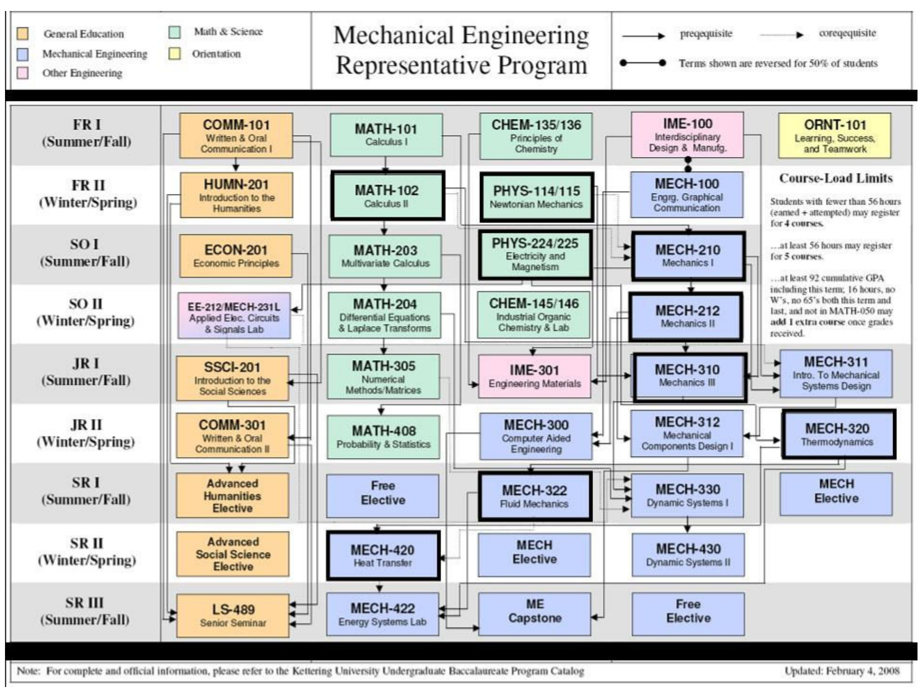

Figure 1: Representative Program in Mechanical Engineering at Kettering University with the Dot Product Trajectory Highlighted

Continuing with the dot product as an example trajectory, the concept is next encountered in the student's Statics class where it is used almost exclusively to determine components of a vector. In some circumstances, the dot product element of selecting components is de-emphasized in favor of a "common sense" or "practical" approach using trigonometry. A similar circumstance exists in Dynamics, the follow-up course to Statics. Depending on a student's course load and an institution's curricular structure, the dot product may not re-appear until Thermodynamics where it is used exclusively to define a flux through a surface. This concept, however, occupies a small fraction of the intermediate-level course and, as such, it is a high-order topic that receives a low exposure. For the dot product, the emphasis in this course is on the motivation of conservation of energy from a control volume perspective but it is almost exclusively applied in a single input – single output form where the dot product has been implicitly applied and where simplifying "rules" (inflows = negative, outflows = positive) are in place. The dot product is therefore not essential for understanding of the simplest or "practical" cases but is important in the generalized motivation of the topic and in advanced implementations of the subject.

The dot product then reappears in a more advanced and central manner in their Fluid Mechanics class as both a flux concept and for determining components of vectors. It is also, however, used as a tool for re-writing equations of motion utilizing the del operator and other elements from vector calculus. In Fluid Mechanics concepts like the divergence also play a central role in the differential analysis subsections of the course.

Introduction: Mode of Failure Analysis

The research being conducted under the NSF's Course, Curriculum and Laboratory Improvement Program (CCLI) consists of pre-instruction and post-instruction assessment of student capability with support topics and skills (integration, differentiation, dot product, equilibrium conditions, etc.) across the curriculum. Student responses to these assessment questions are analyzed to determine the approach which each student took in addressing the problem and to identify aspects of their thinking process: this is especially important in those problems where the students answered the assessment questions incorrectly. The different methods by which students approach these problems are then be categorized and a catalog of the most common "modes of failure" will be developed.

In this way an institutional profile of student competency in selected background skills across the curriculum can be developed. Student responses on pre- and post- assessments can also be compared both within a class and as students progress through the curriculum over the two years of the study. Aggregate data from this longitudinal study will permit assessment of student growth in these areas and it is anticipated that insights will be gained in the solidification of student knowledge as they progress through their academic career. Long term goals related to this research are the development of tools that will permit an institution to: profile student competency in key content and skills, identify and develop targeted instruction or other remedial efforts to improve student learning, and to have a measure for demonstrating the effectiveness of their remedial/re-instruction efforts.

While the content and skills trajectory research seeks to address curricular level efforts in assessment, in addition to mapping content and skill trajectories on a more detailed level, this research also seeks to identify and categorize the methods of content and skills failure within the trajectory structure. Unlike other efforts to evaluate student learning this project will look at failure of learning points rather than success. This analysis will be used to identify where learning breaks down or where gaps exist in student prerequisite knowledge. It is believed that examining modes of failure (or types of errors) will both inform faculty on the baseline capability of their student clients and guide the creation of remedies specific to these different types of failures.

Background and Relationship to Prior Efforts

As a tool for evaluating student learning within a given curriculum, the mode of failure analysis and the content/skill trajectories proposed are invariably compared to other existing programs and criteria: most notably ABET (Accreditation Board for Engineering and Technology) and SUCCEED (Southeastern University and College Coalition for Engineering Education).

Within the SUCCEED structure two methods of curriculum analysis are provided which take structured and hierarchical views of the curriculum: the Knowledge/Skills Method and the Augmented Syllabus Method. The Knowledge/Skills Method[1] encourages the establishment of a sequential process in which the "big picture" is first sketched in broad strokes and then filled in by detail to look at where in the curriculum specific knowledge and skills are addressed. This work defines three (3) levels of skills and four (4) levels of knowledge to determine concentrations of skills and knowledge so that one can see how courses contribute to overall curriculum requirements. Within this structure, skills are defined as learned capacities (as opposed to content) that are fundamental to engineering and cover broad areas of application (engineering design skills, engineering control skills, problem solving skills, organizational skills). Knowledge is also divided by level with level 1 encompassing both prerequisite knowledge elements, such as physics, calculus, and technical writing, and "core" knowledge elements such as engineering drawing and statics. Main engineering knowledge (level 2) is particular to each engineering discipline and level 3 consists of each level 2 element broken down into basic engineering components similar to items in a course outline or textbook chapter.

With the Knowledge/Skills Method, a matrix for a given curriculum is generated where the rows consist of the courses offered and the skills or knowledge areas are represented in the columns. A mark is placed in a cell of the matrix indicating if a skill or knowledge area is taught in that class. Different marks are used to indicate how strongly a skill or knowledge area is included in the course instruction. These matrices are used to analyze a curriculum to determine how requirements are being met, how the knowledge elements and skills are integrated, or how groups of courses are "time-phased".

The second method of evaluating a department's curriculum proposed by the SUCCEED coalition is the Augmented Syllabus Method[2]. The Augmented Syllabus Method addresses the curriculum at the degree to which goals are being met at a course level. It focuses on topical coverage and maps the path to mastery of a student by defining the level of accomplishment necessary at each stage, the pre-requisite knowledge for each topic and the anticipated use of the topics studied. One tool for use in the Augmented Syllabus method proposed by the SUCCCEED coalition is the use of temporal displays of a curriculum, also referred to as phase diagrams[3]. These phase diagrams seek to map a pathway towards mastery and to identify a progression in the depth of instruction on a topic. This phase diagram approach ascribes a four level mastery scale to measure the expected level of student understanding for various topics. In evaluating a department's curriculum, levels of implementation progress from "no exposure" to qualitative exposure (e.g. concepts) and then to quantitative exposure (e.g. homework problem) and finally to use in design or analysis. For example, a phase diagram for the concept of conservation principles published for Clemson University[3] indicates that students will be exposed to the concept (Level 1) in their Sophomore year and then brought to Level 2 by the end of that year. The junior year is mapped as taking students in at Level 2 and bringing them to Level 3 and so on through their senior year.

ABET's approach to curriculum mapping and assessment is both more proscribed than the SUCCEED outcomes and, at the same time, more dependent on the institution for the manner in which student learning is assessed. The approach contained in the ABET, also differs from Project SUCCEED in the degree to which specific content are included for analysis. Within ABET the program outcomes (PO's) set forth in Criterion 3[4,5] address eleven student objectives, (a) through (k), in which an institution must demonstrate that students have attained success. While ABET requires that there must be an assessment and evaluation process that periodically documents and demonstrates the degree to which the program outcomes are attained, within the ABET structure the methods by which these are addressed is up to each department to develop and pursue. For example, (a) "an ability to apply knowledge of mathematics, science, and engineering" can be assessed through a single unified assessment across the board (in which case upper-level students and lower-level students would take the same test) or through applications represented within a given class. If the individual class approach is used, different courses may have different scales or content being examined. For example, one class may assess a student's Algebra skills while another might measure their ability to manipulate and solve Ordinary Differential Equations. As a result, even though the classes may be at different levels (100-level vs. 300-level) there is no way to institutionally assess student understanding of a given concept.

The concept/skill trajectory approach used in this research seeks to identify the dependence of future concepts on specific skills and content development gained earlier. In this respect, the level of detail is considerably higher than that accomplished by either the Augmented Syllabus or Knowledge/Skills method from SUCCEED. For example, instead of examining the influence of Calculus on a subject or a curriculum, the content/skill trajectory research seeks to determine how a student's competency in a specific topic within Calculus impacts their understanding of other science topics. It also does not presuppose that a student's first exposure to the material leads directly to the advancement to the next level of mastery: student progress may be staggered or even regressive when taken in context of a whole school year or summer vacation. In addition, the trajectory based approach allows for the development of specific remedial efforts to progress students from one level of mastery to another.

Second, these trajectories do not have the same linear, hierarchical structure that the Augmented Syllabus or the Knowledge/Skills methods employ. By developing connections between specific content areas an individual trajectory on its own does not provide structure to a curriculum. Instead, these trajectories support success within the larger goals and display what is present rather than what is desired. This is especially important since at many institutions there are students that engage with the curriculum outside of the planned approach (i.e. seniors who end up taking Chemistry I, students who end up repeating a given class, or transfer students). These trajectories can easily be restructured to represent different approaches to the content or to identify problems.

With regards to the relationship between ABET and the Course and Skill Trajectories approach, research currently being done in response to the ABET EC2000 is largely institution specific and pertains to both the scope which they include in their study and the methods by which they have assessed their own institutional practices. For example, Wagner, et al[6] state their hesitancy in using the FE exam (and their arguments against) as a standardized means of assessment and describe their own development of a multiple-choice test method for assessing ABET outcomes. Wang, et al[7] address the application of the ABET assessment to their own classes by explaining how they used student surveys, quiz scores and test scores to evaluate ABET Outcomes (a) – (k). They use this data to conclude that student proficiency with mathematics is central to success in Dynamics. To evaluate their students the researchers looked at scores on individual problems and assessed math proficiency but the sample size for this study was between 10 and 14 students and the authors do not provide details of assessment or examples of student work. In each of these examples, the student's score on a test question, often evaluated without a rubric, was used as a proxy for their understanding of a subject and the satisfaction of the ABET outcome.

The content and skill trajectory approach differs from the ABET approach because it is designed to operate from an assessment standpoint as opposed to evaluation. With each skill or content topic addressed the goal is the identification as to common reasons for students not achieving success on that given topic and the identification of locations within the curriculum where transfer breaks down.

Mode of Failure Analysis: Identification of Failure Modes

An advance distribution of the NSF CCLI longitudinal study was initiated during the Summer and Fall semesters of 2009. Pre-test surveys covering the dot product, partial derivatives, and static equilibrium were distributed to 4 courses in Mechanical Engineering: Dynamics, Thermodynamics, Fluid Mechanics and Heat Transfer. These classes occupy the mid-section of the suggested program of study for Kettering University and include students with primarily junior and senior standing. During the Summer semester of 2009 three problems were assessed for 43 students in the Fluid Mechanics classes only. During the Fall semester of 2009 three problems were assessed for each of the 179 students included in the study.

Pre-test questions for the Summer distribution (Appendix A) were selected for the preview of the longitudinal survey to cover partial derivatives, the dot product and equivalent force systems. Initial analysis of the equivalent force system pre-test question led to a host of failure modes, too many to be useful, and the question was replaced with a more basic static equilibrium question (Appendix B). Identification and faculty attribution of the failure modes were conducted independently by the PI and co-I after each distribution and a comparison was made to standardize the failure modes. A sub-set of the available data is available for presentation.

In the Summer term of 2009 the observed modes of failure were closely aligned with 18 conflicts observed in the dot product problem (41%) and 4 conflicts observed in the partial differentiation problem (9.3%). For the dot product problem, the majority of the conflicts (9) arose because of similarities between two of the modes of failure and were attributed to differing levels of resolution. Of the remaining 9 conflicts associated with the dot product problem, only two discrepancies (4.6%) did not relate to the 'miscellaneous errors' category and were resolved under further review. In the Fall term of 2009 the failure modes were more clearly identified and only two conflicts (0.55%) arose which needed resolving.

Mode of Failure Analysis: Failure Modes

Data from the Summer 2009 distribution of the mode of failure analysis indicates some very clear trends regarding student baseline performance. On the partial differentiation problem there were only two dominant modes of failure (Figure 2) relating to how students addressed the ancillary terms in the problem: those terms which were to be considered as constant with respect to the derivative operation. The most dominant mode of failure (23%) was where students maintained the ancillary (non-x) terms in their solution: in other words, the ancillary terms were treated as a constant and were not operated on by the differentiation.

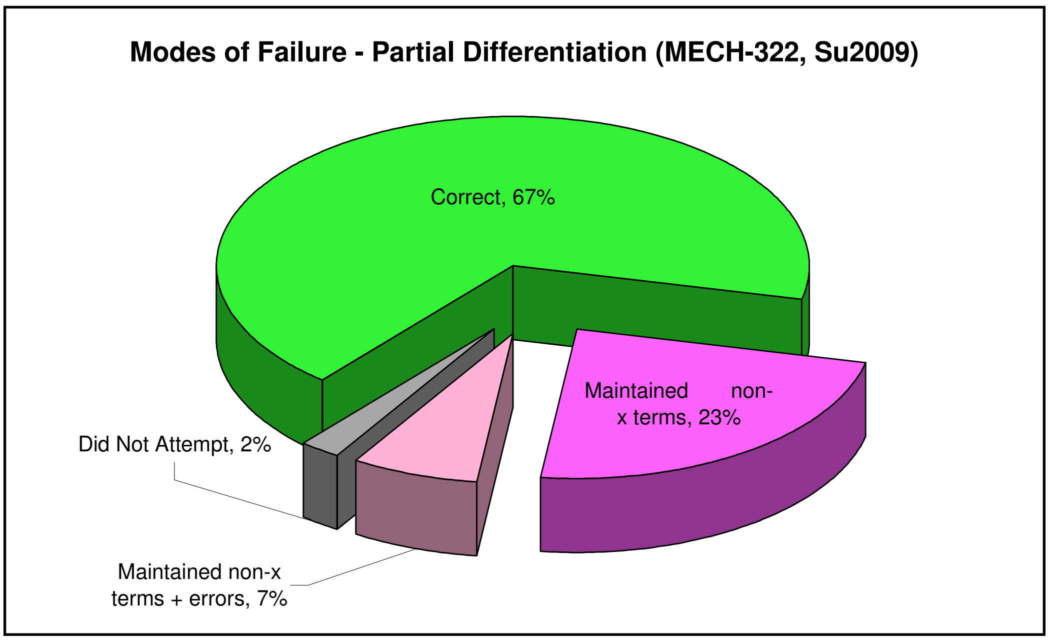

Figure 2: Mode of Failure Analysis – Partial Differentiation (Summer 2009)

With the dot product problem the situation was more complicated (Figure 3). Only 5% of the students taking the pre-test answered the problem correctly. The largest mode of failure, Vector Output, showed students completing the mechanics of the dot product correctly but reporting the answer as a vector. A sizable percentage of the students surveyed did not attempt the problem or produced errors that could not be grouped in to any of the recognized modes or placed in their own mode. The remaining students, 38% of the sample, used one of two methods associated with the cross product to address the problem.

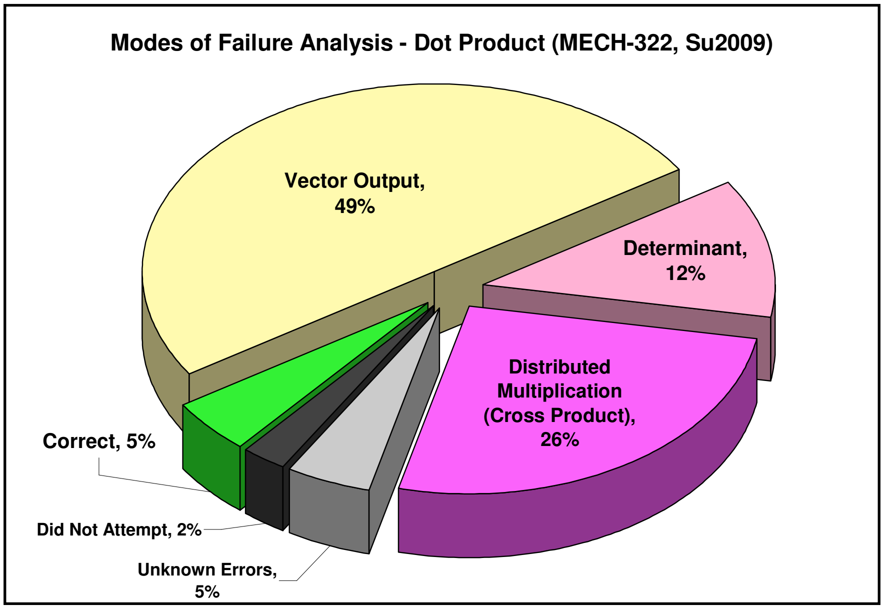

Figure 3: Mode of Failure Analysis – Dot Product (Summer 2009)

In the Fall of 2009 two new modes of failure were observed to be present in the available sample (56 students) for the partial differentiation problem (Figure 4): integration of the function and fractions. The integration mode of failure is where the student integrated the function instead of differentiating the function. The fractions mode of failure consisted of several different and individualized approaches each involving the original function (g(x,t)) being divided by some other product like dg/dx. While each of these new modes occupied a small percentage of the available responses, the overall correct mode was reduced by 8%.

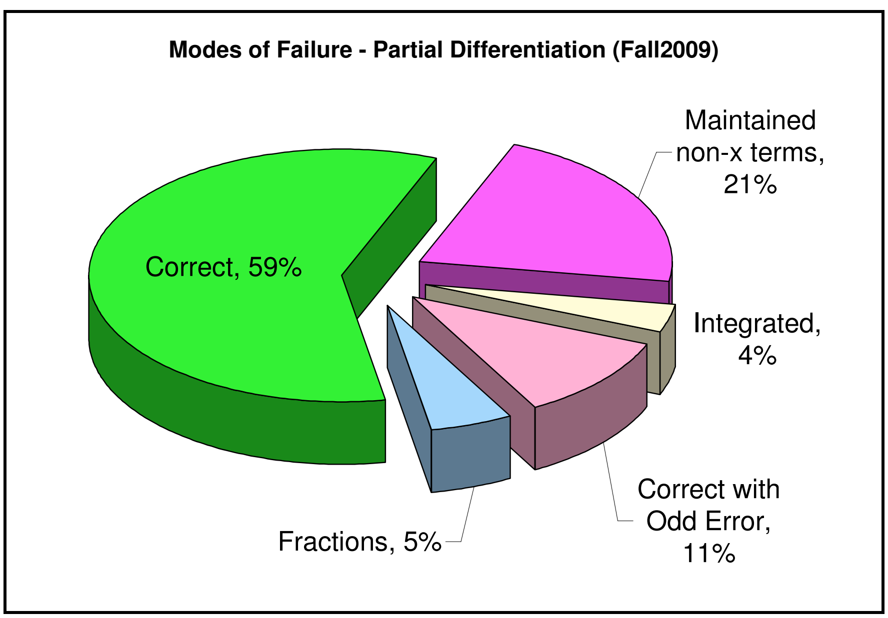

Figure 4: Mode of Failure Analysis – Partial Differentiation (Fall 2009)

With the dot product problem the situation is again more complicated, with more modes being expressed, however, considerable new information is present in the Fall 2009 distribution (Figure 5). With the Fall 2009 distribution of the pre-test questions more students correctly addressed the dot product than during the Summer 2009 distribution. In the Fall of 2009, 25% of the students answering the dot product question got it correct as opposed to only 5% from the Summer 2009. An almost equal number of students represented the dot product output as a vector while getting the mechanics of the dot product correct. Absent from the Fall 2009 distribution is the significant number of students using the distributed multiplication model of the cross product.

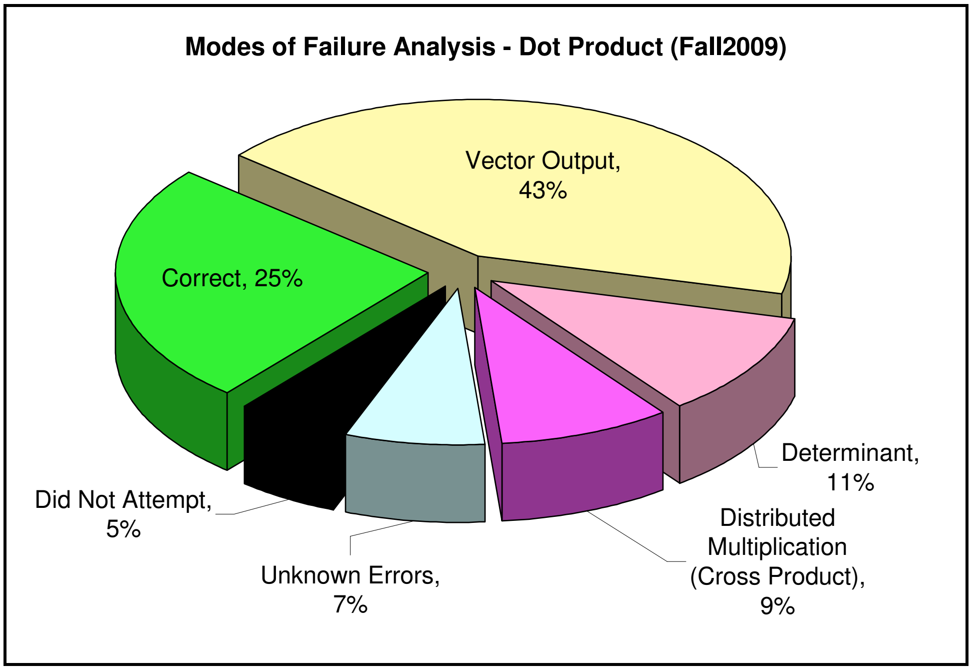

Figure 5: Mode of Failure Analysis – Dot Product (Fall 2009)

Assessment of Mode of Failure Data

The early roll-out of the longitudinal study of pre-requisite knowledge understanding has indicated some limited results that are of interest: a more complete snapshot of student performance and ability will be available at the completion of the two year longitudinal study. In the Fluid Mechanics class the profiles of student ability from term to term were fairly consistent and highlight numerous gaps in the mathematics abilities of the students. An assessment of the content trajectories for the two pre-test questions presented is forthcoming and will perhaps shed light as to why so many students struggled with the content despite having covered the subjects in their Mathematics courses prior to taking Fluid Mechanics.

The consistent issues with the partial differentiation problems, all at the second semester junior year level, indicate that there are some serious and consistent gaps in student pre-requisite knowledge on this topic. In each distribution of the pre-test assessment, only 60% of the students were able to correctly complete the problem. This is surprising because the partial differentiation content trajectory also intersects Thermodynamics, which immediately precedes

Fluid Mechanics in the representative program of study (Figure 1). Of all the errors generated relating to partial differentiation, the maintenance of the non-x terms is the most understandable. Many students recalled that the terms not being operated on were to be considered as constants so their error was most likely due to an incomplete understanding of exactly how these items were to be treated as constant (i.e. constant with respect to the derivative operation on x). A future component of the longitudinal study is the inclusion of student interviews to probe for reasons behind the major modes of failure. Of greatest concern was the occurrence in the Fall term of two new modes of failure: integration and fractions. In one the students integrated the function with respect to the variable x and in the other students introduced a partial fractions-like expansion using the derivative of the function and the function itself. Both of these modes are indicative of a strong lack of understanding of the partial differentiation concept and will have to be monitored closely in future assessments.

Similar conclusions can be drawn from the dot product trajectory. Students here consistently struggle with the content since the pre-tests show that 20% to 40% of the students consistently use vector (cross) product methodologies to address the dot product. It is interesting to note that the cross product methodologies used by the students fall in to two categories: a vector multiplication approach, which is often accompanied by a permutation wheel, and a determinant of a matrix approach. While many of the students were able to get the mechanics correct (Correct and Vector Output modes), the performance on the pre-test indicates that the majority of these students still do not have a firm grasp of the issues underlying vector multiplication. In the Fall, fewer students fell in to the Correct mode of failure than in the Summer, but it is worth noting that the percentage of students who were correct in the mechanics of the dot product, but who represented their answer as a vector (Vector Output mode), increased by a proportional amount. Judgments can be made as to whether the large percentage of students who represent the output of the dot product as a vector is a minor issue (a cognitive slip as opposed to a failure to comprehend), but that is beyond the scope of this study.

Conclusions

At this point in the study information has been shared only with the Mechanical Engineering Department Chair and several interested faculty. After two years of pre-instruction and post-instruction testing it is anticipated that a clearer picture of student performance in various pre-requisite knowledge areas will develop. In addition, the extension of this longitudinal study to additional classes along various trajectories will permit an assessment as to whether these issues are static as students progress through the curriculum or whether there is some dynamics, such as summer vacation or content reduction in the intervening courses, is involved. Regardless, information contained in these modes of failure can be used to develop appropriate and targeted remedial efforts to help the greatest number of students with a minimum of effort.

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Appendix A: Mechanical Engineering Pre-Test (Summer 2009 Distribution)

Summer 2009
0309

Last Name ____________________

Pre-Test: MECH-322 Summer 2009

You have 20 minutes: No calculators for the first <u>two</u> problems

No talking : Show all of your work

1) $g\ (x, y, t) = 3x^2 + 2xt + 3yt^2 + t^3$

find $\frac{\partial g}{\partial x}$

2) $\vec{A} = (4x)\hat{i} + (3xy)\ \hat{j} + (5x)\hat{k}$

$\vec{B} = (3)\hat{i} + (2y)\ \hat{j} + (7z)\hat{k}$

Find $\vec{A} \circ \vec{B}$

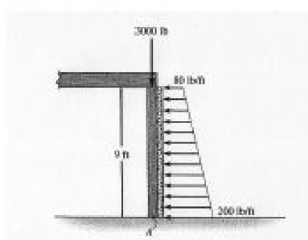
Dot product → scalar inner product

Please indicate by circling YES or NO whether you have a calculator

Calculator?	YES	NO

If you do not have a calculator then set-up the following problems as best you can.

3) The column is used to support the floor which exerts a force on 3,000 lb on the top of the column. The effect of the soil pressure along the column's side is distributed as shown. Replace this loading by an equivalent resultant force and specify where it acts along the column, measured from its base A.



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Appendix B: Mechanical Engineering Pre-Test (Fall 2009 Distribution)

Fall 2009 Last Name ____________________________

Pre-Test - Fall 2009

You have 20 minutes

No talking during the pre-test

No calculators for the first <u>two</u> problems

Please show all of your work.

1) $g(x, y, t) = 3x^2 + 2xt + 3yt^2 + t^3$

find $\frac{\partial g}{\partial x}$

2) Compute the dot product (scalar inner product) of vectors A and B $\left(Find\ \vec{A} \bullet \vec{B}\right)$

$$\vec{A} = (4x)\hat{i} + (3xy)\hat{j} + (5x)\hat{k}$$

$$\vec{B} = (3)\hat{i} + (2y)\hat{j} + (7z)\hat{k}$$

Please indicate by circling YES or NO whether you have a calculator for the pre-test

Calculator?	YES	NO

If you do not have a calculator please set-up the following problem as best you can.

3) A pipe is held in place by a vice. If the bolt exerts a force of 50 lb on the pipe in the direction shown, determine the forces F_A and F_B that the smooth contacts at A and B exert on the pipe.