AC 2007-2695: MODELING COMPRESSIBLE AIR FLOW IN A CHARGING OR DISCHARGING VESSEL AND ASSESSMENT OF POLYTROPIC EXPONENT

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Modeling Compressible Air Flow in a Charging or Discharging Vessel and Assessment of Polytropic Exponent

Abstract

In this work, the classic problem of charging and discharging of a pressurized tank is studied. This experiment allows students to gain a deeper understanding of polytropic processes and compressible flows. The experiment apparatus described in this study allows for direct measurement of the pressure and temperature within the tank, and utilizes a LabView based computerized data acquisition system. To assure accurate measurements of these parameters, a fast-response thermocouple and a high accuracy variable reluctance pressure transducer is employed.

A model was developed to predict the pressure and temperature of the air in the tank during charging and discharging. The model incorporates compressible flow in both sonic and subsonic flow regimes, and models the air as undergoing a general polytropic process. The model was compared with experimental data to empirically determine the polytropic exponent. The values of polytropic exponent obtained through the phenomenological model were compared to those determined by a graphical technique to determine to polytropic exponent. Results show that the polytropic exponent varies with initial pressure and throat area, as well as with time. Thus a constant value for polytropic exponent generally yields an unsatisfactory prediction for temperature and pressure. It is found that a discharge coefficient must be included in the analysis to accurately match the data, due to frictional effects through the throat. Further, the experiment also indicates that heat transfer through the vessel walls plays a major role in the process.

Introduction

The analysis of a pressurized air tank being charged or discharged is one of the most common applications of compressible flow presented in undergraduate fluid mechanics courses. The scenario usually involves an initially pressurized vessel which is suddenly open to a lower outside pressure (such as atmosphere) through a small opening. The goal of this experiment is to predict either the time required to discharge the tank, or the pressure inside the tank, after a specified time. The exercise is useful to students because it is a rather straightforward application of conservation of mass, and introduces the concepts of choked and subsonic flows. Further, the solution integrates aspects of thermodynamics and heat transfer, making for an excellent capstone experiment in thermal sciences.

The most comprehensive solution to the problem is presented by Bober et al.[1] They applied conservation of energy to a discharging tank of air to predict the temperature and pressure inside the tank as a function of time. They analyzed both choked-flow and subsonic regimes, and incorporated the heat transfer through the walls of the tank. The authors modeled the flow through the exit nozzle as isentropic, and the heat transfer as natural convection at the inside and outside wall surfaces. They approximated the heat transfer through the wall as quasi-steady-state. In spite of these simplifications, excellent agreement was found between the model and experimental data. However, the solution itself is complicated, involving the application of conservation of mass to the air, and conservation of energy to the air and the tank wall. The result is three ordinary differential equations to be solved simultaneously for the temperature of the air, the mass of the air, and the temperature of the inside wall of the tank.

A second, simpler approach to the problem avoids modeling the heat transfer explicitly, and instead treats the air inside the tank as undergoing a polytropic process (pv^n = constant). The solution to the problem involves applying conservation of mass to the air inside the tank. This solution has recently been demonstrated by Dutton and Coverdill[2]. They modeled the pressure response of air tanks during either charging from another pressurized tank, or discharging to atmosphere. Transient temperature response was not recorded, but both choked and subsonic regimes were modeled. Two tank volumes were studied, with different nozzle sizes. The authors further limited their analysis to two polytropic processes: isentropic ($n = 1.4$) and isothermal ($n = 1$). They demonstrated that discharging a larger tank (relatively slow discharge process) approached isothermal behavior, while discharging a small tank (relatively fast discharge) approached adiabatic behavior. In contrast, the charging of either tank volume was best approximated by the adiabatic model.

The work of Dutton and Coverdill[2] was limited to isothermal and isentropic processes, which allow for closed-form solution. Although these processes bound the solution, their model does not solve for a general polytropic process. Unfortunately, when modeling the air expansion or compression as a polytropic process, the polytropic exponent n is not typically known a priori. In fact, determining the polytropic exponent is itself a worthy experiment for an undergraduate laboratory.

The polytropic exponent can be determined empirically. California Polytechnic State University has operated a compressible gas tank discharge experiment in various forms for nearly 40 years, the first being designed and built by Alman[3], and later modified by Dobbs[4]. The experiment uses a classic, graphical technique suggested by Hawkins[5] to determine the polytropic exponent directly from a plot of the measured pressure and temperature histories of the air inside the tank. The existence of an empirical technique for obtaining the polytropic exponent provides a means for comparison with a phenomenological model, as well as a means of gaining further physical insight into the process. Moreover, the comparison of these two techniques provides an opportunity to explore a different pedagogy to this classic fluid mechanics experiment.

In this work, the authors develop a model for predicting the pressure and temperature of air charging into or discharging out of a tank. The model incorporates compressible flow in both choked and subsonic flow, and models the air as a general polytropic process of power n. The model is applied to two scenarios: (a) an initially pressurized tank discharged to atmosphere, and (b) an initially evacuated tank being charged from atmosphere. The polytropic exponent, n, is found empirically by matching the model to the data. The results of this method are compared to the graphical technique for calculating the polytropic exponent. The difficulties in determining this exponent, and the challenges of modeling the system without advance knowledge of n, makes this experiment challenging for the undergraduate laboratory, and a pedagogical approach is suggested, along with alternate approaches to facilitate student learning outcomes.

Experimental Facility

A schematic diagram of the tank is presented in Figure 1. The tank is constructed from 20 cm ID (approximately 8 inch) schedule 40 steel pipe, with welded end plates, and constructed with a gasket seal in the middle to allow access to the inside. The tank is pressurized from a compressed air source through an inlet valve. Alternately, a vacuum pump is connected to the valve to evacuate the tank. The tank discharges to or charges from the atmosphere via one of three ports, connected to the opposite end of the tank and controlled by ball valves. The exterior end of each valve is connected to a threaded plug, with a hole machined through it to form a throat. Each throat diameter was chosen to ensure that the throat area is the smallest area in the exit line; that is, to ensure that the throat is where the flow will be choked.

The pressure inside the tank is measured using a Validyne DP-15 variable reluctance pressure transducer, which is connected via a carrier-demodulator to a LabView-based PC data acquisition system. The transducer is calibrated in the laboratory to within ± 3 kPa (approximately 0.05% of full scale). Temperature is measured with a type T micro-thermocouple probe made by Paul Beckman Company. The probe has a junction of approximately 0.05 mm (0.002 in), and has an estimated time constant of 0.02s for the conditions of this experiment. The thermocouple, accurate to ± 1ºC, is shielded using a perforated mylar tube, which was constructed to allow air to pass freely but shield the junction fro thermal radiation. A photograph of the experimental facility is presented in Figure 2.

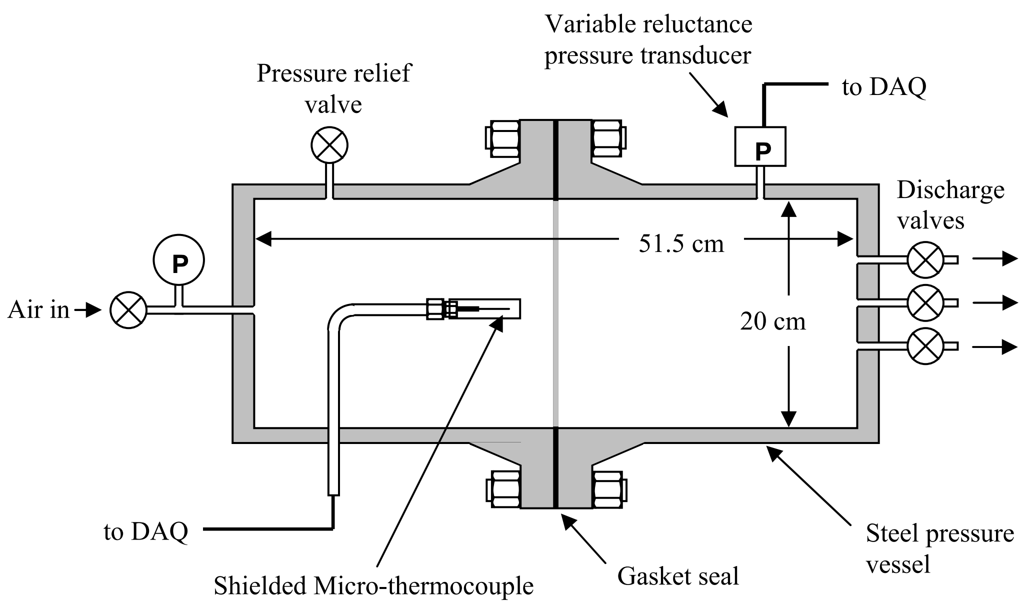


Figure 1. Schematic diagram of air discharge tank and hardware.

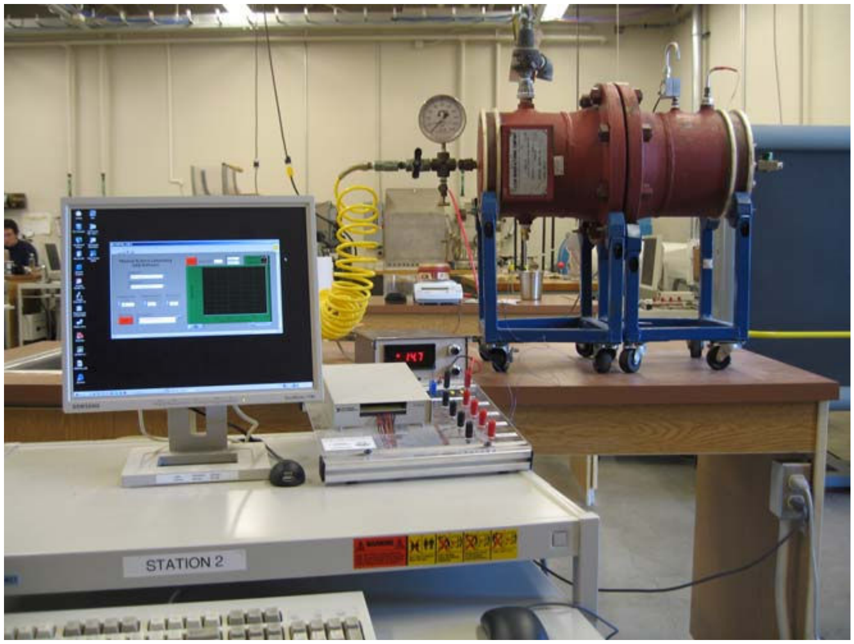


Figure 2. Photograph of facility with computer and data acquisition system.

Empirical Measurement of Polytropic Constant (Graphical Approach)

A direct, experimental method for determining the polytropic constant is suggested by Hawkins[5]. The polytropic relationship,

$$pv^n = p\rho^{-n} = \text{constant}\ , \tag{1}$$

where p is pressure, v is specific volume, and ρ is density, can be combined with the Ideal Gas Law, $pv = RT$, to obtain

$$pT^{n/(n-1)} = \text{constant}\ . \tag{2}$$

This relationship can be linearized by taking the natural logarithm of both sides of the equation, producing

$$\ln p + \frac{n}{n-1}\ln T = \text{constant}\ . \tag{3}$$

Thus, plotting the natural logarithm of the experimentally obtained pressure and temperature (ln(p) against ln(T)) should yield a linear relationship, with slope equal to n/(n-1).

An alternate derivation is to evaluate Eqn. (2) between two states 1 and 2 to obtain

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{n/(n-1)} \quad . \tag{4}$$

Taking the natural logarithm of both sides and rearranging,

$$\frac{n}{n-1} = \frac{\ln(p_2 / p_1)}{\ln(T_2 / T_1)} \ , \tag{5}$$

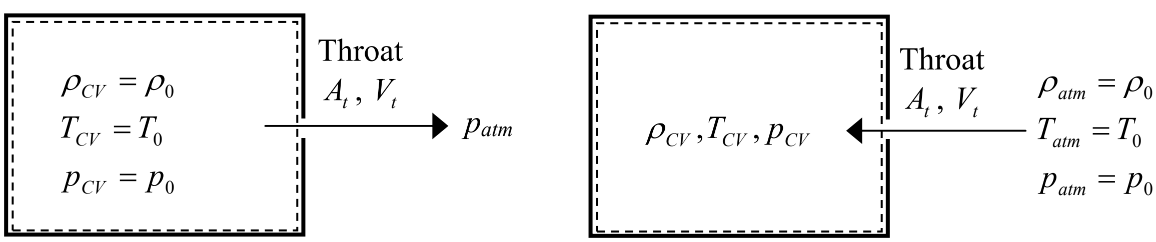
which, by identity, can be written as

$$\frac{n}{n-1} = \frac{\ln(p_2) - \ln(p_1)}{\ln(T_2) - \ln(T_1)} \quad . \tag{6}$$

Equation (6) again demonstrates that the term $n/(n\text{-}1)$ represents the slope of the relationship between $\ln(p)$ and $\ln(T)$.

Development of Phenomenological Model

A model for the charging or discharging process of a vessel filled with air is applied to the control volumes depicted in Fig. 3. The air is assumed to behave as an ideal gas, and the flow through the thoat (exit, in the case of discharging tank) will be approximated as isentropic. However, because of the possibility of heat transfer to or from the surroundings, the air inside the tank is assumed to undergo a more general polytropic process, as shown in Eqn. (1). The model will also allow for the polytropic constant n to vary with time, as will be considered later in this work. Allowing for a general, time-varying polytropic exponent n precludes a closed-form solution; therefore a numerical solution is developed.



(a) Discharging to atmosphere

(b) Charging from atmosphere

Figure 3. Control volumes for analysis of (a) a pressurized tank, discharging to atmosphere, and (b) an initially evacuated tank.

Conservation of mass applied to either control volume of Fig. 3 is

$$\frac{dm_{CV}}{dt} = \pm \dot{m}_t \quad , \tag{7}$$

where m_{CV} is the mass inside a tank of volume $\mathcal{V}$, which can be expressed as $\rho\mathcal{V}$. The term $\dot{m}_t$ refers to the mass flow rate at the throat; a positive value represents a charging process, and a negative value represents a discharging process. Assuming uniform properties within the tank, a rigid control volume, and uniform properties along the throat surface, Eqn. (7) can be written as

$$\frac{d\rho_{CV}}{dt} = \pm \frac{\rho_t A_t V_t}{\mathcal{V}} \quad . \tag{8}$$

For either discharging or charging process, the assumption if isentropic flow through the throat gives the familiar relation between density at the throat to stagnation conditions[6],

$$\rho_t = \rho_0 \left[1 + \frac{\gamma - 1}{2} M_e^2 \right]^{-1/(\gamma-1)} \quad , \tag{9}$$

where γ is the specific heat ratio for isentropic flow of air, and M_e is the Mach number at the throat. In the above, ρ_0 is the stagnation density; in the discharging process, the air inside the tank is assumed to be at stagnation conditions. The velocity at the throat is given by

$$V_t = M_t \left(\gamma R T_t \right)^{1/2} \quad , \tag{10}$$

with R being the gas constant for air. The absolute temperature at the throat T_t is related to stagnation conditions by

$$T_t = T_0 \left(1 - \frac{\gamma - 1}{2} M_t^2 \right)^{-1} \quad . \tag{11}$$

Substituting Eqns. (9) through (11) into Eqn. (8) yields

$$\frac{d\rho_{CV}}{dt} = \pm \frac{\rho_0 A_t M_t (\gamma R T_0)^{1/2}}{\mathcal{V}} \left(1 + \frac{\gamma - 1}{2} M_t^2 \right)^{-(1/2)(\gamma+1)/(\gamma-1)} \quad , \tag{12}$$

Utilizing a finite-difference approximation, Eqn. (12) can be estimated as

$$\rho_{CV,i+1} = \rho_{CV,i} \pm \frac{\rho_{0,i} A_t M_{t,i} (\gamma R T_{0,i})^{1/2}}{\mathcal{V}} \left(1 + \frac{\gamma - 1}{2} M_{t,i}^2 \right)^{-(1/2)(\gamma+1)/(\gamma-1)} \Delta t \quad , \tag{13}$$

thus allowing density of the air in the tank to be computed at time t_{i+1} from knowledge of properties at time t_i.

Modeling of the charging/discharging process proceeds as follows. The tank volume, throat area, initial tank pressure and temperature, and outside air pressure and temperature are given. The Ideal Gas Law is used to compute the initial air density in the tank. The throat Mach number depends on flow conditions:

i. *Choked flow* ($p_{atm} / p_{CV} < 0.528$ for discharging process, $p_{CV} / p_{atm} < 0.528$ for charging):

$$M_{t,i} = 1$$

ii. *Subsonic Flow* ($p_{atm} / p_{CV} > 0.528$ for discharging process, $p_{CV} / p_{atm} > 0.528$ for charging):

$$M_{t,i} = \left\{ \frac{2}{\gamma - 1} \left[\left(\frac{p_{0,i}}{p_b} \right)^{(\gamma-1)/\gamma} - 1 \right] \right\}^{1/2} , \tag{14}$$

where $p_b = p_{atm}$ for the discharging process, and $p_b = p_{CV,i}$ for the charging process.

Once the tank air density at time step $i+1$ has been computed using Eqn. (13), the polytropic relationship, Eqn. (1), is modified to determine the tank pressure at that time step,

$$p_{CV,i+1} = p_{CV,i} \left(\frac{\rho_{CV,i+1}}{\rho_{CV,i}} \right)^{n_i} , \tag{15}$$

where n_i is the instantaneous value of the polytropic exponent, allowed to vary with time in the model. Finally, the tank temperature $T_{0,i+1}$ can be predicted using the Ideal Gas Law,

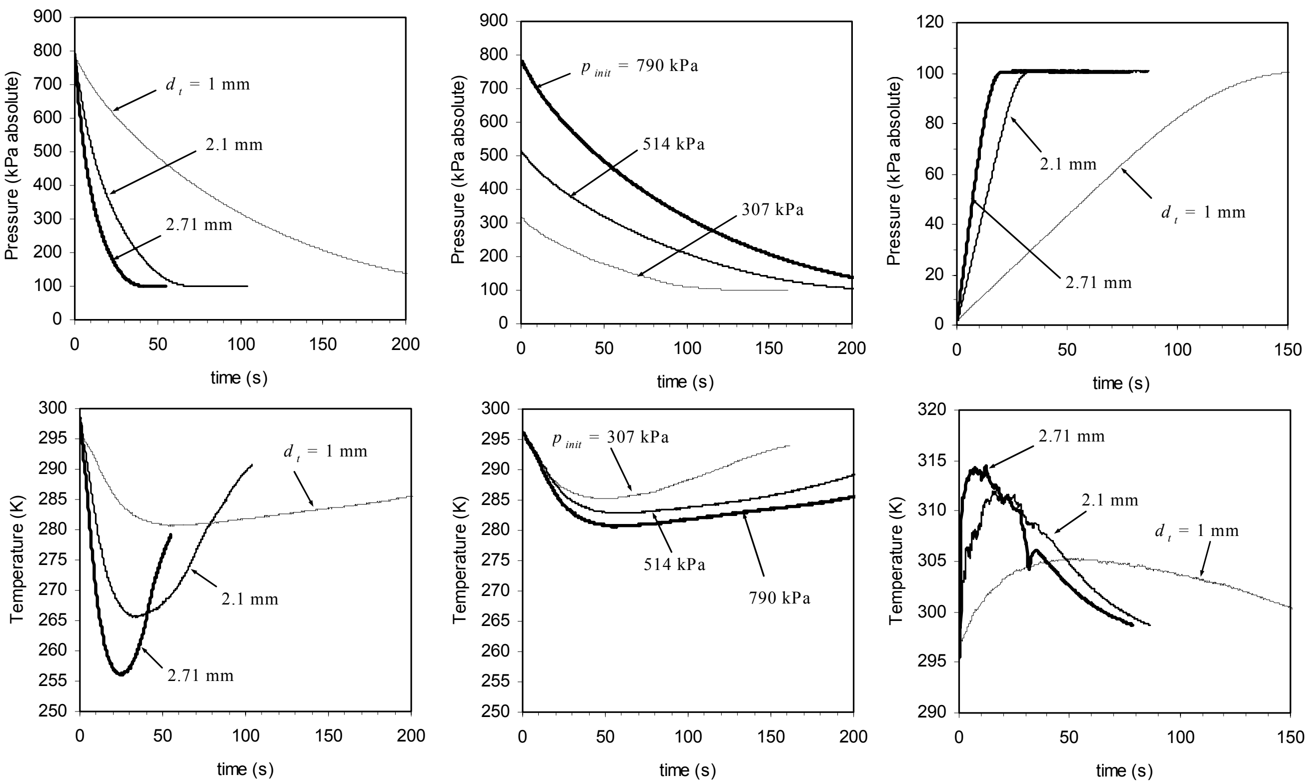
$$T_{CV,i+1} = \frac{p_{CV,i+1}}{\rho_{CV,i+1} R} . \tag{16}$$

Results

Figure 4 depicts pressure and temperature histories obtained from the facility for a variety of throat sizes and initial pressures. Figure 4(a) compares pressures and temperatures measured for an initial pressure of 790 kPa abs (100 psig), with different exit diameters: d_t = 1, 2.1, and 2.71 mm. As expected, the pressure decreases more rapidly for a larger exit diameter. Correspondingly, the temperature drops more rapidly, and to a lower minimum value. The presence of heat transfer is also indicated by the temperature data of the 2.1 mm and 2.71 mm exits, which show a rapid temperature recovery after the tank has discharged. Moreover, the slope of the temperature during recovery is higher for the 2.71 mm exit. That same condition also yields a lower minimum temperature, which would result in a higher potential for heat transfer.

Figure 4(b) compares pressure and temperature plots for the 1 mm throat for three initial pressures, 790 kPa abs (100 psig), 514 kPa abs (60 psig), and 307 kPa abs (30 psig). Again, the tank takes longer to empty when pressurized to a higher initial pressure. In addition, the rate of pressure decrease is higher, which is consistent with the fact that higher tank pressures yield higher exit mass flow rates. Interestingly, the temperature histories are very similar for all three conditions, having nearly identical initial slopes, and reaching similar minimum temperatures.

Finally, Figure 4(c) compares the pressures and temperatures measured during a charging process. As before, the pressure data show the expected trend that a tank with a smaller throat takes longer to pressurize. The temperature data also show that a larger throat results in a larger, and faster, rise in tank temperature. Again, the data for the 1 mm throat show such a slow charging process that the temperature reaches a maximum, and then begins recovering before the end of the charging process.



(a) p_{init} = 790 kPa abs (b) d_t = 1 mm (c) p_{init} = 3.7 kPa abs

Figure 4. Pressure and temperature profiles for selected experimental conditions: (a) varying throat diameter at initial pressure 790 kPa absolute (100 psig); (b) varying initial pressure for throat diameter 0.1 mm; (c) varying throat diameter at initial pressure 3.7 kPa absolute (-14 psig).

Graphical Method for Obtaining Polytropic Exponent

Figure 5 illustrates the graphical method of obtaining the polytropic exponent for an initial pressure of 790 kPa abs (100 psig), and a throat diameter of d_t = 2.1 mm. The plot of ln(p) as a function of ln(T) is fairly linear during the first 15 seconds of the event, or approximately the first quarter of the discharge time. For all the discharging processes studied, a linear curve fit was typically appropriate for the first quarter to the first half of the event. The deviation from linear behavior in these plots suggests that the polytropic exponent n varies with time. This conclusion seems reasonable, since the air temperature varies greatly during discharge, which would affect the heat transfer. In contrast, the plots of ln(p) as a function of ln(T) for the charging events remain fairly linear throughout the entire process, suggesting that the polytropic exponent, and the influence of heat transfer, is fairly constant.

It is instructive to examine how the graphically determined polytropic exponent varies over the conditions studied in this work. Figure 6 plots n versus initial tank pressure for the various throat diameters used in the facility. The first trend observed is that, for the same initial pressure of 790 kPa abs (100 psig), the measured polytropic exponent decreases with increasing throat diameter. This makes sense when considered with the pressure and temperature responses of Fig. 4(a): larger throat diameters result in higher exit mass flow rates. The higher mass flow rate results in a larger drop in air temperature, which provides more potential for heat transfer. Thus the polytropic exponent decreases.

On the other hand, the measured polytropic exponent was not affected significantly by throat size when the tank was initially evacuated. For all throats, the polytropic exponent was close to an isothermal value, approximately 1.01 to 1.02. In fact, the temperature rise during charging is small – on the order of about 20 K. The reason for the nearly isothermal behavior is likely due to the fact that the air entering the tank is coming from constant temperature, ambient conditions.

A trend that particularly interesting is that, for a tank discharging through a particular throat, n appears to increase with initial pressure. This trend seems unexpected, since higher initial pressures yield higher mass flow rates, which should result in lower system temperatures. In turn, the lower temperatures would promote heat transfer, which should make the polytropic exponent decrease. However, Johnson et al.[7] show that the discharge coefficient for sonic nozzles increases with increasing back pressure. Higher discharge coefficients correspond to higher flow efficiency, which tends toward isentropic flow. This may explain the trend seen in the measured polytropic exponent with increasing initial pressure.

Phenomenological Model

The results of the previous discussion reveal that the polytropic exponent is not known a priori, and from a student's perspective is difficult to predict. One way to approach this difficulty is to empirically choose the polytropic constant which best matches the model to the experimental data.

The phenomenological model is compared with experimental data for a throat diameter of 2.1 mm, discharging from an initial pressure of 790 kPa (100 psig), in Figure 7. The model was performed over a range of polytropic exponents, including n = 1.17, the value obtained using the graphical method. Initially, both the pressure and temperature data are modeled closely for n = 1.17. This is consistent with the graphical technique for determining n, which was also effective only during the initial portion of the event. Thereafter, the model predicts a more rapid decline in pressure than measured. In contrast, the model predicts temperatures that are significantly lower than measured.

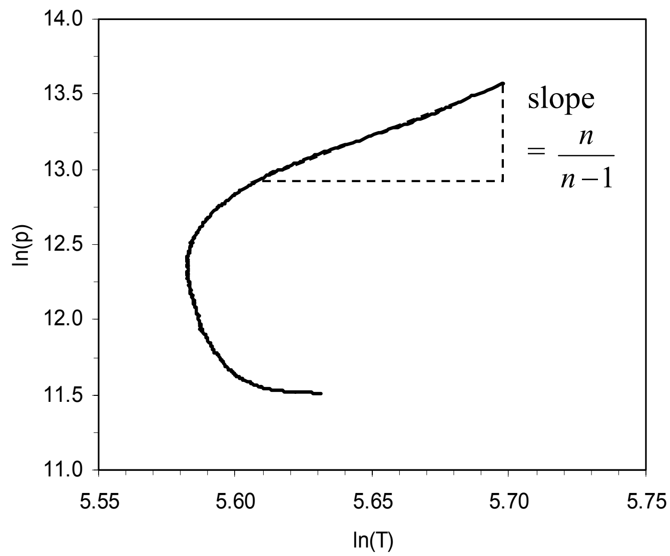


Figure 5. Graphical method for obtaining polytropic exponent n during initial tank discharge, for initial pressure 790 kPa abs (100 psig), d_t = 2.1 mm. The linear portion of the plot spans approximately the first 15 seconds of discharge; the slope shown yields $n \cong 1.17$.

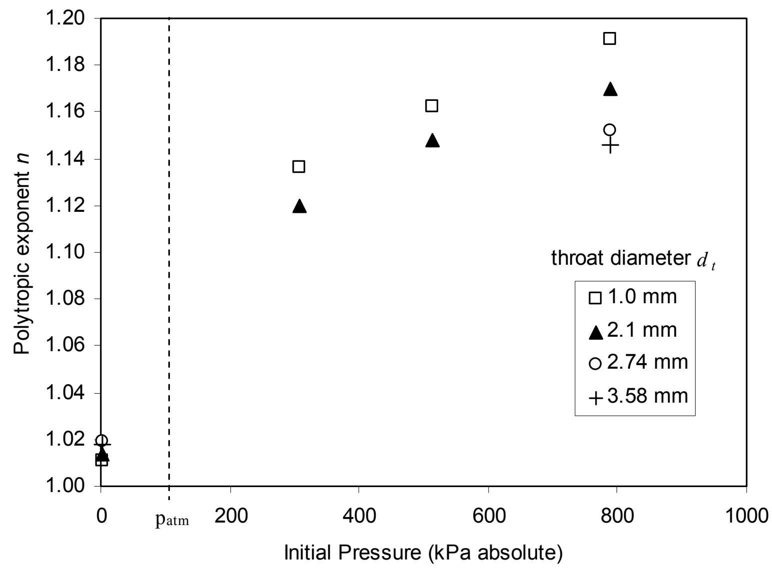
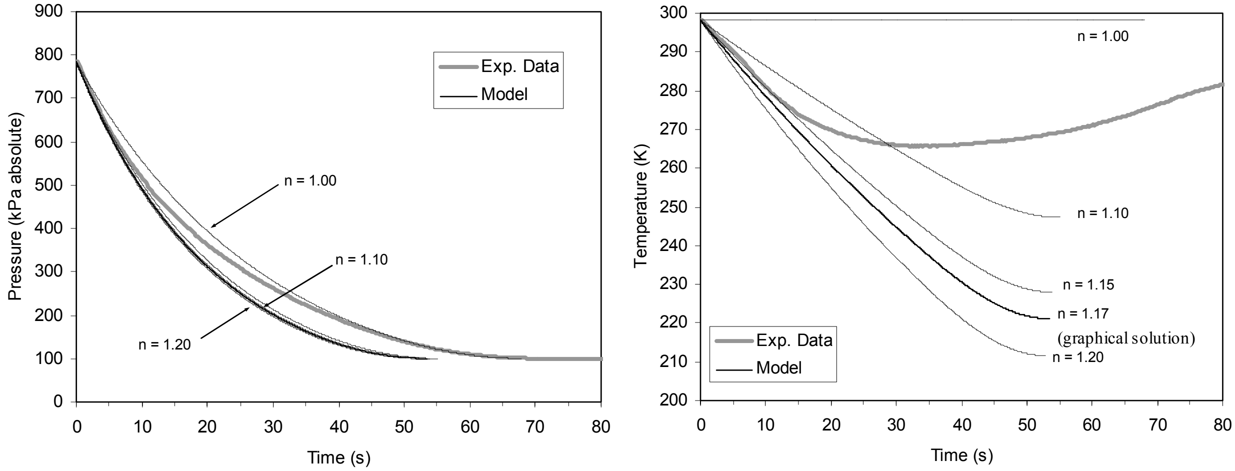


Figure 6. Measured polytropic exponent n (using graphical method) for varying initial vessel pressures and throat sizes. Uncertainties in measured polytropic exponent is approximately ±0.01 for all values.

Examining the model results obtained for various values of n, it is observed that the experimental pressure curve is more closely fit by an $n = 1.17$ model initially, but the isothermal model appears to match the experimental data more closely near the end of the process. Although the modeled pressure is only slightly sensitive to the value of n, the temperature prediction is extremely sensitive to n, and it appears that no single value of n satisfactorily describes the experimental temperature.



(a) Pressure response　　　　　(b) Temperature response

Figure 7. Comparison of model with experimental data for p_{init} = 790 kPa absolute (100 psig) and d_e = 2.1 mm. Includes parametric study of the effect of varying polytropic constant.

One explanation for the discrepancy between the predicted and measured temperatures is that the relative influence of heat transfer changes with time, since the temperature of the air in the vessel varies. This effect would make the polytropic exponent a transitory value. To gain insight into this effect, the data obtained for p_{init} = 790 kPa (100 psig) and d_t = 2.1 mm is reexamined. In Figure 8, the graphical method is used to calculate n, but this time n is found from the instantaneous slope of $\ln(p)$ vs. $\ln(T)$ data. The result is a measure of n as a function of time, and Figure 8 shows the value of n decreases. Indeed, if this time-varying polytropic exponent is incorporated into the model, the resulting semi-empirical model more closely approximates the behavior of the system. This result is demonstrated in Figure 9.

The semi-empirical model, with varying polytropic exponent, still under-predicts the pressure and temperature response slightly. This discrepancy could be due to the effects of friction. Following Johnson[7], a discharge coefficient could be incorporated into the mass flow rate; discharge coefficients for airflow through sonic nozzles range from approximately 0.94 to 0.97, and increases with Reynolds number. To examine whether friction alone could explain the discrepancy in the model, a discharge coefficient was added to the modified model. A discharge coefficient value was selected that best matched the experimental data. A value of 0.92 produced excellent results, suggesting that the presence of friction could explain the discrepancy between the modified model and the data.

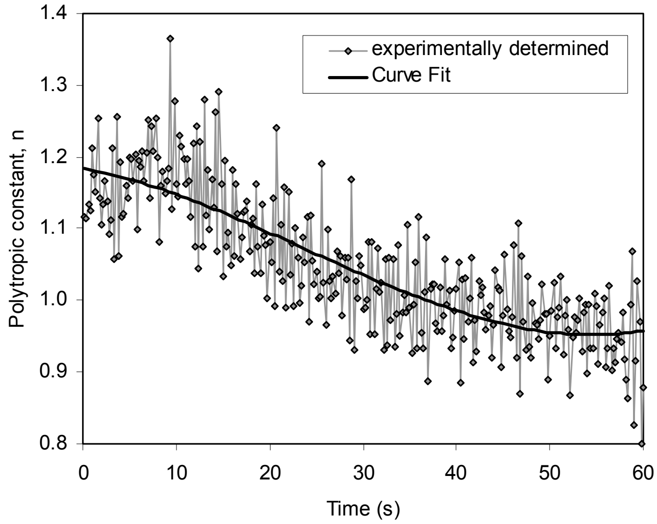
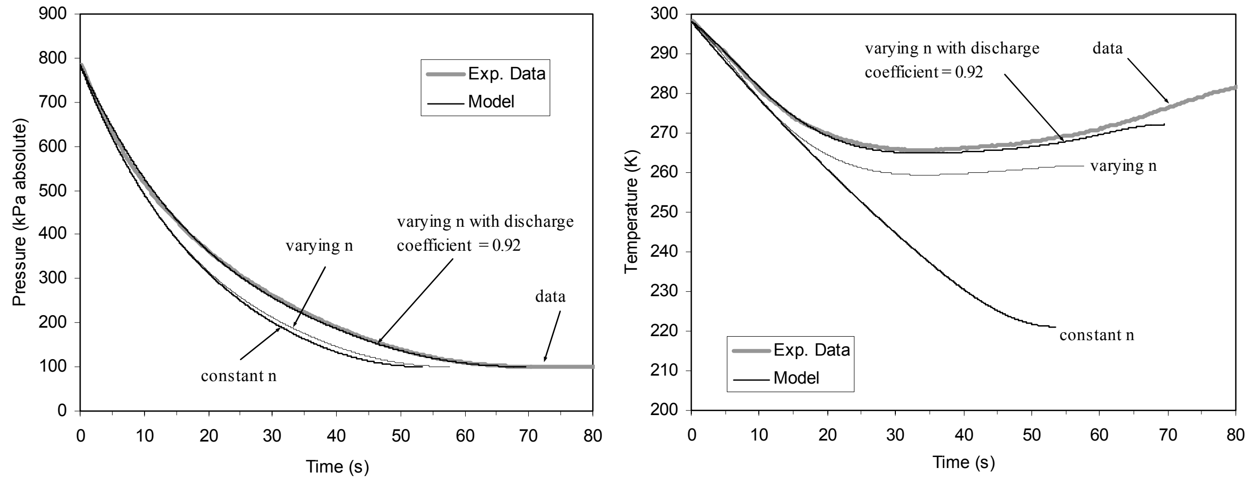


Figure 8. Variation of instantaneous polytropic constant n, determined by taking local slope of ln(p) vs. ln(T) curve at each time step (Eqns. 5 or 6). Data depicted above was from condition p_{init} = 790 kPa absolute (100 psig) and d_e = 2.1 mm.

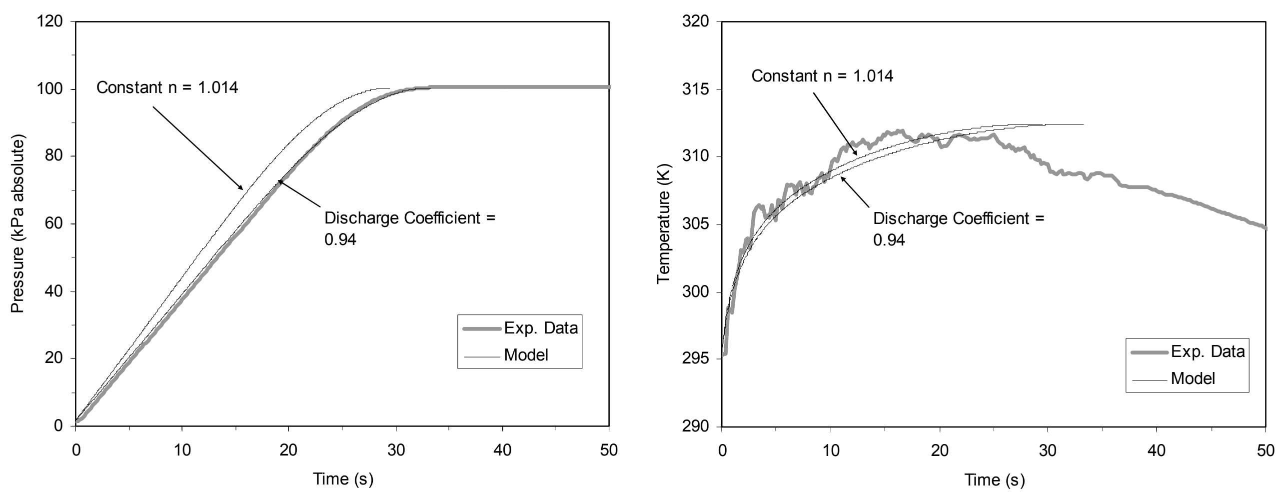
Finally, the model is applied to the charging process, illustrated in Figure 10. In the charging scenarios, a constant value on n was satisfactory, with the value of n determined by the graphical analysis of the experimental data. The use of a constant value for n is supported by a plot of the instantaneous n value as a function of time, shown in Figure 11. The model slightly overpredicts the pressure response, but incorporating a discharge coefficient of 0.94 provided an excellent fit to the data. The fact that the discharge coefficient would be higher under these conditions than the conditions of Figure 9 makes sense, since the inlet velocity during charging from ambient pressure is lower than the exit velocity for a back pressure of 790 kPa, and hence the effect of friction should be correspondingly lower.

It should be noted that incorporating an empirically-determined, time-varying polytropic exponent, and fitting a discharge coefficient to the model to approximate frictional losses, is not fundamentally rigorous. These modifications to the model are merely used to explore and demonstrate the influence of varying heat transfer and friction.



(a) Pressure response (b) Temperature response

Figure 9. Modified model, showing effect of time-varying polytropic constant, and assumed discharge coefficient. Model compared with experimental data for p_{init} = 790 kPa absolute (100 psig) and d_t = 2.1 mm.



(a) Pressure response (b) Temperature response

Figure 10. Comparison of model with experimental data for p_{init} = 3.7 kPa absolute (-14 psig) and d_t = 2.1 mm. Model incorporated a polytropic constant value of n = 1.014.

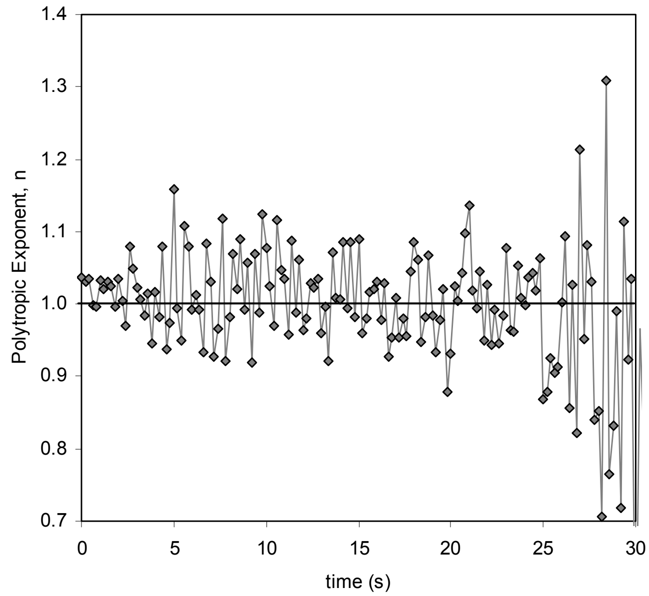


Figure 11. Variation of instantaneous polytropic constant n, when tank is initially evacuated: p_{init} = 3.7 kPa absolute (-14 psig) and d_t = 2.1 mm.

Conclusions

In this work, a model was developed to predict the pressure and temperature of air in a tank during charging and discharging. The model incorporates compressible flow in both choked and subsonic flow regimes, and models the air as undergoing a general polytropic process. The model was compared with experimental data to empirically determine the polytropic exponent. The values of polytropic exponent obtained through the phenomenological model were compared to those determined by a graphical technique to determine to polytropic exponent.

The experimental data, modeling effort, and comparison of techniques to determine the polytropic exponent resulted in several observations. For the discharging processes studied in this work:

- The polytropic exponent, determined using the graphical technique, was found to be a function of operating conditions in the vessel. Larger throat diameters yielded lower values of n, while higher initial pressures yielded higher values of n.
- The graphical technique for calculating the polytropic exponent based on temperature and pressure histories was suitable only for the initial period of discharge. In fact, the polytropic exponent was found to vary with time. This is consistent with the fact that the polytropic exponent n is an indicator of the influence of heat transfer which, for an uninsulated vessel, varies with time as the conditions inside the tank vary.
- The phenomenological model underpredicts the pressure and temperature histories when a constant value of n was used in the model. The discrepancy may be explained,

however, by the variation in polytropic exponent (and hence, heat transfer) with time, as well as the presence of flow friction.

Examining the initially evacuated vessel being charged from ambient air:

- The polytropic process was found to be nearly isothermal, with *n* being approximately constant with time. The graphical method could therefore be used to determine *n* over nearly the entire the process, not just initially.
- The phenomenological model matched the experimental temperature and pressure data very well, and the small discrepancy between model and data could be explained by the existence of flow friction.

The goal of this work is to develop an experiment and solution approach that is adaptable to the undergraduate laboratory. The difficulty of applying this model is that the polytropic exponent is not known, or easily determined, at the outset of the analysis. Moreover, the solution is highly sensitive to the value of *n*, and since *n* varies with time, the model results – particularly the temperature variation – does not satisfactorily compare with experimental data without a detailed modeling of both heat transfer and flow friction. However, for the initially evacuated tank being charged with ambient air, the polytropic exponent is approximately constant, and modeling yields a pressure and temperature prediction that compares well with measurements without a great deal of additional analysis.

The following pedagogical approach is recommended. During the laboratory experience, the students are initially asked to predict the time it will take to charge an evacuated pressure vessel or discharge a pressurized vessel using their knowledge of compressible fluid mechanics and thermodynamics. The students then perform the experiment of charging or discharging the pressure vessel and record the data during this process. Comparisons are made between the predicted flow rates and the experimental data. The students will find that regardless of their original assumptions for the process (isentropic or otherwise), their prediction will not adequately represent the collected data. As a result of this finding, the students will apply two methods for more accurately predicting the flow: a numerical approach that allows for both the sonic and subsonic regimes to be accurately modeled as a polytropic process, and a graphical method to directly determine the polytropic exponent. Both methods must be matched to the experimental data in order to determine the correct polytropic exponent. Even using these methods, it is found that a discharge coefficient must be included in the analysis to accurately match the data, due to frictional effects through the orifice. Further, the experiment also indicates that heat transfer through the vessel walls plays a major role in the process.

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References

[1] Bober, W., Heinzman, J.D., and Laptchak, D.M., “Non-Steady Nozzle Flow: An Educational Experiment,” *Int. J. Mechanical Engineering Education*, Vol. 7, No. 3, pp. 131-136, 1979.

[2] Dutton, J.C. and Coverdill, R.E., “Experiments to Study the Gaseous Discharge and Filling of Vessels,” *Int. J. Engineering Education*, Vol. 13, No. 2, pp. 123-134, 1997.

[3] Alman, Donald, “The Emptying Process of a Pressure Vessel,” Senior Project 69-0319, California Polytechnic State University, San Luis Obispo, 1969.

[4] Dodds, Robert, “Polytropic Blowdown Test Stand,” Senior Project 78-0545, California Polytechnic State University, San Luis Obispo, 1978.

[5] Hawkins, G.A., *Thermodynamics*, John Wiley and Sons, New York, 1946.

[6] White, F.M., *Fluid Mechanics,* 5th Edition, McGraw Hill, New York, 2003.

[7] Johnson, A.N., Espina, P.I., Mattingly, G.E., Wright, J.D., and Merkle, C.L., “Numerical Characterization of the Discharge Coefficient in Critical Nozzles,” NCSL Workshop and Symposium, Session 4E, pp. 407-422, 1998.