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Modeling of Pipe Flow and Observation of Laminar-Turbulent Transition in Smooth Pipes

Abstract

An undergraduate experiment has been developed to measure the mass flow rate of water exiting a constant-head tank through a tube. There are three tubes that can be investigated independently, with each tube having different entrance geometry. The scenario is a common problem found in undergraduate fluid mechanics textbooks, and loosely based on a classic experiment by Osborne Reynolds. The design of the experiment, and the pedagogical structure, provide a diverse set of educational objectives to be attained. Students are directed not only to develop a model to predict the mass flow rate of the exiting water, but also to predict the accuracy of the resulting model using uncertainty analysis. The experiment is designed to obtain laminar-turbulent transition, and the students use their model to measure the upper-limit transition Reynolds number. The result is an experiment that demonstrates a fundamental application of fluid mechanic – pipe flow theory. Further, the experiment promotes the role and importance of uncertainty analysis in engineering experimentation, and provides an avenue for students to conceptualize laminar and turbulent flow and the physical significance of the Reynolds number. A detailed description of the experiment is presented, along with the development of the pipe flow model and associated uncertainty analysis. The turbulence-based model compares well to the experimental data in the turbulent regime, and the data predictably deviates during transition. The Reynolds number of transition was demonstrated to vary from the accepted value of 2300, depending on tube inlet geometry. Finally, experimentally determined values of pipe friction factor were plotted against Reynolds number, and found to closely match the classic Moody Diagram. A pedagogical approach is developed along with the experiment facility, and is also described in detail.

Introduction

The development of an undergraduate engineering laboratory is challenging, because a laboratory serves two sometimes distinct sets of goals. The first are generally classroom-specific goals: to demonstrate physical phenomena developed in the classroom, to compare theoretical models to experimental data, and to develop an approach to analyzing and designing complex engineering systems. The second goals are laboratory-specific: to introduce methods of measurement and instrumentation, to collect, organize, analyze, and interpret data, and to develop an approach to engineering experimentation. Woven into these goals is the objective of promoting teamwork, communication skills (written and oral), and at the same time achieving learning objectives like those of Bloom’s Taxonomy.

The difficulty of attaining such a diverse set of objectives can lead to some goals being underemphasized – often, ironically, the laboratory-specific goals. Frequently, the complexity of an experiment, and the sheer amount of data collected, focuses student attention on “crunching data.” As a result, the goal of the students often becomes the mere completion of the assignment, instead of any thoughtful analysis of the results. Furthermore, some aspects of experimentation are neglected; an example of this is the topic of uncertainty analysis, which is of fundamental importance to engineering experimentation and well-suited to the laboratory. Numerous studies,
such as those of Allie et al.\textsuperscript{2} and Deardorff\textsuperscript{3}, demonstrate that uncertainty analysis is given little thought in engineering education in general.

In this work, the authors present an experiment developed for a fluid mechanics laboratory. The goal is to develop a simple experiment to demonstrate pipe flow and the effect of major and minor frictional losses, and to solidify a physical understanding of laminar and turbulent flow and the role of the Reynolds number. However, it was also desired to incorporate uncertainty analysis, and to demonstrate its role in interpreting the results of an experiment. At the same time, the authors sought to achieve an open-ended approach to the experiment, in order to promote a sense of “discovery” and to encourage thoughtful analysis of the data.

The experiment presented in this work is simple in concept and operation. Students are asked to predict the mass flow rate of water exiting a constant-head tank, through each of three long tubes, each with different entrance geometry. The problem, illustrated in Figure 1, is commonly found in undergraduate fluid mechanics textbooks, and the experiment itself is loosely based on the pioneering work of Osborne Reynolds\textsuperscript{4}. Reynolds used a similar apparatus to examine the structure of laminar and turbulent pipe flows. The analysis of this experiment a fairly straightforward application of pipe flow theory, except that in addition to predicting the mass flow rate of the water, students are directed to predict how accurately their model will compare with experimental data. Predicting the accuracy of their model, as well as the accuracy of the measurements, requires uncertainty analysis. The results of uncertainty analysis are used to identify the major causes of uncertainty, and to interpret whether the differences between the predicted and measured mass flow rates are “important.”

![Figure 1. Schematic of water draining out of a constant-head tank through an exit tube.](image)

In addition to developing a model, analyzing its accuracy, and then verifying the model by comparison with experimental data, the students can also apply the model to discover something new. Specifically, the experiment has been designed so that, if the tank is allowed to empty, at certain water heights in the tank the flow that exits the tube undergoes laminar-turbulent transition. The students record the height of the water when transition occurs, and can apply their model to determine the velocity at that moment. From this information the students
can calculate the transition Reynolds number, a value which they discover is not necessarily equal to the expected value of 2300. The observation of transition, and the analysis of the Reynolds number, reveals a physical picture of laminar and turbulent flow, as well as reinforces the physical interpretation of the Reynolds number. So what begins as a simple analysis of a pipe flow system reveals much more about the nature of fluid flow, the Reynolds number, and the role and importance of uncertainty analysis in engineering experimentation.

**Experimental Facility**

A schematic diagram of the facility is presented in Figure 2. The system consists of a clear acrylic tank that drains water through one of three 76.2 cm long, 0.670 cm ID, clear plastic tubes. The tank is constructed from 1.27-cm-thick (half-inch-thick) acrylic, with inside dimensions 40.6×45.7×50.8 cm (16×18×20 in) and a volume of approximately 94 liters. Each exit tube has a different entrance geometry: rounded, square, and reentrant. The tube entrance geometries are depicted in Figure 3. The tubes are connected to the tank by O-ring compression seals attached directly to the tank. The water drains through these tubes to a 130-liter reservoir, and returned to the tank using a submersible pump. A photograph of the facility is presented in Figure 4.

Figure 2. Schematic diagram of facility.
Figure 3. Diagram of drain tube inlet geometries and seals.

Figure 4. Photograph of facility.
The water height in the tank is held constant through one of four standpipes, which drain back to the reservoir. The height of the tank (and thus the range of water heights) was chosen carefully so that, depending on the water height, laminar, transition, or turbulent flow could be achieved in the exit tubes. The standpipes and fill line are connected to the tank via PVC bulkhead fittings. The standpipes are attached with threaded pipe fittings, and additional standpipes of various heights are available to achieve water heights from about 2.5 cm (1 in) to 38 cm (15 in) above the centerline of the exit tubes. During operation, one of four water heights can be selected by opening one of the standpipe valves. The tallest standpipe has no valve, so that if the remaining valves are closed, the last standpipe automatically drains, preventing the tank from overflowing. The water height in the tank is measured with a clear plastic scale attached to the side of the tank, as seen in the photograph of Figure 4.

Inside the reservoir, water from the pump is directed through one of two paths, which provide different mass flow rates. In the first path, the water is directed through an orifice, which reduces the mass flow rate, and a check valve that prevents backflow when the pump is turned off. This path is used during operation of the experiment, when data is being collected. The second path bypasses the orifice, used when higher flow is needed to quickly fill the tank at the beginning of the experiment. The water enters the tank through a J-shaped pipe, directing the inlet water to the bottom of the tank, which reduces disturbance to the water. To help visualize operation of the system, the plumbing between the tank and reservoir is constructed of clear, schedule 40 PVC (1.90 cm, or 0.75 inch ID).

Analysis

1) Development of Pipe Flow Model

Analysis is performed on the control volume depicted in Figure 1. State 1 is taken to be to free surface of the water in the tank, which is maintained at a constant height \( h \) above the drain tube centerline; state 2 is defined at the tube exit. Assuming steady, incompressible flow, Conservation of Energy yields the pipe flow equation,

\[
\left( \frac{p}{\rho} + \alpha \frac{\bar{V}^2}{2} + g z \right)_1 - \left( \frac{p}{\rho} + \alpha \frac{\bar{V}^2}{2} + g z \right)_2 = h_{i,T},
\]

where \( p \) is the pressure at the control volume surface, \( \rho \) is the density of the water, \( \alpha \) is the kinetic energy coefficient of the flow at each location, \( \bar{V} \) is the mean velocity, \( g \) is gravity, \( z \) is the elevation at each location, and \( h_{i,T} \) is the total head loss across the pipe. The above equation can be simplified by recognizing that the pressure at surfaces 1 and 2 are both atmospheric and therefore cancel. The elevation \( z_1 \) is defined as zero in the chosen control volume, and the velocity at surface 1 is assumed to be negligible. Therefore Eq. (1) reduces to

\[
gh - \alpha \frac{\bar{V}^2}{2} = h_{i,T},
\]
where \( z_2 = h \). The kinetic energy coefficient, \( \alpha \), depends on the flow regime: for laminar flow, \( \alpha \) is equal to 2, while for turbulent flow \( \alpha \) ranges from approximately 1.03 to 1.08, depending on the shape of the velocity profile\(^5\).

Two effects contribute to the total head loss, \( h_{l,t} \): the first is the major loss due to pipe friction,

\[
h_{l,M} = f \frac{L V^2}{D} \quad ,
\]

where \( f \) is the pipe friction factor. In the laminar regime, \( f \) is given by

\[
f_{\text{lam}} = \frac{64}{\text{Re}_D} \quad ,
\]

where \( \text{Re}_D \) is the Reynolds number based on inside pipe diameter. For the turbulent regime, the Blasius correlation has been chosen, assuming smooth pipe:

\[
f_{\text{turb}} = \frac{0.3164}{\text{Re}_D^{1.25}} \quad .
\]

In both flow regimes, developing-length effects have been neglected as a first approximation. The second head loss is a minor loss due to the entrance of the pipe,

\[
h_{l,m} = K \frac{V^2}{2} \quad .
\]

For the reentrant tube, \( K \) is approximately 0.78; for the square-edged tube, \( K \approx 0.5 \). For the rounded entrance, \( K \) depends on the radius of the rounded edge, \( r \), relative to the inside diameter of the tube, \( D \). For this entrance, \( r/D \) is approximately 0.2, and therefore \( K \approx 0.04 \) (following Fox and McDonald\(^5\)).

Substituting Eqs. (3) and (6) into Eq. (2) and solving for the exit velocity yields

\[
V_2 = \left[ \frac{2gh}{\alpha + K + f (L/D)} \right]^{0.5} \quad ,
\]

from which the mass flow rate can be calculated using

\[
m_{\text{predicted}} = \rho V_2 A_2 = \rho V_2 \pi D^2 / 4 \quad .
\]

Once the geometry and working fluid are selected, the solution to Equations (7) and (8) requires two pieces of information: the water height, \( h \), and the flow regime (laminar or turbulent). Since
the friction factor depends on Reynolds number, and the Reynolds number itself is a function of
the exit velocity, Eq. (7) in its present form cannot be solved explicitly for \( \bar{V}_2 \). However, it can
be solved simply by iteration or with an equation solver.

2) Simplified Model

A simplified model could be developed by ignoring the effects of friction in Eq. (2). Without friction, the head loss \( h_{i,T} \) is zero, and the flow is uniform (hence \( \alpha = 1 \)). Eq. (2) reduces
to Bernoulli’s equation, and the solution yields

\[
\bar{V}_2 = \sqrt{2gh},
\]

which is also called Torricelli’s Law. The purpose of the simplified model is as a first, order-of-
magnitude solution, as well as to examine the relative effect of fluid friction by comparison with
the advanced model.

3) Measured Mass Flow Rate

The analytical model will be compared to measured values of mass flow rate. The
simplest method to measure mass flow rate is to collect water from the exiting stream over a
period of time, and measure the resulting mass (the “bucket-stopwatch” method). The mass flow
rate is determined by

\[
m_{\text{measured}} = \frac{m_f - m_i}{\Delta t},
\]

where \( m_i \) is the initial mass of the (empty) bucket, \( m_f \) is the final (filled) bucket, and \( \Delta t \) is the
elapsed time.

4) Uncertainty Analysis

Prior to the experiment, an uncertainty analysis is performed in order to assess the
precision to be expected from (1) the predicted mass flow rate, and (2) the measured mass flow
rate. It is important to perform this analysis prior to the experiment (and establish the technique
as important to students) for several reasons. First, uncertainty analysis is a tool to assess the
precision of the results of an experiment; it is a design tool that is used to identify sources of
uncertainty in order to design a more precise experiment (albeit in this case the experiment has
already been built). Second, uncertainty analysis establishes a baseline for comparing the
measurements to the model and judging the model’s accuracy. For example, any differences
between a model’s prediction and experiment results are generally not ascribable to any physical
effect or faulty assumption if the differences are within the uncertainty bounds of either quantity.
Moreover, when uncertainty analysis is not performed, it is difficult to determine whether the
differences are “important.”
Uncertainty analysis is first performed on the analytical model. Approximating the uncertainties in the quantities of Eq. (7) to be independent, a general expression for the uncertainty in a function \( R(x_1, x_2, \ldots, x_k) \) is given by Coleman and Steele,

\[
U_R = \left[ \left( \frac{\partial R}{\partial x_1} U_{x_1} \right)^2 + \left( \frac{\partial R}{\partial x_2} U_{x_2} \right)^2 + \ldots + \left( \frac{\partial R}{\partial x_k} U_{x_k} \right)^2 \right]^{1/2},
\]

where \( x_1, x_2, \ldots, x_k \) are variables contributing uncertainties to the function \( R \). In the case of Eq. (7), uncertainty analysis takes the form

\[
\bar{V}_2 = \bar{V}_z(g, h, \alpha, K, f, L, D)
\]

and substitution of Eq. (7) into Eq. (11) yields a resulting value for the uncertainty, \( U_{\bar{V}_z} \), in the exit velocity. The uncertainty in the predicted mass flow rate can be determined by performing the same analysis on Eq. (8), and the same analysis is used to determine the uncertainty in the measured mass flow rate, Eq. (10).

The estimated uncertainties in the constitutive measurements used to determine the uncertainty in the predicted and measured mass flow rates are listed in Table 1. The uncertainty in gravity (about 0.3 \%) is a generally accepted value, which is primarily due to the dependency of gravity on latitude. Uncertainties in the quantities \( h, L \), and \( m_i, m_f \) were estimated from the resolutions of the respective measuring scales, while the elapsed time \( \Delta t \) was estimated from the reaction time in operating the stopwatch. The inside diameter, \( D \), of the tubes were measured by filling each tube with water and measuring the resulting volume. The resulting equation,

\[
D = \left( \frac{4m}{\rho \pi L} \right)^{1/2},
\]

resulted in a mean tube diameter, which was of more interest than local diameter. Uncertainty analysis yielded a measured diameter within \( \pm 0.003 \) cm. The uncertainty in water density, \( \rho \), accounts for variation with temperature, and the uncertainty in friction factor was selected as \( \pm 15\% \), a generally accepted value for the Blausius correlation. Finally, uncertainties in the entrance loss coefficient, \( K \), and the kinetic energy coefficient, \( \alpha \), were chosen as reasonable approximations.

The resulting uncertainties in the predicted and measured mass flow rates vary from condition to condition. However, to get an initial estimate, a “typical” water height of 25 cm is used. Assuming turbulent flow, uncertainty analysis shows that the predicted mass flow rate is on the order of \( \pm 6\% \), with the uncertainty in the friction factor playing the largest role. On the other hand, the measured mass flow rate when \( h \) was about 25 cm yields an uncertainty of approximately \( \pm 1\% \). The bucket-stopwatch is accurate to this level because the bucket was
allowed to fill for a long period of time -- about 200 s -- so that about 7 kg of water was collected. Thus the relative effect of the uncertainties is reduced.

Table 1. Estimated measurement uncertainties.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimated Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravity, ( g ) (9.81 m/s(^2))</td>
<td>± 0.03 m/s(^2)</td>
</tr>
<tr>
<td>water height, ( h )</td>
<td>± 0.16 cm</td>
</tr>
<tr>
<td>pipe length, ( L )</td>
<td>± 0.5 cm</td>
</tr>
<tr>
<td>mass of water and bucket, ( m_i, m_f )</td>
<td>± 0.005 kg</td>
</tr>
<tr>
<td>filling time, ( t )</td>
<td>± 1 s</td>
</tr>
<tr>
<td>pipe inside diameter, ( D )</td>
<td>± 0.003 cm</td>
</tr>
<tr>
<td>water density, ( \rho )</td>
<td>± 0.2 %</td>
</tr>
<tr>
<td>friction factor, ( f )</td>
<td>± 15 %</td>
</tr>
<tr>
<td>entrance loss coefficient, ( K )</td>
<td>± 20 %</td>
</tr>
<tr>
<td>kinetic energy coefficient, ( \alpha )</td>
<td>± 10 %</td>
</tr>
</tbody>
</table>

Results

Figure 5 depicts the measured mass flow rate of water for each exit tube as a function of tank water height. Also depicted on the plot is the mass flow rate predicted by Torricelli’s Law (Eq. 9), which neglects friction. The data show that, at a given water height, the tube with the rounded entrance experiences the highest mass flow rate, followed by the square-entrance tube, then the reentrant tube. This result is expected, since the rounded entrance imparts the least friction loss, followed by the square entrance and reentrant geometries. Further, it is seen that, although the model based on Torricelli’s Law captures the general shape of the data, the model over-predicts the mass flow rate by approximately 100 percent, predominantly due to the assumption of frictionless flow.

Figures 6 and 7 compare the generalized mass flow rate model (including friction) with the measured mass flow rates obtained from the square entrance and reentrant geometries, respectively. The dashed lines represent the uncertainty bounds on the model; the uncertainty bands on the experimental data are not shown, but are within the size of the symbols. In both graphs, the model compares well to the data, even when transition flow was observed (this occurred at the lowest water height for both tube geometries, as shown in the figures). It should be noted that the model matches the data well, despite the fact that developing-length effects were ignored.

The transition between laminar and turbulent flow was readily identifiable, and marked by intermittent oscillations between the two flow regimes. Figure 8 illustrates the difference between the regimes as observed at the exit of the rounded-entrance tube. At the same water height in the tank, the laminar flow velocity is higher than that of turbulent flow, since there is less frictional resistance in laminar flow. Reynolds\(^4\) referred to these laminar-turbulent oscillations as “flashes;” later Prandtl and Tietjens\(^7\) identified structures within the flow that they
called “puffs” and “slugs,” which are unstable turbulent regions in the flow. The nature of these structures remains a topic of advanced study in the field.8, 9

Figure 5. Measured mass flow rate as a function of tank water height for three tube entrances, and comparison with model based on Torricelli’s law.

Figure 6. Predicted and measured mass flow rate as a function of water height for square entrance. Dashed lines represent uncertainty bounds of model.
Figure 7. Predicted and measured mass flow rate as a function of water height for reentrant tube entrance. Dashed lines represent uncertainty bounds of model.

Figure 8. Photographs of (a) laminar and (b) turbulent flow emanating from the rounded-entrance tube at a height of approximately 24 cm above the centerline of the tube.
Figure 9 compares the model to the mass flow rates measured from the rounded-entrance tube. Several features are apparent from the graph. First, transition was observed over a wider range of water heights (and mass flow rates) than for the other two tubes. The wider range of mass flow rates undergoing transition can be attributed to the fact that the rounded entrance disturbs the flow much less than the square or reentrant, thus promoting transition at higher values of mass flow rate. Second, the model slightly under-predicts the mass flow rate in the turbulent flow regime. However, the data obtained in the transitional flow regime deviates significantly from the shape of the turbulent-flow model (the transition data appears to “bulge” upward). The fact that the mass flow rate during transition is higher than that expected by the model can be explained by the fact that the model assumes turbulent flow, and the transition flow is an intermittent oscillation between laminar and turbulent flow. Since the mass flow rate increases when the flow becomes laminar, the average mass flow rate is increased. In fact, if the model is adjusted to assume laminar flow, it is seen that the measured mass flow rate during transition is between the laminar and turbulent models. This result is depicted in Figure 9.

![Figure 9](image-url)

Figure 9. Predicted and measured mass flow rate as a function of water height for rounded entrance. Dashed lines represent uncertainty bounds of model.

The fact that the model under-predicts the turbulent data in Figure 9 is interesting, and to develop an explanation for this discrepancy, the authors reexamined a fundamental assumption in the model. The presence of a developing length was neglected. Following White,

\[
\frac{L_c}{D} \approx 4.4 \operatorname{Re}_{D}^{1/6},
\]

(13)
is a generally accepted correlation for the developing length, $L_e$, for turbulent flow. For the Reynolds numbers encountered in this study, the ratio $L_e/D$ ranges from approximately 16 to 20. The overall $L/D$ ratio for the exit tubes is approximately 114, so the developing length takes up less than 20% of the overall length. Furthermore, correlations presented by Benedict\textsuperscript{11} for the “apparent” friction factor in the developing region predict higher values of friction factor than those for fully developed flow. Including the apparent friction factor in the model would yield a higher overall friction factor; however, the data in Figure 9 suggest that the overall friction factor in the tube is lower. One issue with the “apparent” friction factor correlations presented by Benedict is that they assume the flow to be turbulent from the entrance. However, the behavior of the flow in the developing region of a tube is quite complicated, as discussed above.

The discrepancy between predicted and measured mass flow rates might be attributable to an additional behavior that was observed during the experiment. The velocity of the flow exiting the rounded-entrance tube was seen to oscillate slightly from higher to lower velocity, even as the flow exiting the tube remained turbulent. It is interesting to note that these oscillations preceded the onset of laminar flow at the exit of the tube. In fact, when the flow at the exit became laminar, it did so at the peak of one of those turbulent velocity oscillations. It would seem that the flow consisted of a laminar length, followed by a turbulent length in the tube. If this is the case, oscillations in the length of the laminar region would explain the velocity oscillations, as well as the increased mass flow rate that was observed. Moreover, the fact that the model more closely matches the data obtained from the square-entrance and reentrant tubes could be that the entrances disturb the flow enough to promote turbulence, and minimize the laminar behavior. Indeed, the small oscillations in the turbulent flow were not observed in those tubes.

Despite the observations and reasoning described above, the cause for the slight discrepancy between the data and the model could be due to the model itself. The model utilized the Blasius correlation for the friction factor (Eq. 5), but its estimated uncertainty (about ±15%) is the largest contributor to the uncertainty in the model. The choice of friction factor correlation, therefore, can have a large impact on the predicted mass flow rate. To illustrate this effect, the model was modified to incorporate the Nikuradse model for friction factor\textsuperscript{11},

$$f = 0.00332 + 0.221 \text{Re}^{-0.237}.$$ \hspace{1cm} (14)

This correlation gives slightly lower values of $f$, and thus higher predicted mass flow rates from the model. The resulting model then compares more favorably to the rounded-entrance data, as can be seen in Figure 10. It should be noted that no improvement in modeling can be claimed for the data of the square-entrance and reentrant geometries (Figures 11 and 12, respectively).
Figure 10. Comparison of models based on Nikuradse and Blasius friction factor, and laminar flow correlation, with data for rounded entrance geometry.

Figure 11. Comparison of models based on Nikuradse and Blasius friction factor correlations with data for square entrance geometry.
As a further educational tool, the results of this experiment can be used to replicate the classic Moody Diagram, which relates friction factor to Reynolds number. Rearranging Eq. (7), the experimental friction factor can be calculated from the data as

$$f = \left( \frac{L}{D} \right) \left( \frac{2gh}{V^2} - \alpha - K \right), \quad \text{(15)}$$

where $\alpha$ was chosen as 1.03 for turbulent flow, as before. For transition flow, a value of 1.5 was assumed as a rough approximation, as the flow is, on average, somewhere between laminar and turbulent regimes. (A better method for estimating $\alpha$ during transition would be to record the time that the flow resides in either regime, and defining an intermittency factor$^{12}$, a weighted average for $\alpha$ could then be calculated. This approach, however, was not performed in this study.) The resulting values of friction factor are plotted in Figure 13, along with the models based on Blasius, Nikuradse, and laminar flow. The results are quite good, and demonstrate a significant shift in the friction factor toward the laminar model at the same time that transition is observed. In addition, at the lowest value of Reynolds number, when all tubes experienced transition, the measured friction factors appear to converge.

One final, but important observation is the value of the transition Reynolds number observed for the three tubes. For the square-entrance and reentrant geometries, transition was observed at a Reynolds number of approximately 2500, close to the accepted value of 2300. For
the rounded-entrance tube, however, transition occurred up to \( \text{Re} \approx 6000 \). This discrepancy is an important result, because students often believe that the transition Reynolds number is a fixed value. Of course, transition to turbulence can be delayed to even higher Reynolds numbers by reducing the disturbances in the flow further.

The visual observation of transition and the measured transition Reynolds number provide an opportunity for students to develop a physical concept of the Reynolds number and its role in laminar and turbulent flows. The oscillating laminar and turbulent flow that occurs during transition can be explained partly by the difference in wall friction between the two regimes, and by the physical definition of the Reynolds number as the ratio of inertia forces to viscous forces in the fluid. At the same water level, a laminar flow will produce more mass flow than a turbulent flow, because the wall friction is lower in laminar flow. At higher flow velocities, though, disturbances in the flow can overtake the viscous forces in the fluid to cause turbulent flow. If the flow transitions to turbulent, however, the higher wall friction reduces the velocity in the flow, and the flow can transition back to laminar as the viscous forces again dominate.

What is confusing to many students is the apparent contradiction that in laminar flow, viscous forces dominate, yet friction is reduced. This experiment provides an opportunity to clarify that the Reynolds number pertains to inertia and viscous forces at a fluid particle level, while the wall friction is a macroscale effect.

![Graph showing friction factor comparison](image)

Figure 13. Measured friction factor, and comparison to models of Nikuradse, Blasius, and laminar flow. The empty symbols (□, ○, △) refer to measurements taken during transition.
The pedagogy for this experiment is modeled after the work of Herb et al.\textsuperscript{13}, who applied the Learning Cycle of Kolb\textsuperscript{14} and the 4MAT system of McCarthy\textsuperscript{15} to engineering instruction. Specifically, the authors have adopted an approach to the experiment that can be described as follows:

1. \textit{Modeling the System}. The students are directed to examine the system, make appropriate assumptions, and then develop a model to predict the mass flow rate of the water from each of the exit tubes as a function of water height in the tank. This process is done with a minimum of instructor input, for as open-ended an analysis as possible. However, the students are directed to create two models for the system: a simplified model, neglecting fluid friction, and an advanced model that includes friction effects. The purpose of the simplified model is as a first, order-of-magnitude solution, as well as to examine the relative effect of fluid friction by comparison with the advanced model. At the same time, the students examine how the experiment will be operated and how the mass flow rate will be measured (it is accomplished by weighing an amount of water collected in a bucket over a measured period of time).

2. \textit{Predicting the Accuracy of the Model}. The students are directed to perform uncertainty analysis on the predicted and measured mass flow rate, and evaluate how accurate they expect their results to be.

3. \textit{Testing the Model}. The experiment is run, and data are collected. The data are used to plot the mass flow rate of exiting water as a function of tank water height. In practice, only four water heights are studied, which were carefully chosen so the resulting flow is turbulent. Four data points is sufficient to demonstrate the efficacy of the models while reducing the data collection to a reasonable level of effort.

4. \textit{Determining the Transition Reynolds Number}. With the model for mass flow rate successfully verified, the model can be used as a tool to “discover” the transition Reynolds number. By shutting off the fill pump and allowing the tank to drain slowly, students can observe when the turbulent flow first begins its transition from turbulent to laminar flow. Because the water level is decreasing, the mass flow rate cannot be measured with the “bucket-stopwatch” method. Instead, students record the water height when transition begins, and use their model to calculate the average mass flow rate. (The tank drains slowly enough that transient effects are negligible.) Thus, having developed a model, they can now apply it to answer a new question.

\textbf{Conclusions}

The experiment developed in this work presents a simple problem to undergraduate fluid mechanics students, which is to predict the mass flow rate of water exiting a constant-head tank. The scenario, however, provides physical insight and promotes an engineering approach to
experimentation that is not easily revealed in the classroom. Comparison of the model to the experiment reveals several important effects:

- The model, an application of pipe flow theory, compares well to the mass flow rates measured from the square-entrance and reentrant geometry tubes. The model slightly under-predicts turbulent mass flow rate from the rounded-entrance tube. Observations suggest that the difference might be attributed to the possibility of a laminar developing length; however, the variation in empirical friction factor models was shown to be a major factor as well.
- During transition, particularly for the rounded-entrance tube, the mass flow rate deviates significantly from the turbulence-based pipe flow model. The mass flow rate is higher than predicted, which is expected given that the transition flow actually alternates between laminar and turbulent flow.
- The experiment was designed to result in transition flow at certain water tank heights. Transition was marked by alternating laminar and turbulent flow, which is readily observable. This feature allows students to test their understanding of the nature of the Reynolds number and the structure of laminar and turbulent flows.
- The transition Reynolds number was approximately 2500 for the square-entrance and reentrant tubes, close to the expected value of 2300. However, the rounded entrance tube yielded a value of approximately 6000, demonstrating that by reducing disturbances, a flow can attain higher velocities (and hence higher Reynolds numbers) before transitioning to turbulence.
- Further analysis of the data gives measured values of friction factor, which, plotted against Reynolds number, give a chart that closely matches the classic Moody Diagram.

Along with achieving the technical goals of the course, the pedagogy was developed to reveal more than simply the verification of a pipe flow model. In particular,

- Performing uncertainty analysis reveals how accurate the model is expected to be prior to running the experiment. The technique also allows for judging whether the differences between a model and experimental data are “significant.”
- Students are asked not only to develop a model and verify it, but they are also directed to use the model as a tool to discover something new – in this case, to measure the transition Reynolds number. Furthermore, students discover that the transition Reynolds number is not a fixed value, but rather an empirical one. This study is an excellent example of how the Learning Cycle approach can be applied in the laboratory.

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