Modeling the Mouse Trap Car

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Abstract:
Most students have the ability to build a car powered by a mouse trap.   However, a typical student who has completed their sophomore dynamics course will still have trouble modeling and analyzing their design. This paper presents a structure to aid in completing the modeling and analysis of a mouse trap car project. It discusses a twelve step design process that could be provided to students to guide them through difficulties with the design analysis before they start building.

Introduction:
I have used the design and construction of a mouse/rat trap race car as a team design project to cap a course in dynamics. Students enthusiastically embrace this project. Why is that? Just like their instructors, they like to build things. Just like their professors, they like to play with toys. It's not hard to understand that if you have your students design and build a toy they get to test and play with, and make it a competitive event against other teams, you will have a project that the students can get excited to participate in.

However, after using this activity a number of times, it became apparent that the focus of the student efforts tended to be top heavy on the construction side and not focused enough on the design side. To compensate for this misplaced focus, the project format was changed to emphasize the design and communication component of the project. The major communication component added was a poster presentation of their car design in conjunction with the building of the model. The poster presentation was scheduled concurrently with the race competitions which are held during the last week of the class. The poster was to include requirements such as team information (team name and members), photos and/or drawings of their design/prototype, a list of safety measures to be followed when working with their car, and a complete analysis of the dynamic principles behind their design. In addition to forcing the students to formalize their design and analysis, these posters allowed the other students to examine other possible solutions to the same problem they were working on. This was a graded project where 80% of the grade is allocated to the poster, 15% to the construction and performance of the vehicle, and 5% to their group interaction and participation. While the construction and racing may have been the "fun part", the design and communication of their work made up the majority of the student's project grade.

After running this projects for two semesters, it was found, that even after putting this emphasis on the design and analysis component, the sophomore students still had much difficulty with the modeling and analysis of the car. It became apparent that they had difficulty breaking the problem down into smaller tasks that they felt comfortable

working through. To address this problem, a stepwise process was put together to help the students proceed through the modeling and analysis, testing, and construction. These steps include:
1) Establish a formal brainstorming and decision process to create and then chose a number of 2-dimensional generic models.
2) For each model, define and specify a list of the parameters in the model which may be varied. Give an approximate range of each of the parameters.
3) Develop the equations for each model which will find the distance of travel versus the spring rotation.
4) Run an experiment to find the moment of force exerted by the spring versus the spring deflection angle.
5) Run an experiment on a demonstration car to determine approximate effect of rolling and/or viscous friction on the vehicle.
6) Develop a force-acceleration model using Newton's 2nd Law with the spring moment and friction relationships found in the steps 4 and 5.
7) Determine car velocity and car position versus time by integrating the acceleration model. Use small discrete time steps assuming constant acceleration over each step to integrate this numerically using Euler's Method.
8) Change model parameters to study model performance and the sensitivity of performance to changes in individual parameters.
9) Compare the different models and pick one model to be built.
10) Build prototype.
11) Test prototype and compare to generic model.
12) Race in the competitive finale.
The focus of this paper is to present an example of how these steps can be used to work through the modeling and analysis of a typical mouse/rat trap car. The modeling and analysis are covered by steps 1 to 7 of the twelve step method.

Step 1: Choose a generic model.
Students should be worked through a formal brainstorming and decision making process to help them generate and then narrow down a number of possible model choices. While the student teams should set up several different models to study, just one will be demonstrated here. This example will consist of a rear wheel driven car which has a cord or ribbon directly wrapped around the rear axle and pulled by the arm connected to the spring. The simplified drawing of this model of this is shown in figure 1.
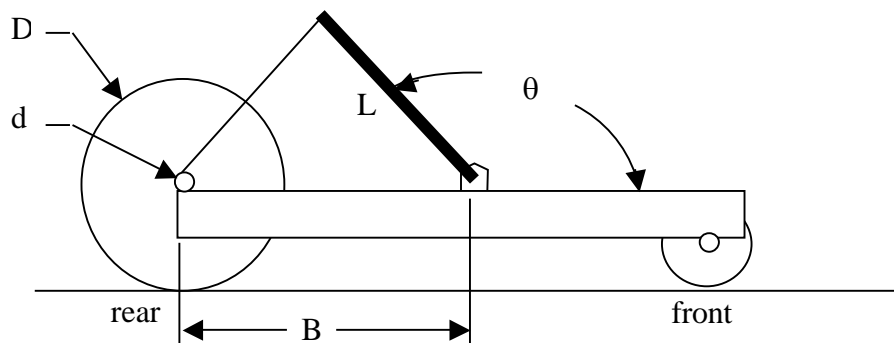

Figure 1

Step 2: Specification of parameters.
This step of the modeling process consists of defining any variable parameters and establishing what the working ranges might be. While there are some definite limits and specifications to what some of the parameters are, your students may have to make estimates or judgments that limit what other parameters might be. For example, on the model used here, the diameter of the rear wheel needs to be at least greater than the sum of the thickness of the mouse/rat trap base and the radius of the rear axle. Therefore, it has a minimum value. Parameters will be based primarily on geometric considerations and race performance requirements.

The parameters associated with the example car design are listed in Table 1.

Table 1: Variable parameters and approximate ranges.

Description:	Symbol	Approximate Minimum Value	Approximate Maximum Value
Spring Deflection Angle	θ	0 degrees	180 degrees
Length of Arm	L	1 inch	12 inches
Distance from Arm Fulcrum to Rear Axle	B	1 inch	12 inches
Rear Axle Diameter	d	1/8 inch	3 inches
Rear Wheel Diameter	D	1.5 inches	12 inches
Mass of Car	m	1/3 lb_m	2 lb_m

Step 3: Develop an equation that describes the relation between spring deflection angle and the car position.
This step will usually involve a purely geometric analysis of the car. This relationship may be shown for the demonstration model. As the spring pulls on the cord, which is wrapped around the rear axel, it causes the axle to rotate. This moves the car forward. For this analysis, assume there will be no slippage of the rear wheel along the floor.

For the car modeled here, start with the arm at its maximum position, when θ_{max} is extended to 180 degrees. At maximum spring deflection, the cord distance, S_o, shown in figure 2a, is given by

$$S_o = B - L \qquad\qquad\qquad (Eq. 1)$$

For any other angle of spring deflection, the cord forms side, S, of a triangle, which is shown in Figure 2b. Using law of cosines to find S gives

$$S = \sqrt{L^2 + B^2 - 2LB\cos(180 - \theta)} \qquad\qquad (Eq. 2)$$

The total pulled distance of the cord, L_{pulled}, is the difference of these two lengths.

$$L_{pulled} = S - S_o \qquad\qquad\qquad \text{(Eq. 3)}$$

At the rear axle, this pull length causes the axle to rotate through an arc length equal to L_{pulled}. The wheel rotates through the same angle as the rear axle. For the same angle of motion, the wheel rotates through an arc length that is larger than the axle arc length by the magnitude of the ratio of the wheel diameter over the axle diameter. With no slippage of the wheel on the ground, this wheel arc length is equal to the distance the vehicle will have moved, x, shown in figure 3.

$$x = \frac{D}{d} L_{pulled} \qquad\qquad\qquad \text{(Eq. 4)}$$

Combining Eq. 1, 2, and 3 with Eq. 4 gives

$$x = \frac{D}{d} \left\{ \sqrt{L^2 + B^2 - 2LB\cos(180 - \theta)} - (B - L) \right\} \qquad\qquad \text{(Eq. 5)}$$

This is the equation which gives the distance the car moved as a function of the defined model parameters, D, d, L, B, and θ.
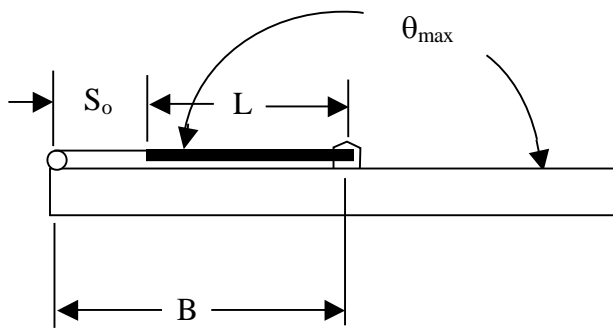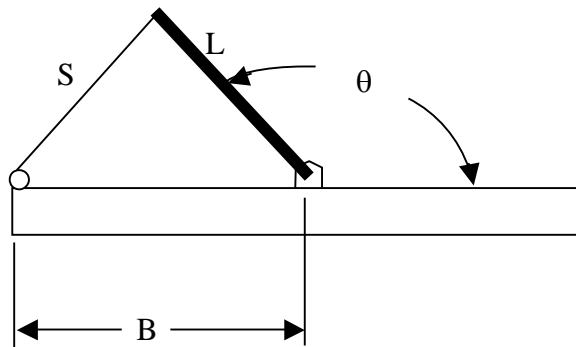

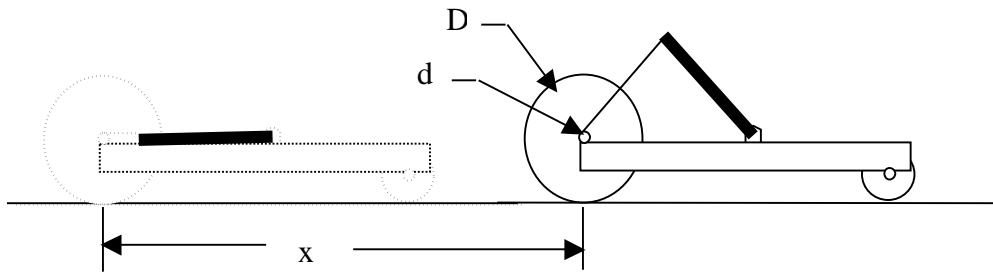Figure 2a.   Figure 2b.

Figure 3

Step 4: Experimentally determine the spring moment versus deflection angle.
To find the moment acting in the spring, a simple static test can be run using some clamps, a weight hanger, and a set of weights. Start by clamping the mouse/rat trap in each of the positions shown by figure 4a, 4b, and 4c. Apply weights to the hanger until the arm is balanced at 0, 90, and 180 degrees respectively. The data for a test of a rat trap used in a previous design project is given on Table 2.

Table 2. Weight versus spring deflection data for rat trap with length, L = 3.25 inches.

Spring deflection angle	Applied weight	Moment: $M = W*L$
0 degrees	1.6 lbs	5.2 in-lb
90 degrees	4.3 lbs	14.0 in-lb
180 degrees	6.2 llbs	20.2 in-lb

Fitting a line to the data gives a relationship between the moment of force produced by the spring and the deflection angle.

$$M = a\, \theta + b \qquad\qquad (Eq.\ 6a)$$

$$M = (0.0833\ \text{in-lb/deg})\, \theta + 5.2\ \text{in-lb} \qquad (Eq.\ 6b)$$

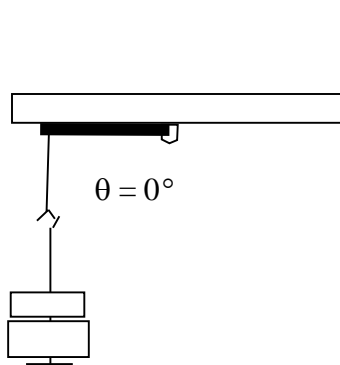
$\theta = 180°$

$\theta = 90°$
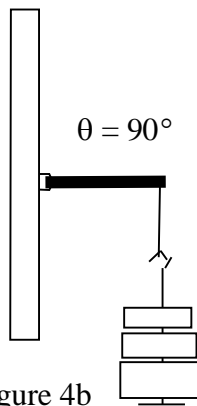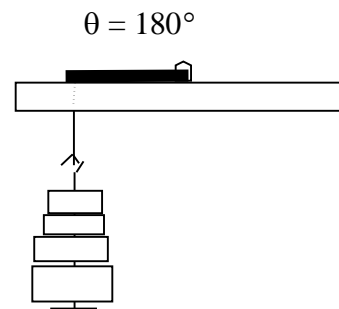
$\theta = 0°$

Figure 4a

Figure 4b

Figure 4c

Step 5: Experimentally determine effect of friction.
One method to approximate the frictional forces on the mouse trap car can be found by finding the amount of weight needed to keep a test car moving at constant speed. While the test car is not expected to be exactly like the model to be built, it will give the approximate size of the frictional forces to be found. The frictional force is expected to be a combination of contact and vicious friction effects.

To experimentally find the effect of the contact frictional forces, hook up a string to the car and run it over a pulley. Hang just enough paper clips on the string to make the car move with a slow but constant speed. For a very simple model of the frictional force you can simply use the weight of the paper clips as the frictional force.

$$F_{f\,contact} = W_{clips} \qquad\qquad (\text{Eq 7a})$$

If you wish to include the effects of viscous friction, you may take additional readings with extra paper clips added to the pulley weight. During these runs, after initially speeding up, the car will reach a terminal velocity that can be measured. Take enough readings with different amounts of weight to define a plot of weight versus velocity. A best fit through the data should give you an approximately linear fit for the low speeds expected by your car, such that

$$F_f = F_{f\,contact} + k\ v \qquad\qquad (\text{Eq. 7b})$$

where k represents the viscous drag coefficient and v is the instantaneous velocity. Since the car the student teams will eventually build will be different from the test car used to measure this data, this data is only meant to provide an approximate value of the friction force, to help in the design of their car. After their design is built, the friction should be remeasured with the actual team car, and the effect of friction on the actual vehicle should be used to update the model.
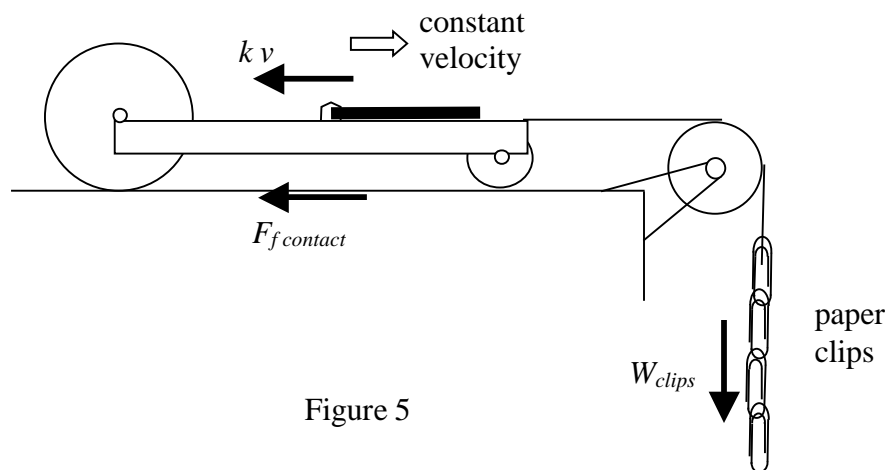

Figure 5

Step 6: Develop a force-acceleration equation for the model.
The force-acceleration model can be developed by use of Newton's 2nd Law. This step will find an equation which will predict the acceleration of the mouse/rat trap race car as a function of the spring deflection and the other defined variable parameters.

The first step is to break the model into a number of free body diagrams that show the forces and moments acting on the model. These are shown for our generic model in figures 6, 7, and 8.

In figure 6, the spring moment, M, sets up a tension, T, in the cord. The term, $\sin(\gamma)$, is found using the law of sines

$$\sin(\gamma) \ = \ B/S * \sin(180-\theta) \tag{Eq. 8}$$

From Newton's 2nd Law, the sum of the moments acting about the fulcrum gives

$$\Sigma M_o \ = \ I_o \alpha_o \tag{Eq. 9}$$

$$M - T L \sin(\gamma) \ = \ I_o \alpha_o \tag{Eq. 10}$$

Solving for the tension in the cord, T, gives

$$T \ = \ (M - I_o \alpha_o)/ (L \sin(\gamma)) \tag{Eq. 11}$$

Assuming that the inertial term, $I_o \alpha_o$, is much less than the moment, M, since the arm has very little mass, this equation can be approximated as

$$T \ = \ M / \ L\sin(\gamma) \tag{Eq. 12}$$

Figure 7 shows the free body diagram of the rear axle and wheel. Neglecting any axle friction, the moments of force acting about the rear axle may be summed up as

$$\Sigma M_r \ = \ I_r \alpha_r \tag{Eq. 13}$$

$$T \ (d/2) \ - F_{drive} \ (D/2) \ = \ I_r \ \alpha_r \tag{Eq. 14}$$

Assuming again that the inertia terms, $I_r \ \alpha_r$, will be small compared to the forces (this may or may not be true depending upon the choice of the wheel material), then the drive force, F_{drive}, can be found as

$$F_{drive} \ = \ T \ (d/D) \tag{Eq. 15}$$

$$F_{drive} \ = \ (M \ d) / \ (L \ D \ \sin(\gamma)) \tag{Eq. 16}$$

Figure 8 shows the free body diagram of the entire vehicle. Writing Newton's 2nd Law for the vehicle in the horizontal direction gives

$$\Sigma F_x \ = \ m \ a_x \tag{Eq. 17}$$

$$F_{drive} \ - \ F_f = m \ a \tag{Eq. 18}$$

therefore, solving for the acceleration gives

$$a = (F_{drive} - F_f)/ m \qquad\qquad (Eq.\ 19)$$

$$a = [M\ d\ /\ (L\ D\ \sin(\gamma)) - F_f\ ]\ /\ m \qquad\qquad (Eq.\ 20)$$

which contains of the variable parameters, $d,\ D,\ L$, and m. It is also a function of the spring moment, M, which itself is a function of the spring angle, θ. The friction force, F_f, may be found since it is either a function of velocity, v, or is treated as constant.
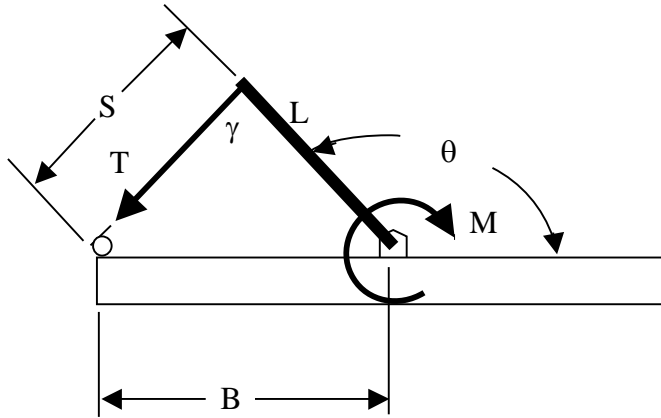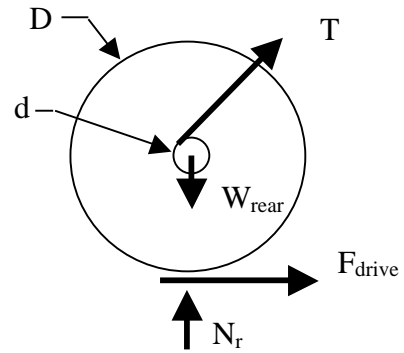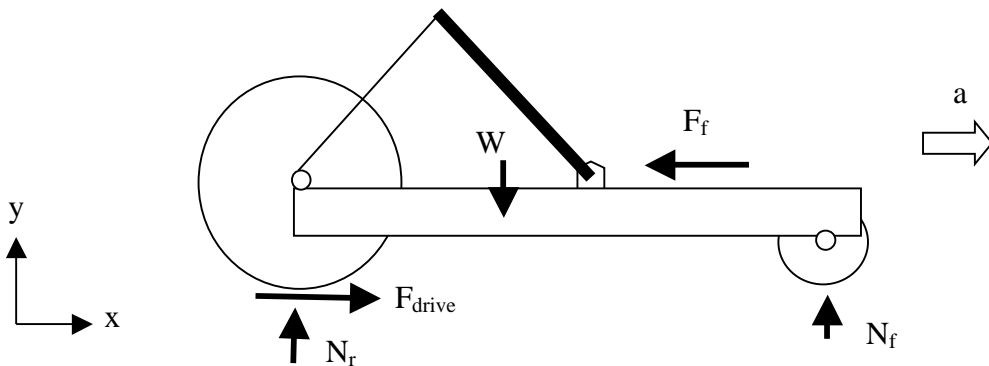

Figure 6

Figure 7

Figure 8

Step 7: Determine car velocity and position.
The velocity, v, of the car is found by integrating the acceleration with respect to time, t.

$$a = dv/dt \qquad\qquad (Eq.\ 21)$$

Since acceleration is not constant over the entire motion, one way to complete this integration is to integrate over very small time periods, treating the acceleration as constant over each small period, and then reevaluating it at the next step.

Using a small time step, Δt, the acceleration may be expressed as

$$a \ = \ (v_{i+1} \ - \ v_i \ )/ \ \Delta t \qquad\qquad\qquad \text{(Eq. 22)}$$

which gives $\qquad v_{i+1} \ = \ v_i \ + \ a \ \Delta t \qquad\qquad\qquad \text{(Eq. 23)}$

These steps are repeated to find the position, x, of the car, by integrating the velocity with respect to time.

$$v \ = \ dx/dt \qquad\qquad\qquad \text{(Eq. 24)}$$

Once again, using the same small time step,

$$v_i \ = \ (x_{i+1} \ - \ x_i \ )/ \ \Delta t \qquad\qquad\qquad \text{(Eq. 25)}$$

so $\qquad x_{i+1} = x_i + v_i \, \Delta t \qquad\qquad\qquad \text{(Eq. 26)}$

Time, t, increments by

$$t_{i+1} = t_i + \ \Delta t \qquad\qquad\qquad \text{(Eq. 27)}$$

At each new value of time, t_{i+1}, the values of both the position, x_{i+1}, and velocity, v_{i+1}, are known. The new position, x_{i+1}, is used with Eq. 5 to find the new spring deflection angle. This in turn can be used to define the new value for the drive force, F_{drive}, and the acceleration, a. The new velocity, v_{i+1}, can be used to determine a new friction force, F_f, if the viscous model for friction was used.

This use of the Euler method to integrate both the acceleration and the velocity gives a set of data which provides a way to plot both position and velocity versus time. This is the information with which we may simulate the run time behavior of our generic model. At the completion of Step 7 a viable model has now been defined.

Steps 8-12: Refining, choosing, building, and testing the mouse/rat trap race car.
The discussion of Steps 1 to 7 was the primary topic this paper targeted. Normally, in an actual project, there is still a fair amount of work to complete. Step 8 would have the students explore how altering the mass, pull-arm length, pull-arm angle, rear axle diameter, and rear wheel diameter would affect run time performance of their car over a given distance.

Step 9 of the process would have them look at what other features or aspects they should consider beside the kinematic and dynamic behavior of the car. Before they build their model, they should consider such things as model strength, methods of construction, available materials, model reliability, ease of use, and safety considerations. They should have some plan to weigh the relative importance of the different features that will allow them to select a design which is a good balance of all the most important features.

Step 10 is the stage where the model becomes a real car. Working with tools is great experience, and all engineers should have some ability to actually build things with their own hands. Figure 9 shows one of the cars built by a student team.

Figure 9.

Step 11 in the process takes the completed vehicle and tests it to see if it matches the performance that the model predicted.

Step 12 is the final competition between the different teams and their projects. If a team has not managed their time well and has not completed their project, they will not compete. A competition date creates a very real and inflexible deadline for the completion of the project.

Conclusion:
The purpose of this paper has been to present one method to help students work through the design and analysis of a mouse/rat trap race car. Mouse/rat trap race car design works well as a design project at the sophomore level especially after completing a dynamics class. Some of the reasons that it is a good choice include
    1) It is easy and inexpensive to build.
    2) There are a wide variety of different designs possible.
    3) It lends to a highly competitive event or testing process.
    4) The testing equipment required is cheap, accessible, and simple to use.
    5) The model is deceptively simple to analyze, if you break it down into simple steps.
    6) It reinforces a number of different dynamics concepts.
    7) It creates a sense of interest and excitement.
However, it needs to be recognized that freshman and sophomore students will typically have difficulty with the modeling and analysis parts of this project. They will most commonly focus on the construction phase unless additional priority and guidance is

given to the design and analysis phases. To address this concern, the twelve step process presented in this paper was created to help clarify the tasks of design, analysis, and construction. Having students work through the twelve step process bridges the gap between the dynamics concepts and the model they will build. By understanding the twelve steps as they work through them, the students create the ties between the concepts and their design. The understanding gained in this process, shows them how the principles of dynamics are a strong and powerful tool.