MOSL: An Innovative Approach to a Supplementary Course of Mathematics in Engineering

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1. Introduction

Supplementary education has been the traditional method used by professors to help students who do not have adequate preparation for college. According to Grubb [4], supplementary or remedial courses are defined as a set of activities intended to meet the needs of students who do not initially have the skills to perform well at a regular level. Currently, most universities offer these type of courses in a variety of formats. In Latin America, the deficiencies of most high school math courses are shocking. This is especially true in developing countries, such as Guatemala, where 9 out of 10 high school students fail the standardized test provided by the government [8]. Therefore, offering supplementary mathematics courses for first-year college students is a necessity for universities with engineering schools. It is also an opportunity to reevaluate the way in which the mathematics courses are taught, since there are plenty of alternatives that could be implemented and then studied for their effectiveness.

A common practice for identifying students in need of supplementary education is by giving them a standardized test; this is not the case for some Latin American colleges. Once these students are identified, they are usually split into groups where those with a score below a minimum threshold receive a supplementary course before the beginning of their academic program. A drawback of this approach is that those students are usually delayed by a semester at the beginning of their career. Moreover, those remedial courses do not count towards their degree, and the students do not put a significant effort into them, since the only reward is being able to take the first mathematics course towards their degree.

Galileo University is a private university in Guatemala, which was originally founded as an institute of engineering and computer science. It stands out due to its technological approach in higher education, offering multiple specializations within engineering. Over the past several years, Galileo University has put significant effort into improving the retention of students that do not meet the minimum requirements in the area of mathematics for engineering. In this paper we present one of the methods that has had success in closing the gap between students lacking the minimum required knowledge and those who are well prepared. This non-traditional supplementary course has the particularity of being taught concurrently with the first engineering calculus course. One of the most important advantages of this method is that underprepared students benefit from the positive influence of their peers, due to their interaction with higher ability students as recognized by [1]. A second benefit is that the fundamentals learned in the supplementary sessions are used immediately in the first calculus course; hence, the students are able to recognize the importance and application of those concepts before they progress in such a course.

The rest of this paper is organized as follows. In Section 2 we discuss the experiment design including the methodology of the proposed approach and the details about the structure of each course session. Section 3 presents the statistical methods used to analyze the collected data, along with the main results. Finally, we draw some conclusions and discuss our future work in sections 4 and 5.
2. Experiment design

In the first semester of 2016, 250 students were enrolled in the first engineering calculus course, referred to as, Calculus 1. This course was focused on differential calculus and its applications. To conduct this study, among those 250 students we sampled 160. In this sample, we included students from different specializations within engineering such as electronics, mechatronics, computer science, industrial and management.

To identify students lacking high school mathematics knowledge, we ran an initial non-standardized test which, for brevity, we refer to it simply as INST. INST evaluated basic concepts, including operatory and problem solving skills in the following 4 areas:

- Arithmetic,
- Algebra,
- Equations, and
- Trigonometry.

It is important to mention that a study guide was provided to all the students so that they were aware of the topics covered in the test and, hence, could be prepared for such an evaluation. An example of the problems included in the INST are shown in Figure 1.

![Fig. 1. (a) Sample questions in the INST, original version in Spanish. (b) Translation to English.](image-url)

Since our goal was to detect those students with the highest probability of failure in calculus, the problems selected to construct the INST evaluated only the most basic concepts in the areas previously mentioned. Even more, our test was divided in 4 sections, where each one contained 10 questions about basic concepts, operatory skills and word problems (applications). Those students who did not obtain a satisfactory grade (less than 60 out of 100 points) in this test were enrolled in the Math Operatory Skills Laboratory (MOSL). MOSL is our approach to a supplementary mathematics course, which we describe in detail in the following subsection.
2.1. Math Operatory Skills Laboratory

As previously mentioned, MOSL is a supplementary course offered concurrently with Calculus 1. This course (MOSL) pursues a double objective: (1) instruct the students in mathematics, and (2) increase their engagement in academic work. This is because, as stated in [12], the more a student is engaged in academic work, the greater his or her level of knowledge acquisition and general cognitive development. In addition, it is important to highlight that MOSL is worth 10% of the Calculus 1 grade.

2.1.1. MOSL Structure

In this section, we give details with respect to MOSL structure. This course was taught in the first term in 12 weekly sessions, each one lasting 2 hours. It was divided into 4 units: (1) the field of real numbers, (2) algebra and the properties of real numbers, (3) mathematical models and equations, and (4) basics of geometry and trigonometry. A summary of the topics covered in each of these units is provided in the following table.

TABLE 1. Topics covered in MOSL sessions.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topics Covered</th>
</tr>
</thead>
</table>
| 1. The field of real numbers | 1.1. The real number line  
1.2. Arithmetic of integer numbers and hierarchy of operations  
1.3. Arithmetic of rational numbers  
1.4. Proportions and ratios |
| 2. Algebra and the properties of real numbers | 2.1. Fundamental properties of real numbers  
2.2. Integer and rational exponents  
2.3. Properties of exponents  
2.4. Algebraic expressions  
2.5. Basic operations with polynomial expressions  
2.6. Factorization of polynomials  
2.7. Basic operations with fractional expressions  
2.8. The connection between semantics and syntax in algebraic expressions |
| 3. Simple mathematical models and equations | 3.1. Linear, quadratic and other important types of equations  
3.2. Systems of linear equations  
3.3. Applied problems |
| 4. Basics of geometry and trigonometry | 4.1. Angles between parallel lines cut by a transversal line  
4.2. Basic concepts of triangles  
4.3. Similar and congruent triangles  
4.4. Trigonometric ratios in right triangles  
4.5. Applied problems |
A key characteristic of our approach is that each session was designed in such a way that a variety of topics from different units are taught at the same time. Thus, the outline of topics presented in Table 1 is not a timeline. The main reason for using this approach is to show the student that in mathematics, every single topic is connected. For example, you can teach arithmetic of rational numbers and show that similar triangles are an application of the previous topics. Moreover, this approach is useful in giving priority to those topics that the students are using and applying in Calculus 1. It is worth emphasizing that the topics covered in MOSL are not new to students, thus we were able to use the aforementioned scheme. Below, we summarize the main topics covered in each unit and present a Gant diagram (see Figure 2) to show the timeline of the topics covered in MOSL.

2.1.2. MOSL Methodology

Each MOSL session is managed by two people, one instructor and one teaching assistant. Both of them are characterized by their good relationship with students and their experience with underprepared students. As Mitchel observed in his work [7] “whereas a teacher may have no control over student’s incoming personal interests, that same teacher may be capable of having a noticeable influence on the student’s outgoing personal interests by the end of the school year”. For this reason, we consider that the motivation transmitted to the students from the instructors is crucial to their learning process.

The course sessions are divided in three parts, each of which is implemented with a combination of active learning techniques. In part 1, the students take a short self-assessment test, that includes sample problems related to the central topic of the particular session. During the second part of the session, the instructor solves and discusses the problems by through interacting with the students. Finally, in the last part of the session, the students analyze, discuss and solve a MOSL worksheet in groups. Let us discuss each MOSL part in detail.

- **Part 1 – Self-assessment test**: Just-in-time teaching (JITT) is a pedagogical strategy based on feedback [10]. The students respond to conceptual questions before each class, then the instructor adjusts the lesson contents in reaction to the misconceptions revealed. Based in
JITT, in our case, a short self-assessment test was administered at the beginning of the session, see Figure 3 for an example. When all the students finished, the instructors took a few minutes to identify the problems which, in their opinion, need to be addressed in the MOSL part 2; this was carried out based upon the difficulties demonstrated by the students.

![Figure 3: (a) Sample questions in a MOSL self-assessment test, original version in Spanish. (b) Translation to English.](image)

**Part 2 - Discussion:** The instructor solves a selection of problems for the students. Such a selection depends on the results obtained in part 1. It is worth mentioning that each sample of problems is selected so that its solution involves recalling the fundamentals of the topic under discussion, maintaining a balance between novel and familiar work. In novel work the priority is the semantics, which is associated with the deeper end of the learning spectrum. On the other hand, the familiar work, which is associated with the syntax, covers the superficial end of the learning spectrum (see [2]). As a result, the main objective of this part of the session is to reinforce basic principles, so that the students are able to solve the problems contained in the worksheet to be administered in part 3.

**Part 3 - Worksheet:** Finally, the third part of each MOSL session consists of the solution of a set of problems. The MOSL worksheet contains meticulously selected exercises and word problems (see Figure 4 for an example). The worksheet is solved in groups of a maximum of 4 students, with support from the instructor and teaching assistant. We place the students in groups and oversee their work using a combination of cooperative, collaborative and team based learning, rather than having the students sit in the same place doing their assignments individually [9]. If the instructors detect a mistake or a student requests their help, they sit together with the group of students and give them a set of “hints” rather than simply solving the problem for them. We teach our students that in order to learn mathematics they should never ask for the solution to the problem, instead they should do it on their own so that they learn by solving the problem [3]. After all, in this part we want to engage more students in learning mathematics. We want the teachers to become facilitators of learning experiences and improve the self-efficacy of students by boosting their confidence and help-seeking abilities [11]. It is worth mentioning that the errors committed by the students are very interesting to analyze, since from them we can obtain their thinking process for tackling the difficulties in mathematics problem solving.
3. Statistical Analysis

To test the effects of MOSL in our students’ by the end of the semester, all 160 students belonging to the sample took again the same initial non-standardized test. For shortness, we called this test Final Non-Standardized Test (FNST). In order to accomplish our goal, we conducted two types of statistical tests to the data obtained from the results of INST and FNST. First, we conducted several independent t-tests to compare the results of the students who passed the INST (from now on called Regular Students) and the students who were assigned to MOSL (i.e. those who failed the INST); the details of the measured variables are summarized in Table 2. To test the effects of MOSL, i.e. measure the difference between INST and FNST in the MOSL and Regular students, we ran two paired t-tests (see Table 3 for more details). All the results in this paper were generated through SPSS 24 and interpreted using a significance level of 0.05.

**TABLE 2. Independent t-tests.**

<table>
<thead>
<tr>
<th>Variable Measured</th>
<th>Groups of Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>INST grades</td>
<td>Regular students vs. MOSL students</td>
</tr>
<tr>
<td>FNST grades</td>
<td>Regular students vs. MOSL students</td>
</tr>
</tbody>
</table>

**TABLE 3. Paired t-tests**

<table>
<thead>
<tr>
<th>Group Measured</th>
<th>Variables of comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOSL students</td>
<td>INST grades and FNST grades</td>
</tr>
<tr>
<td>Regular students</td>
<td></td>
</tr>
</tbody>
</table>

3.1 Independent t-tests

Table 4 shows the descriptive statistics of the grades obtained in the INST classified by Regular and MOSL students. It is worth to highlight that the difference between the means of the grades between the two groups is 31.61 points, which shows a considerable disparity between the two
groups (see Fig. 5). However, notice that the standard deviation is about the same, showing that the homogeneity between the two groups is sustained.

TABLE 4. Descriptive statistics of INST grades for MOSL and Regular students.

<table>
<thead>
<tr>
<th>n (group size)</th>
<th>Mean grade</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular students</td>
<td>96</td>
<td>76.9852</td>
</tr>
<tr>
<td>MOSL students</td>
<td>64</td>
<td>45.3722</td>
</tr>
</tbody>
</table>

Fig. 5: Histogram of the INST grades from Regular and MOSL students.

The results of the independent $t$-test performed to the INST grades comparing Regular and MOSL students are shown in Table 5. This test shows that there is a statistically significant difference between the two groups.

TABLE 5. Independent $t$-test to compare the means of INST grades between MOSL and Regular students.

<table>
<thead>
<tr>
<th>Test results</th>
<th>$t$</th>
<th>Degrees of freedom</th>
<th>p-value</th>
<th>Mean Difference</th>
<th>Std. Error Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal variances assumed</td>
<td>19.8420</td>
<td>158</td>
<td>0.0000</td>
<td>31.6130</td>
<td>1.5932</td>
</tr>
</tbody>
</table>

The descriptive statistics of the grades obtained in the FNST classified by Regular and MOSL students are presented in Table 6. Note that the difference between the means is significantly smaller than that presented in Table 4, such a difference is about 20.931 (see Fig. 6).
Nevertheless, the standard deviation is about the same, showing that the homogeneity between the two groups is maintained, as in the INST grades case.

TABLE 6. Descriptive statistics of FNST grades for MOSL and Regular students.

<table>
<thead>
<tr>
<th>n (group size)</th>
<th>Mean grade</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular students</td>
<td>96</td>
<td>79.9154</td>
</tr>
<tr>
<td>MOSL students</td>
<td>64</td>
<td>58.9844</td>
</tr>
</tbody>
</table>

Fig. 6: Histogram of the FNST grades from Regular and MOSL students.

The independent $t$-test between Regular and MOSL students performed to the FNST grades are shown in Table 7. This test shows that there is a statistically significant difference between the mean of the two groups.

TABLE 7. Independent $t$-test to compare the means of FNST grades between MOSL and Regular students.

<table>
<thead>
<tr>
<th>Test results</th>
<th>t</th>
<th>Degrees of freedom</th>
<th>p-value</th>
<th>Mean Difference</th>
<th>Std. Error Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal variances assumed</td>
<td>10.1000</td>
<td>158</td>
<td>0.0000</td>
<td>20.9310</td>
<td>2.0686</td>
</tr>
</tbody>
</table>
3.2 Paired t-tests

To test the effects of MOSL, we first compare the INST and FNST grades of the Regular students and those of the MOSL students with paired t-tests. In Table 8 the descriptive statistics of the aforementioned grades of the Regular students are shown. Even though there is a small difference between the means of the grades in the INST and the FNST of the Regular students, such a difference is statistically significant. This result is shown in Table 9.

TABLE 8. Descriptive statistics of the INST and the FNST grades for Regular students.

<table>
<thead>
<tr>
<th>Mean grade</th>
<th>N</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>INST</td>
<td>76.9852</td>
<td>96</td>
</tr>
<tr>
<td>FNST</td>
<td>79.9154</td>
<td>96</td>
</tr>
</tbody>
</table>

TABLE 9. Paired t-test to compare the means of INST and FNST grades for the Regular students.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
<th>t</th>
<th>Degrees of freedom</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FNST - INST</td>
<td>2.9301</td>
<td>10.2847</td>
<td>1.0497</td>
<td>2.7910</td>
<td>95</td>
</tr>
</tbody>
</table>

Now, we will focus on the grades of those students who were assigned to MOSL. The results show that there is a statistically significant difference between the grades of the initial and final tests of MOSL Students. The descriptive statistics are summarized in Table 10, while the paired t-test results are presented in Table 11. Note that the average grade of MOSL students increased about four times the average grade of the Regular students.

TABLE 10. MOSL students INST and FNST descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>Mean Grade</th>
<th>N</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>INST</td>
<td>45.3722</td>
<td>64</td>
<td>9.5827</td>
</tr>
<tr>
<td>FNST</td>
<td>58.9844</td>
<td>64</td>
<td>14.8206</td>
</tr>
</tbody>
</table>

TABLE 11. Paired t-test to compare the means of INST and FNST grades for the MOSL students.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
<th>t</th>
<th>Degrees of freedom</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FNST - INST</td>
<td>13.6120</td>
<td>14.8730</td>
<td>1.8591</td>
<td>7.3220</td>
<td>63</td>
</tr>
</tbody>
</table>

In summary, MOSL possible effects in our students were tested. The results obtained from the independent t-tests (presented in Section 3.1) highlight the statistically significant differences between MOSL and Regular students in the INST and the FNST average grades. In Section 3.2, the results of the paired t-tests revealed the statistically significant difference in the grades of INST and FNST for our two groups, the MOSL and the Regular students. Such results evidence a notable reduction in the gap (in terms of average) between the two groups of students.
4. Discussion and Conclusions

According to our diagnostic test (INST), 40% of the first-year engineering students in the study do not fulfill the minimum requirements of knowledge in mathematics to succeed in an introductory calculus course. This high percentage of underprepared students is astonishing, considering that they are engineering students who may use mathematics on a daily basis while and after pursuing their degree. Moreover, the difference between the means in the INST of these students and the rest is significant.

Recall that our approach, MOSL, is focused on problem solving and operatory skills and was taught concurrently with the first engineering calculus course. Also, this supplementary course was worth 10% of the total Calculus 1 grade. The main result of the implementation of this supplementary course is shown in Table 6, where it demonstrates that the gap between MOSL students and Regular students, in terms of grades averages, was reduced by approximately 34%.

Our experience in previous years strongly suggests that MOSL had a positive impact in our students’ achievement. This impact could be the result of the peculiarity of our approach. To conclude the discussion, we outline the advantages and disadvantages of MOSL.

Advantages

- We taught the course content assuming that there is no previous knowledge of arithmetic, algebra, equations and trigonometry.
- The course use teaching methodologies such as collaborative, cooperative, and team-based learning.
- Most supplementary courses do not confer academic credits to the students who have to take them; consequently, some of the students do not enroll in these kind of courses [5]. With this issue in mind, MOSL is a possible solution. As already mentioned, it is worth 10% of the grade of the Calculus 1 course, which serves as a motivation for them to complete all MOSL coursework.
- Generally, the students in need of supplementary education have at least a semester delay. To avoid such a situation, MOSL was taught concurrently with the Calculus 1 course.

Disadvantages

- The individual effects of each of the aforementioned methodologies utilized to teach the course was not measured.
- There is enough statistical evidence to demonstrate that our approach has a positive effect on the reduction of the existing academic gap between students, however the students in need of more meticulous remedial education may not succeed with the support provided by MOSL alone.

5. Future Work

In our future work we will consider other variables such as students’ performance in subsequent courses, motivation to graduate, and demographic variables. Also, we want to measure the
individual effects of all the teaching methodologies used during the course sessions in order to provide more evidence regarding the effectiveness of MOSL. Levin [6] points out that motivation and teamwork, two key points of MOSL, are extremely important in the design of successful interventions for unprepared students. Therefore, we will also include a measure of the importance that motivation and teamwork have in the context of Latin-American supplementary education.

6. Acknowledgments

We would like to thank Universidad Galileo for the support provided for this research. In particular, we want to acknowledge all the ideas and invaluable input provided by Eduardo Suger, Ph.D., President of Universidad Galileo. Last but not least, we want to thank Byron Carranza, Alejandro Mendizabal and Michaelle Pérez for their great help in the creation, teaching and management of MOSL.

7. References


