MULTIPLE HEAT TRANSFER PROCESSES OF A TEA BREWING SYSTEM: THEORETICAL AND EXPERIMENTAL INVESTIGATIONS

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Abstract

This paper analyzes multiple heat transfer processes of an instant tea brewer system. Three processes are identified and analyzed using both theoretical and experimental analysis. The first process is an internal flow problem when the tap water is heated by coils with a constant surface heat flux boundary condition. The second process involves a natural convective heat transfer when the heated water flows through a rubber tube. The transition tube transfers the heated water to the spout where the tea is brewed. The third process is a transient process, in which the brewed tea drips into a pitcher filled with ice. Our goal is to determine the amount of time the brewed tea would remain at a comfortable drinking temperature. Theoretical equations are formulated in each heat transfer process with certain assumptions. The mass flow rate and the temperatures at various locations were measured to provide necessary boundary conditions or to validate the theoretical predictions. The conservation of energy and the lumped capacitance method are applied to determine the tea temperature as a function of time. Our theoretical predictions are well aligned with the experimental data.

Introduction

Heat Transfer is a core subject in mechanical engineering undergraduate curriculum. One of the important elements in learning objectives for Heat Transfer is to apply theoretical knowledge to analyze real-world problems, both experimentally and analytically. In this paper, multiple heat transfer processes of an instant tea brewing system are investigated with both theoretical and experimental analysis.

In the United States it is common for southern state residents to brew their tea at home. Most recipes for brewing tea are very simple. One hot brew recipe includes boiling two cups of tap water, then pouring the boiled water over several tea bags, letting it sit for a few minutes, stirring, removing the tea bags, and finally adding two more cups of water. A newer method is a cold brew method. This method is to simply add water and tea bags to a pitcher; let the tea bag dilute; remove it and the process is complete. Another method is a hot and cold brew using an iced tea maker. This process begins with adding water to an integrated reservoir; the water is then heated by coils and flows through the system to brew the tea. The hot tea then flows out of the system, through the spout, and is cooled by ice in the decanter.

This paper discusses the heat transfer processes involved in an iced tea maker system. The goal is to determine the amount of time the brewed iced tea would remain at a suitable drinking temperature. In the following, the iced tea maker system is introduced first. Three heat transfer processes and the mathematical models for each process are described. Experiments were conducted to measure the mass flow rate, tea temperature at spout, tea temperature in pitcher and the time to remain at a suitable drinking temperature. Finally, results are presented with detailed discussions and the comparison with experimental results.

System Process

Figure 1 is a Mr. Coffee® 3 Quart Iced Tea Maker. The system begins with filling the water compartment with tap water. Once the reservoir is filled, the machine is turned on and the water filtering through an outlet in the reservoir as shown in Figure 1. The water travels along the coils as shown in Figure 2. The system then absorbs the water up through a black transition tube and drips the water onto the tea bag located in the upper compartment of Figure 2. The tea is then dispensed into three (3) quarts of ice in the decanter. At this point the tea is then cooled with ice and poured for drinking pleasure.

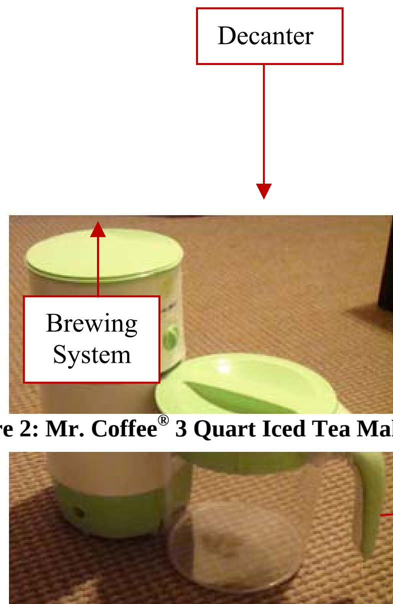

Figure 2: Mr. Coffee® 3 Quart Iced Tea Maker

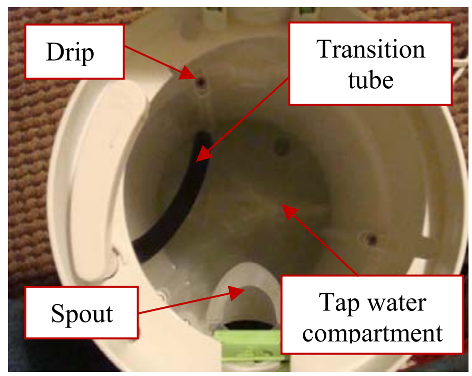

Figure 1: Inside Iced Tea Maker

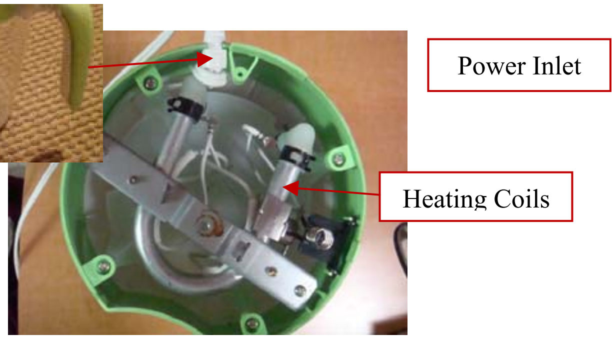

Figure 3: Heating coils of Iced Tea Maker

Three heat transfer processes are identified in this system, as shown in Figure 4: (1) tap water flows through a tube heated by coil; (2) hot water passes through a transition tube that is surrounded by tap water; and (3) the hot tea is mixed with ice cubes in a pitcher to reach a thermal equilibrium.

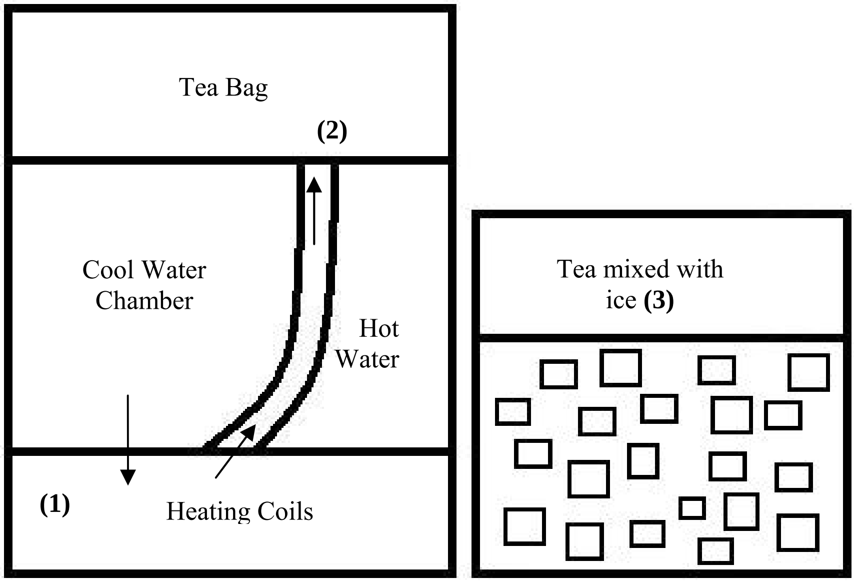

Figure 4: Schematic of Heating and Cooling Processes

Mathematical Model

1. Internal flow with constant surface flux

In the first process, the problem can be simplified as a water flow with a constant surface heat flux, as shown in Figure 5. Assuming the uniform flux and steady-state conditions, the mean temperature out of the coils can be calculated as follows

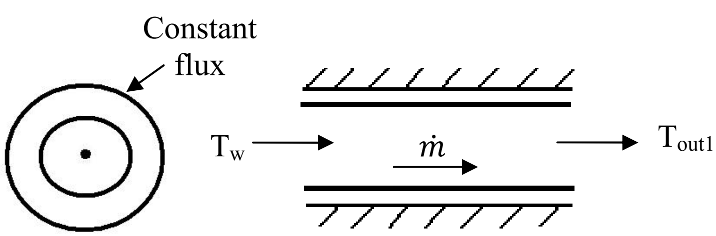

Figure 5: Internal flow in heating coils

$$T_{\text{out1}} = T_{\text{w}} + \frac{P}{\dot{m}Cp}\,\eta_{\text{coils}} \tag{1}$$

where,

T_{out1}- mean temperature out of the coils,
T_w- tap water temperature,
P - power supplied to the system,
Cp - specific heat of water,
$\dot{m}$ - mass flow rate of the water through the system, and
η_{coils} - efficiency of the heating coils.

2. Water flow in black rubber tube under natural convection

The hot water out of the coils passes through the tube, which is surrounded by tap water in the tap water compartment. To simplify the problem, the outer surface temperature of the transition tube is assumed to be same as the tap water temperature T_w. Also, the internal flow is assumed to be laminar flow, and fully developed. In this case, the outlet temperature can be calculated as follows,

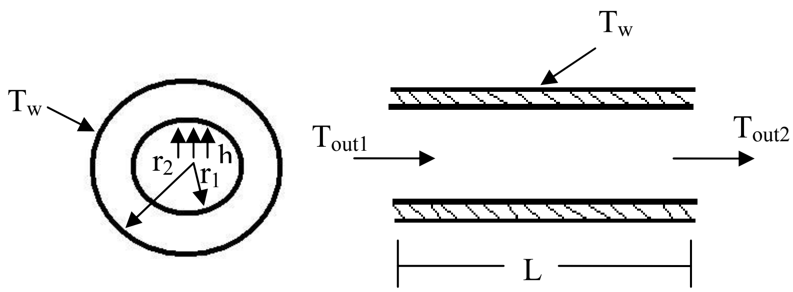

Figure 6: Internal flow in transition tube

$$T_{\text{out2}} = T_w - (T_w - T_{out1})e^{\left(-\frac{\pi D_{\text{tube}} L_{\text{tube}}}{\dot{m} C_p} U\right)} \quad (2)$$

where, $UA = \frac{1}{R_{\text{tot}}}$ (3)

D_{tube} - tube diameter,
L_{tube} - length of the transition tube,
T_{out2} - outlet temperature of the water through the tube, and
U is the overall heat transfer coefficient, which can be calculated from a thermal circuit as shown in Figure 7.

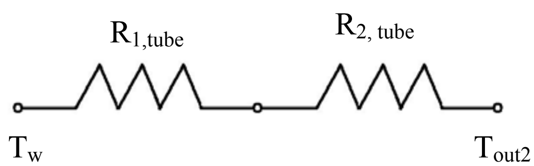

Figure 7: Thermal Circuit for Transition Tube

Thermal resistances in Figure 7 can be calculated as follows,

$$R_{1,\text{tube}} = \frac{\ln\left(\frac{r_{1,\text{tube}}}{r_{2,\text{tube}}}\right)}{2\pi k_{\text{tube}} L_{\text{tube}}} \quad (4)$$

$$R_{2,\text{tube}} = \frac{1}{h_{\text{tube}} A_{\text{tube}}} = \frac{1}{\pi D_{\text{tube}} L_{\text{tube}} h_{\text{tube}}} \quad (5)$$

where h_{tube} is the heat transfer coefficient associated with internal water flow. Since laminar flow is assumed h_{tube} can be calculated from

$$h_{\text{tube}} = 3.66 \frac{k_w}{D_{\text{tube}}} \quad (6)$$

where k_w is the thermal conductivity of water.

From the first process T_{out1} can be calculated by equation (1) and then, the T_{out2} is calculated by equation (2).

3. Energy balance

In the third process, the system can be expressed by the conservation of energy. The hot tea is mixed with ice cubes. Such a process is a transient process, which means the temperature changes over time. This allows the calculations for the amount of time the tea will remain cool enough to drink. In this analysis, the temperature gradient within the decanter is neglected. In this process, thermal energy released by the hot tea and the thermal energy into the system from ambient will be stored by the mixture of ice and tea. The energy storage includes three elements, ice temperature change through ice initial temperature to the melting temperature of ice, ice infusion, and the temperature change from the melted ice temperature to the final temperature. The thermal equilibrium cab be written as

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$$Q_1 + Q_2 + Q_3 = Q_4 + Q_5 \quad (7)$$

$$Q_1 = m_{\text{ice}} C_p \Delta T = m_{\text{ice}} C_{p,\text{ice}} (T_{\text{melt}} - T_{\text{ice}}) \quad (8)$$

$$Q_2 = m_{\text{ice}} h_{\text{sf}} \quad (9)$$

$$Q_3 = m_{\text{ice}} C_p \Delta T = m_{\text{ice}} C_p (T_{\text{f}} - T_{\text{melt}}) \quad (10)$$

$$Q_4 = m_{\text{w}} C_p (T_{\text{out3}} - T_{\text{f}}) \quad (11)$$

$$Q_5 = qt = \left(\frac{T_{\text{amb}} - T_{\text{f}}}{R_{\text{tot}}}\right) t \quad (12)$$

where,

Q_1- energy stored to heat the ice from T_{ice} to 0˚C,

Q_2- energy stored from melted ice,

Q_3- heat stored when ice melts from 0˚C to T_{f},

Q_4- heat released when tea at T_{out3} is cooled to T_{f}, and

Q_5- heat provided by ambient.

In equations (7) to (12) , m_{ice} is the mass of ice, h_{sf} is the heat infusion coefficient, m_w is the mass of tea, T_{melt} is the ice melting temperature (0°C), T_{ice} is the initial ice temperature, T_{f} is the final mixture temperature (3°C), T_{amb} is the ambient temperature, t is the time to reach the T_{f} for the mixture, R_{tot} is the thermal resistance of the decanter as shown in Figure 8. From Figure 8 the thermal resistances can be calculated as follows

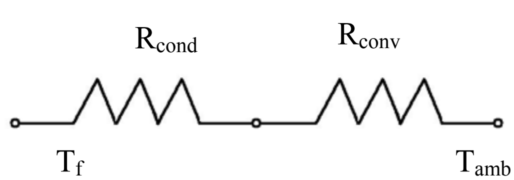

Figure 8: Thermal Circuit for Decanter

$$R_{\text{tot}} = R_{\text{cond}} + R_{\text{conv}} \quad (13)$$

$$R_{\text{cond}} = \frac{\ln\left(^{r_2}/_{r_1}\right)}{2\pi k_{decanter} L} \qquad (14)$$

$$R_{\text{conv}} = \frac{1}{h_{amb} A} = \frac{1}{h_{amb} \pi D L} \qquad (15)$$

where r_1, r_2, D, k and L are the internal radius, external radius, diameter, thermal conductivity, and length of the decanter.

4. Material properties

Material properties are listed in Table 1.

Table 1: Fluid and Material Properties

$C_{p\text{-}40}$	$C_{p\ 3}$	$C_{p\ 90}$	h_{sf}	h_{amb}	k_w	$k_{decanter}$	k_{tube}
$\frac{J}{kg * K}$	$\frac{J}{kg * K}$	$\frac{J}{kg * K}$	$\frac{J}{kg}$	$\frac{W}{m^2 * K}$	$\frac{W}{m * K}$	$\frac{W}{m * K}$	$\frac{W}{m * K}$
2040	4211	4203	3.3E+05	5	0.59	.59	.13

Experiment

The entire iced tea maker system was ran to collect data for this experiment. The Mr. Coffee® iced tea maker has a mark on the pitcher that indicates a volume of five (5) cups, which can be used to fill the water of the tea maker. The pitcher also has a three (3) quart marking on it that indicates the amount of ice needed. These values will be used for other calculations

$V_{\text{w}} = 5 \text{ cups} = 1.18 \text{ L}$

$m_{\text{w}} = 1.18 \text{ kg}$

$m_{\text{ice}} = 1 \text{ kg}$

The mass flow rate of the system can be calculated assuming that for 1.18 kg of water per 10 minutes

$\dot{m} = 0.00197 \frac{\text{kg}}{\text{s}}$

Table 2 and Table 3 display the measured geometries of the decanter and of the transition tube.

Table 2: Measured Values of the Decanter

r_1	r_2	L
m	m	m
0.0746	0.0762	0.18

Table 3: Measured Values of the Transition Tube

$r_{1,tube}$	$r_{2,tube}$	L_{tube}
m	m	m
0.005	0.00635	0.23

Two separate experiments were conducted to collect the data of the tea temperatures at the exit from the system and the mixture temperature in the pitcher with and without ice. First, the experiment was run without ice. Table 4 shows the measured temperatures of tap water, tea temperature at exit and tea temperature in the decanter. It can be seen that there is about 10°C temperature differences between T_{out3} and T_{exit}. The tea cools by air while dripping into the decanter. Table 5 shows the experimental data for running the system with ice. As expected the mixture temperature is 0°C, and the tea temperature at the exit is around 80°C.

To measure the amount of time that the mixture temperature is below 3°C, the experiments were repeated three (3) times. During the experiment, the ambient temperature of the room was approximately 20°C. Table 6 shows the averaged experimental results. The tea remains cool enough within three and half hours.

Table 4: Experimental Data running tap water through system without ice

T_w	T_{exit}	T_{out3}
˚C	˚C	˚C
15.6	78.3	70

Table 5: Experimental Data running tap water through system with ice in the pitcher

T_w	T_{exit}	T_{out3}
˚C	˚C	˚C
15.6	80	0

Table 6: Experimental Results running tap water through system with tea in the brewing basket and ice in the pitcher

$T_{w,i}$	$T_{w,b}$	$T_{w,p}$	T
˚C	˚C	˚C	hr
15.6	80	0	3.5

Results and Discussion

In understanding the complete procedure, first the exit temperature out of coils is examined. According to the system specification, the power provided by the system is 725Watts. Assuming that the power efficiency of the system ranging from 70% to 90%, Figure 9 plots the heated water temperature against the efficiency, according to equation (1). The outlet temperature is between 75°C to 95°C. This validates the system design since the water should be heated to a certain temperature to meet tea brewing requirement but below 100°C. Figure 10 plots the results of the outlet temperature as function of mass flow rate with an efficiency of 85%. Again, it shows the system is designed to allow the mass flow rate to generate the hot water at certain temperature without overheating.

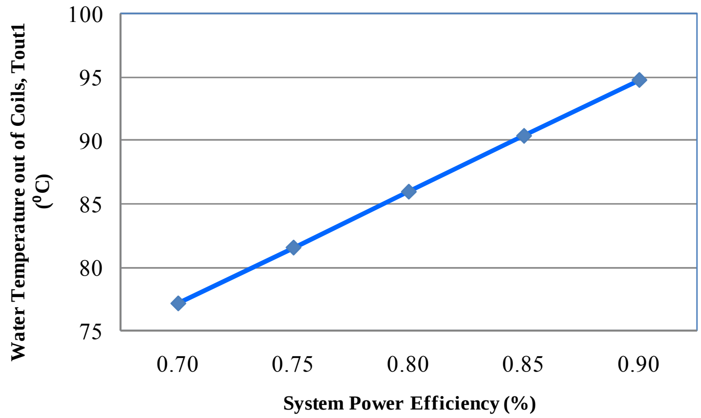

Figure 9 Water Temperature of Coils, T_{out1} as a Function of Efficiency

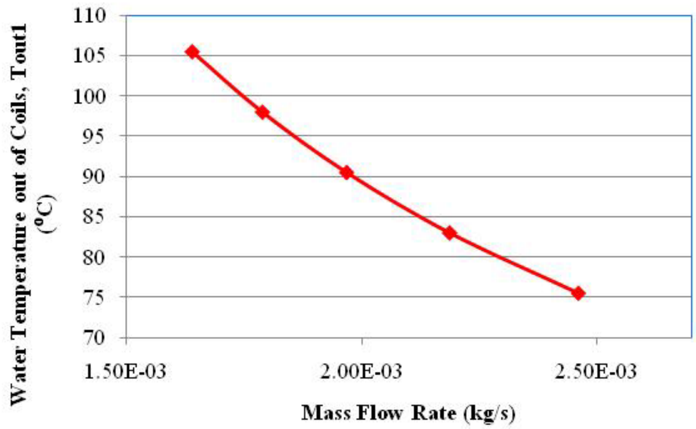

Figure 10 Water Temperature of Coils, T_{out1} as a Function of Mass Flow Rate

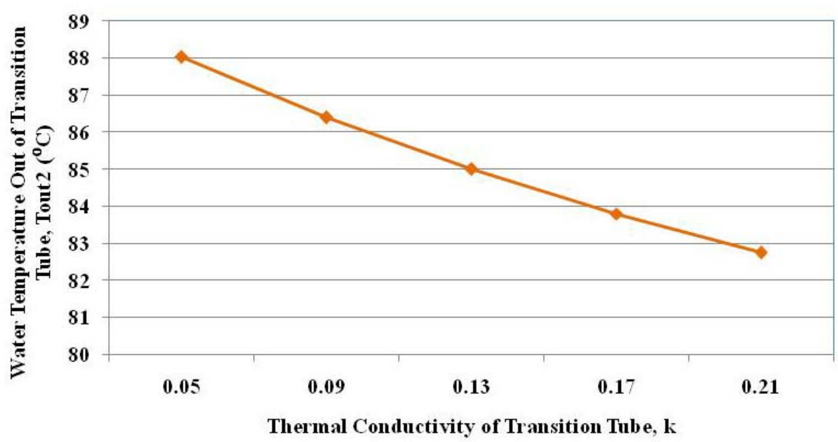

Figure 11 Water Temperature of Transition Tube, T_{out2} as a Function of Thermal Conductivity

Figure 11 shows the results that temperature loss through the transition tube is minimal; even there is a large temperature gradient between the hot water and surrounding tap water. This is because that the tube is made of rubber material that is has low thermal conductivity and tube length is very short.

Based on the analytical results of the first and the second process, it is concluded that the hot water temperature can reach the tea brewing temperature requirement. During tea brewing process, the tea temperature will be dropped. From the experiment it can be seen that after brewing the tea droplet temperature is about 80°C.

Next the thermal equilibrium in the decanter is analyzed through equations (7)- (12). Since the exact initial ice temperature is not known, several scenarios of ice initial temperatures are assumed. The tea initial temperature is assumed to be 70°C, and the final mixture temperature is set 3°C. Table 7 lists the results of thermal energies Q_1 to Q_5 and the time needed to reach the equilibrium. The results of the analytical calculations and the experiment are closely aligned.

Table 7 Comparison between Analytical and Experimental Results

T_{ice}	Q_1	Q_2	Q_3	Q_4	Q_5	t (hr)
-15	30600	334000	12633	331261.4	45971.6	1.8
-20	40800	334000	12633	331261.4	56171.6	2.2
-25	51000	334000	12633	331261.4	66371.6	2.6
-40	81600	334000	12633	331261.4	96971.6	3.8
Experimental Result						3.5

Conclusion

1. The heat transfer processes of an ice tea maker are analyzed in a real life situation. The system is observed at different stages. In the theoretical calculations, it is validated that the system is designed with a design power and the control of brewing time to allow the water is heated up to a brewing temperature without overheating (boiling water) or under-heating. It is also concluded that the heat loss through the transition tube is not significant.
2. Experimental data make it possible to compare analytical results with real life data. The results of the analytical calculations and the experiment are closely aligned.
3. This project provides a very good opportunity to apply the theoretical knowledge learnt in the class to solve real world problems. The heat transfer processes in this system involves with internal flow with a constant heat surface flux, an internal flow in a natural convective condition, thermal circuit, and thermal equilibrium with a transient process.

References

[1] Incropera, F. P., Dewitt, D. P., & Bergman, T. L. (2006). *Introduction to Heat Transfer.* Wiley.