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Newton's Law and Accelerometer Integration Applied to Impact Analysis

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Abstract

This paper presents an experimental technique which illustrates to the student an application of Newton's law, the use of an accelerometer, and numerical double integration of the acceleration to obtain displacement. The simple impact experiment consists of a falling rigid projectile striking a sample. Data analysis utilizes Newton's law and double integration of the accelerometer output to obtain the dynamic load-deflection curve for the sample. An oscilloscope, digital data transfer to a PC, and manipulation of the data using spreadsheet software are incorporated in the experiment.

Introduction

The rate of loading is well known to strongly influence the mechanical behavior of many materials. However, there is a tendency in the design of dynamic applications to utilize readily available material property data determined from quasi-static type tests. For example, synthetic and natural gut strings used in tennis rackets are commonly compared using slow test results from a tensile testing machine where the load-deflection behavior is typically very nonlinear and with high hysteresis. In application, the string is pre-tensioned and subjected to a loading cycle (ball impact) of just a few milliseconds. Dynamic tests under similar conditions to the actual application have shown that the behavior is very linear with negligible hysteresis. (1) The conclusion is that the designer should always use material properties which have been determined under rates of loading and other conditions which are similar to the application.

In this study a weight-drop impact apparatus with an accelerometer attached to the falling weight is used to obtain a dynamic force-deflection curve for the sample. The concept is similar to that used in an earlier study on shock absorption of athletic shoes.(2) An oscilloscope is used to capture the acceleration-time data and a PC is used to process the data. When used as a classroom or laboratory demonstration, or laboratory/project assignment, this experiment illustrates the following to the student:

1. Determination of the impact force-time record using Newton's law.
2. Evacuation of the displacement-time record by double integration of the acceleration and enforcing proper initial conditions.
3. Technique and advantages of smoothing the experimental data.
4. Convenience of spreadsheet software to process and analyze the raw data, and to plot the results.
5. The energy absorbing and loading-rate effects of using a material such as foam.

Test Apparatus and Procedure.

The test arrangement consists of a cylindrical steel projectile instrumented with an accelerometer which is guided under free fall by an acrylic tube. The projectile strikes a sample and the accelerometer output is recorded on a digital oscilloscope. The digital data is transferred to a PC equipped with appropriate acquisition and spreadsheet software. A schematic of the apparatus is shown in Fig. 1.

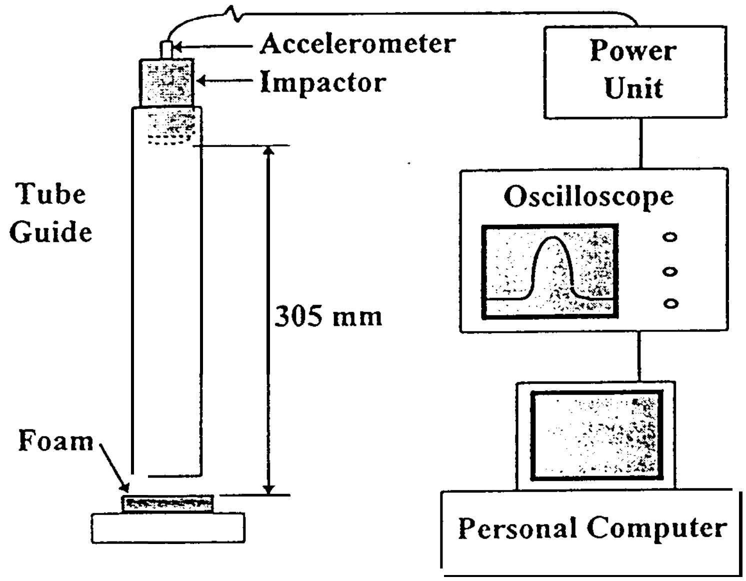

Fig. 1. Impact apparatus with associated instrumentation.

The test utilized 127 g, 275 g, and 543 g masses each having a slightly rounded impact surface. A PCB model 303 accelerometer with 10 mV/g sensitivity was attached to the top end of the mass by a stud mount. The mass was released 305 mm above a foam sample 28-mm in diameter and 12-mm thick. The mass falls at -g acceleration (neglecting air drag and friction) and strikes the sample. On impact, the mass deceleration becomes very large, peaks out, and returns to -g at rebound. The free-body-diagram of the mass in contact with the sample is shown in Fig. 2.

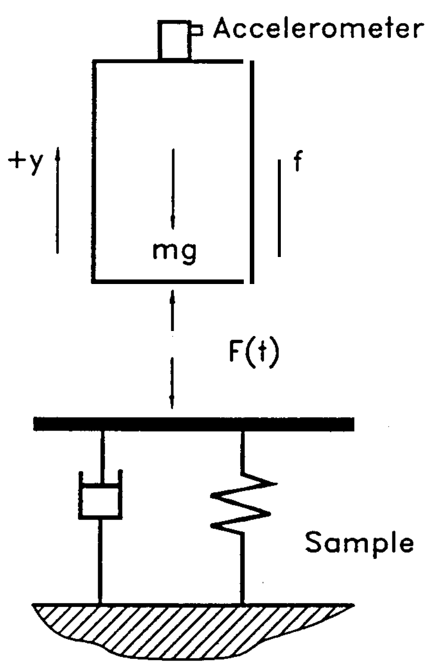

Fig. 2. Free-body diagram and sample model for impact test.

The application of Newton's law for the falling mass at impact gives

$$F(t) = m(a + g) - f \quad (1)$$

Here F(t) represents the contact force, f represents the forces due friction, air drag and other losses, mg is the mass/accelerometer weight, and the sample is represented by a parallel damper-spring viscoelastic type model.

As the mass strikes the sample and bottoms out, the friction and air drag type terms change direction and, more importantly, become negligible so that the peak contact force due to the impact can be written

$$F_{max} = m(a_{max} + g) \quad (2)$$

After shifting the relatively small lg, the acceleration curves for the three masses areas shown in Fig. 3.

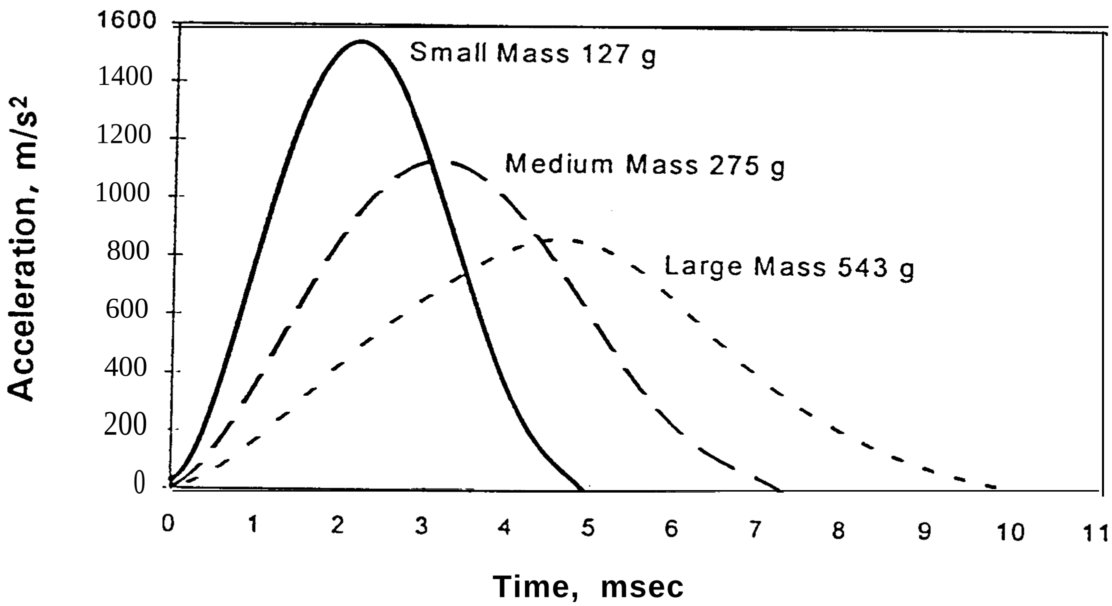

Fig. *3.* Acceleration curves for impact of foam sample by three different masses.

Accelerometer Integration Check

The integration of accelerometer output to get velocity and again to get displacement can be done either electronically or by direct integration of the signaL (3) The electronic circuit approach is suitable for continuous vibration situations but does not work well for transient acceleration where the initial conditions (v_o, x_o) determine the constants of integration. In the present application, the direct integration of the signal is used where $v_o = -\sqrt{(2gh)}$ and x_o = O. Because accelerometers have a tendency to "zero shift" which would cause significant error in the integration procedure (4), a calibration procedure was developed to check the double integration determination of displacement with that determined directly with a noncontacting Photonic displacement sensor. The arrangement is shown in Fig. 4 where a micrometer displacement stage was used to check the displacement output of the Photonic sensor using the actual projectile curved surface.

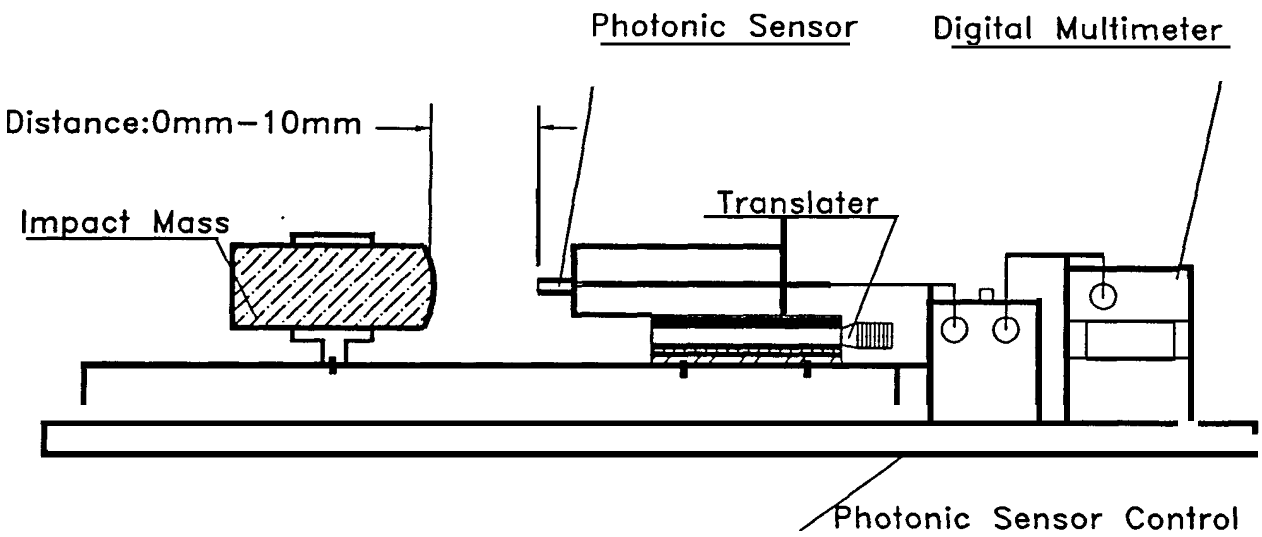

Fig. 4. Calibration arrangement using Photonic sensor.

An impact experiment was then conducted where the double integration approach for displacement was compared with the direet Photosonic sensor measurement. Results agreed quite well, as shown in Fig. 5, and the double integration determination of displacement was assumed to be sufficiently accurate for this accelerometer model and application.

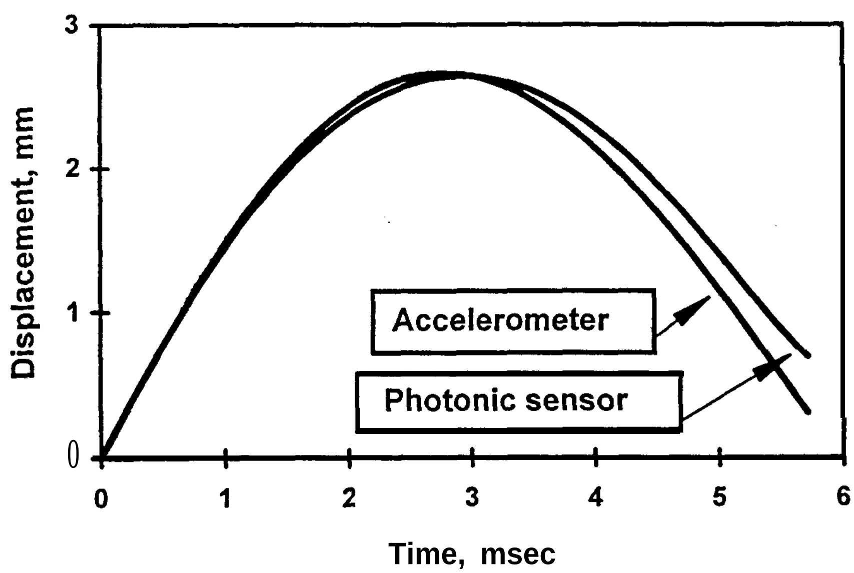

Fig. 5. Dynamic displacement comparison of Photonic sensor and double integration.

acceleration units, and to shift the acceleration data to begin at zero time and zero acceleration (deceleration) in preparation for numerical integration. The data was also smoothed by performing an averaging of ten successive values to obtain the smoothed mid-data point value. The acceleration curve was integrated with respect to time by the trapezoidal rule and the initial velocity enforced to be $v_o = -\sqrt{(2gh)}$ by shifting the curve. The velocity curve was integrated again and initial displacement enforced to be zero. Newton's law, eq. 1 with f assumed negligible, was applied to get the dynamic contact force. The plot of force vs deflection for the sample was constructed as shown in Fig. 6 for the three projectile masses. The sample force and displacement are taken to be positive downward.

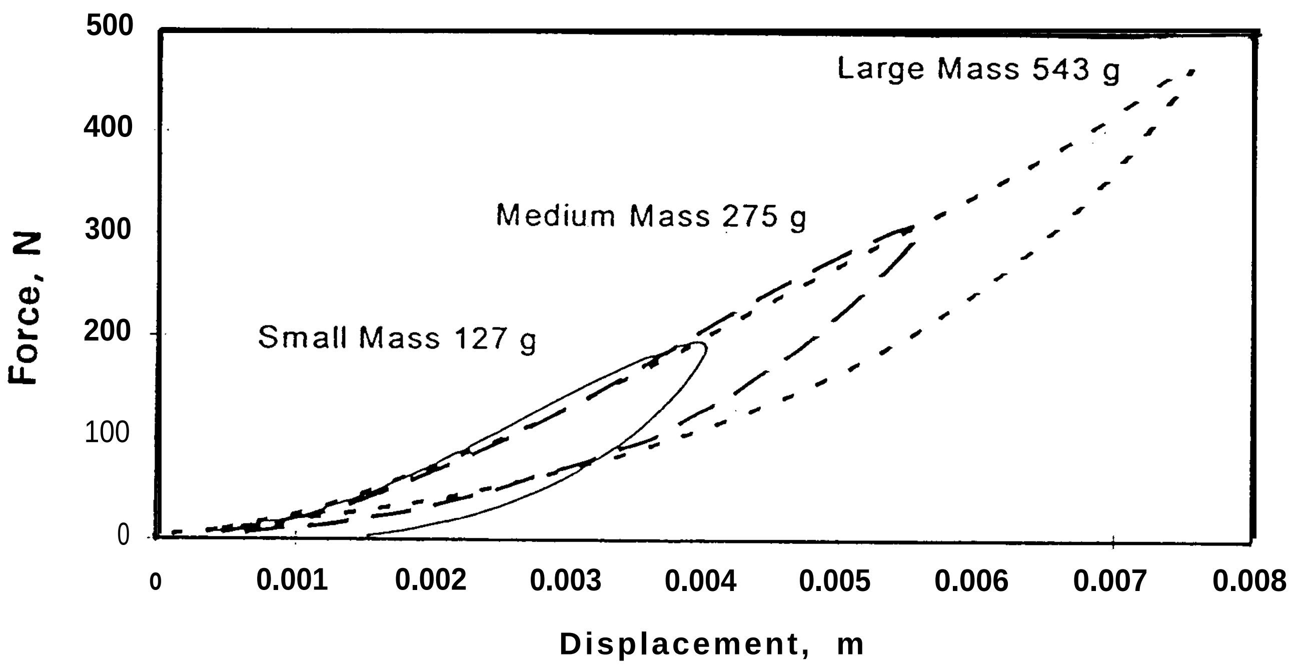

Fig. **6. Dynamic force-deflection curve for foam sample.**

Discussion

As expected, the results of Fig. 6 indicate increasing impact force with mass size while the general force-deflection curve shape is similar for each mass. Also, The curves clearly show the hysteresis loop of energy loss in the loading cycle with the deflection not returning quickly to zero at rebound due to the rate effect. While not illustrated here, another effect attributed to high loading rate is that the force peak does not occur exactly at the time of zero velocity of the mass as might be expected. The calibration check on the double integration determination of dynamic displacement indicates good accuracy so that load and displacement of the sample can be determined with a single transducer output.

This rather simple experiment demonstrates a number of dynamics and materials behavior principles which can significantly increase the depth of understanding of the student. In a dynamics course, the results can be presented as an application of Newton's law illustrating the importance of the free-body-diagram, and double integration of the acceleration measurement with initial conditions enforced. The actual experiment or a presentation and discussion of the results would also be appropriate in the transient vibration portion of a vibrations course. The complete experiment could be assigned for an experimental mechanics course laboratory or project which would include student involvement in all the details required for the data acquisition and processing, computer generated plots, and formal report writeup.

References

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CLARENCE CALDER received his BS degree in Mechanical Engineering from Oregon State University in 1960. He earned an MS degree from Brigham Young University in 1962 and completed a doctorate in Engineering Science at the University of California-Berkeley in 1969. Professor Calder has taught at Washington State University, worked as a research engineer at Lawrence Livermore Nationals Labs, and has been on the Mechanical Engineering faculty at Oregon State University since 1978. He has been very active in the Society for Experimental Mechanics and serrved as President from 1987-88. In addition to teaching in the applied and experimental mechanics areas, Professor Calder conducts experimental research in the areas of wave propagation, impact, laser ultrasonics, smart composites, and sports mechanics.

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