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Node Centrality and Ranking Tool

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Abstract –The paper is about the topology of a network. Find a center is always important in geometry. There are different kinds of center but in general centers minimize or maximize a property of the geometrical figure. The ranking tool is just a way to figure out how far or close to a center is a particular node in the network. Node centrality is the crucial importance in different areas like transportation, wireless communication, Internet, virology and more. Each time when we have a network, knowing where are the centers locations in a network is the crucial importance to improve, optimize, protect or attack a network.

Keywords
Network, Centrality, Node, Link, Topology, Degree, Closeness, Betweenness, and Eigenvector

Introduction

Any network can be characterized by a vector of nodes $N = \{v_1, v_2, \cdots v_n\}$ and an adjacency matrix A which elements are the links l_{ij} between the nodes and, l_{ij} is the link between the nodes is v_i and v_j. In this paper we assume that all of the links do not have a preferred direction, and also all of them have the same value. In addition all of the nodes have the same importance. This means that all of the nodes will be treated as if they have the same characteristics. Under these conditions the matrix A is symmetric.

In any network a critical factor is which node is the more important among all nodes. Just thinking about if we have a disease it will spread jumping from one node to other. In this case cut the links or nodes that are more important will be delay or stop the disease. Or in the case of a computer network figure out which node is the most important one will be use to improve that node to increase our security, or if that is not our network that will be a good node to start and attack. In a traffic network this information will be use to decide which roads or intersections need to be enlarged in order to improve the network in a more efficient way. In a communication network this information can help us to reduce cost and make the less critical nodes more accessible in price.

For all of these reasons and, many more, it is determined which is the importance of the nodes will be useful. Here is when a node centrality and ranking tools make and apparition. We use the centralities to characterize each node in a particular network. Then we assign each node a rank base of how central is this node. In this paper we will talk about 4 node centralities measures :Degree, Closeness, Betweenness, and Eigenvector centrality. Then we will use these criteria independently and jointly to evaluate for different sample networks.

Degree Centrality (DC)

In this first Centrality we will rank as most important node the one which has more connections. For example in the case of a disease the most important thing is limit the links that pathogen can be use to spread the infection. It is easy to image that in this case the number of connections that a node has is the most important thing.
We define dc_i as:

$$dc_i = \sum_{j=1}^{n} dc_{ij} \qquad (1)$$

$$where\ dc_{ij} = \begin{cases} 1\ if\ there\ is\ a\ link\ beteween\ i\ and\ j \\ 0\ if\ there\ is\ not\ a\ \ link\ beteween\ i\ and\ j \end{cases}$$

In this case the network can be characterize by the matrix *DC* or an equivalent vector *DC*

$$DC = \begin{bmatrix} 0 & dc_{12} & \cdots \\ dc_{21} & 0 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \equiv DC = \begin{bmatrix} dc_1 \\ dc_2 \\ \vdots \end{bmatrix}$$

Closeness Centrality (CC)

Now let's say that the most important thing is that a node is close to another node. Or in other words how many channels are between these nodes using the closes path. This idea has relevance, for example, in a distribution network. The most important warehouse will be the one that is close the others warehouses. We can move material from one warehouse to another one and obviously the transport will be cheaper if the warehouses or nodes are close in distance. Of course in this particular example the links will not have the same value. But for now we will use the same values for any link. We define cc_i as

$$cc_i = \left(\sum_{j=1}^{n} cc_{ij} \right)^{-1} \qquad (2)$$

$$where\ cc_{ij} = d(v_i, v_j)$$

$d(v_i, v_j)$ is the distance between the nodes v_i and v_j; and it is the sum of the links that we need to cross for going from the node v_i to the node v_j
In this case the network can be characterize by the matrix *CC*

$$CC = \begin{bmatrix} 0 & cc_{12} & \cdots \\ cc_{21} & 0 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Take in account that

$$cc_{ij} = 0 \Leftrightarrow i = j\ or\ cc_{ij} \geq 1 \Leftrightarrow i \neq j$$

Betweenness Centrality (BC)

In this case the importance of the node depends if the node is part of a path that is use for others nodes. First we define $\|G(v_j, v_k)\|$ as the cardinality of the set that include all of the geodesics between to nodes *j* and *k*. (a geodesic is the minimal distance between two nodes). Also we defined $\|G_{v_i}(v_j, v_k)\|$ which is the cardinality of the subset that include all of the geodesics between nodes *j* and *k* that include the node *i* in their path. This will be relevant in the case of a communication network. If we have a router or antenna that handle the information between many nodes this particular node will be highly important. In this case be close or have connections is not as relevant as be part of the path between nodes. We define bc_i as:

$$bc_i = \sum_{j \neq i \neq k} \frac{\left\| G_{v_i}(v_j, v_k) \right\|}{\left\| G(v_j, v_k) \right\|} \tag{3}$$

The centrality of the node *i* dependent of its relationship with the other nodes for *bc* we do not have a characteristic matrix we have a tensor of order 3 in which the elements of the tensor are:

$$bc_i^{jk} = \frac{\left\| G_{v_i}(v_j, v_k) \right\|}{\left\| G(v_j, v_k) \right\|} \Rightarrow BC = BC_i^{jk} \tag{4}$$

Take in account that

$$0 \leq bc_i^{jk} \leq 1$$

Eigenvector Centrality (EC)

If we use the characteristic matrix *A=DC* which is the matrix defines using equation (1) by the theorem:

> Theorem: (Perron-Frobenius) If a matrix A has a real eigenvalue λ_{max} greater than or equal to the magnitude of any of its other eigenvalues. There is a nonnegative eigenvector v_A associated with λ_{max}

The value ec_i is simple the value of the component *i* of the vector v_A. λ_{max} is also called the spectral radius of *A* and the vector v_A of length 1 is called the Gould vector. This model is particularly use in virology networks. If you put the disease in the most ranking node the disease will be spread faster around the network.

Combine Centrality

Sometimes is useful and more precise use more than one centrality value. In this analysis we use the linear combination of the four centralities describe above. And define.

$$cm_i = w_1 \cdot dc_i + w_2 \cdot cc_i + w_3 \cdot bc_i + w_4 \cdot ec_i \tag{5}$$

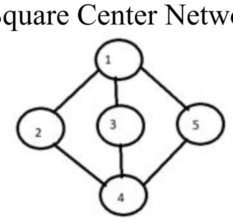
This combine centrality *cm* uses some weights w_n to characterize the degree of importance of each centrality value. Take in account that none of the centralities were normalizing and each value is limited by:

$$\begin{aligned} 1 &\leq dc_i \leq n-1 \\ \frac{1}{(n-1)^2} &\leq cc_i \leq \frac{1}{n-1} \\ 0 &\leq bc_i \leq \binom{n-1}{2} \\ 0 &< ec_i < 1 \end{aligned} \tag{6}$$

This normalization values will be useful if we want that all of the centralities values will be in the same scale.

Examples **Table 1 relative weighs use in all examples**

a) Square Center Network



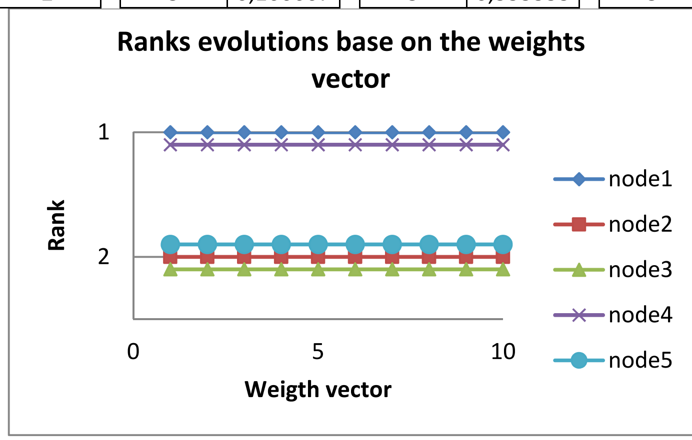
set number	Relative weights base on the normalization value			
1	1	1	1	1
2	4	0	0	0
3	0	4	0	0
4	0	0	4	0
5	0	0	0	4
6	1	1	1,75	0,25
7	1	1	2	0
8	2	0,5	0,5	1
9	0	0,5	3	0,5
10	0,8	1,2	1	1

Node in Rank order	DC value
1	3
4	3
2	2
3	2
5	2

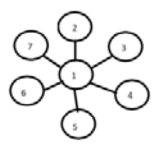
Node in Rank order	CC value
1	0,2
4	0,2
2	0,166667
3	0,166667
5	0,166667

Node in Rank order	BC value
1	1,5
4	1,5
2	0,333333
3	0,333333
5	0,333333

Node in Rank order	EC value
1	0,5
4	0,5
2	0,408248
3	0,408248
5	0,408248



b) Star Network

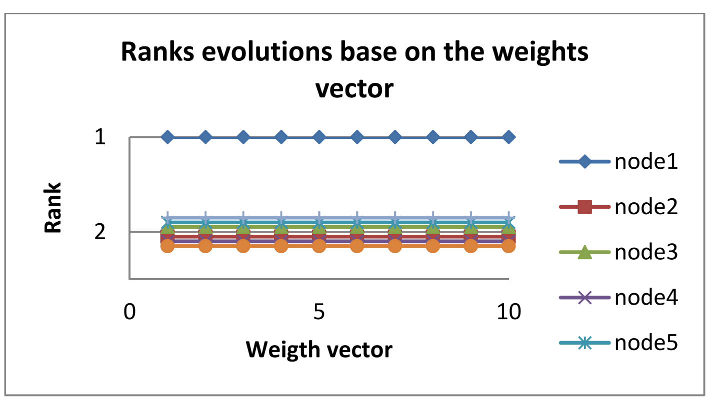


Node in Rank order	DC value
1	6
2	1
3	1
4	1
5	1
6	1
7	1

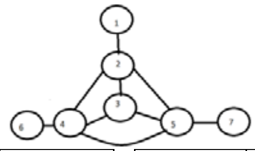
Node in Rank order	CC value
1	0,16666667
2	0,09090909
3	0,09090909
4	0,09090909
5	0,09090909
6	0,09090909
7	0,09090909

Node in Rank order	BC value
1	15
2	0
3	0
4	0
5	0
6	0
7	0

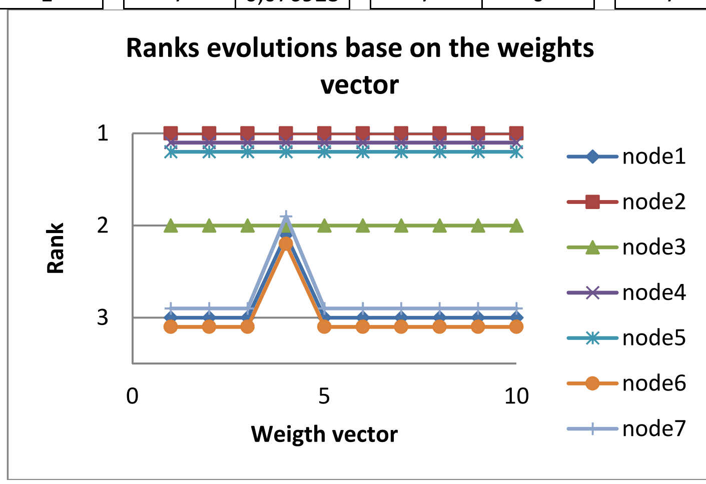
Node in Rank order	EC value
1	0,70710678
2	0,28867513
3	0,28867513
4	0,28867513
5	0,28867513
6	0,28867513
7	0,28867513



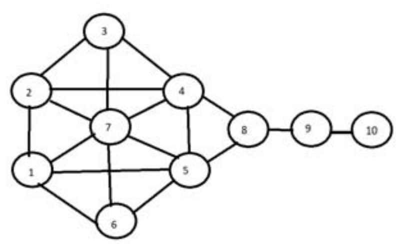
c) Triangular Center Network



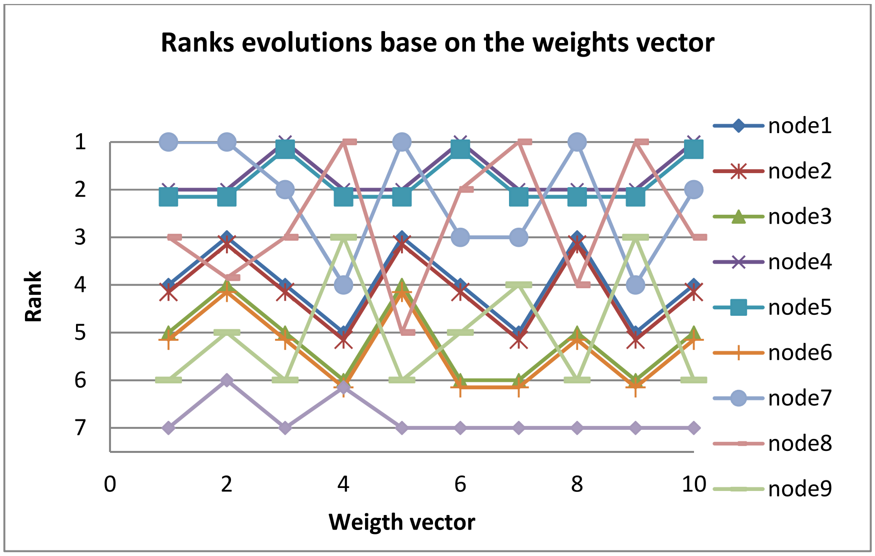
Node in Rank order	DC value	Node in Rank order	CC value	Node in Rank order	BC value	Node in Rank order	EC value
2	4	2	0,125	2	5	2	0,491123
4	4	4	0,125	4	5	4	0,491123
5	4	5	0,125	5	5	5	0,491123
3	3	3	0,111111	1	0	3	0,455296
1	1	1	0,076923	3	0	1	0,151765
6	1	6	0,076923	6	0	6	0,151765
7	1	7	0,076923	7	0	7	0,151765



d) Kite Network



Node in Rank order	DC value	Node in Rank order	CC value	Node in Rank order	BC value	Node in Rank order	EC value
7	6	4	0,071429	8	14	7	0,481021
4	5	5	0,071429	4	8,333333	4	0,397691
5	5	7	0,066667	5	8,333333	5	0,397691
1	4	8	0,066667	9	8	1	0,352209
2	4	1	0,058824	7	3,666667	2	0,352209
3	3	2	0,058824	1	0,833333	3	0,285835
6	3	3	0,055556	2	0,833333	6	0,285835
8	3	6	0,055556	3	0	8	0,195861
9	2	9	0,047619	6	0	9	0,048073
10	1	10	0,034483	10	0	10	0,011163



Conclusions

If a particular node has a lower rank that other node in all of the centralities, that node has a lower rank in any combination of the nodes as long as all the weigh are positive. In the kite example we can observe that the node 8 always has a rank superior the node 9. We can also observe that the nodes 4 and 5 are always on rank 1 or 2, it is ease to notice that these nodes acts a junction between the body and the tale of the kite. We can observe also that there is not a unique rank. The rank is mostly base on the nature of the problem, the same network can have different centers base on different parameters. Another interesting thing is the fact that for different centralities the network can be represented by a vector, matrix or tensor this is mostly because a particular characteristic of the network can be a function of the node, the links or the relationship between the node and the links.

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