

## **Numeric Tolerances in Online Learning Management Platforms: A Case Study in Heat Transfer**

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## Abstract

The Nusselt number physically represents the enhancement in heat transfer through a fluid layer due to convection relative to conduction. However, the Nusselt number is often thought of as a dimensionless heat transfer coefficient in undergraduate heat transfer curricula. To obtain a value for the convection heat transfer coefficient, students must select an appropriate Nusselt number correlation. Unfortunately, in undergraduate heat transfer textbooks, as many as five Nusselt number correlations are presented for fully developed turbulent flows in smooth tubes. Furthermore, the Nusselt number correlations often have the same, or similar, Reynolds and Prandtl number criteria. Therefore, there may be more than one appropriate Nusselt number correlation for any given problem. An excess of appropriate Nusselt number correlations is problematic when evaluating student work in online learning management systems because numeric tolerances are often too small to account for variations in Nusselt numbers. For example, there is a 96.9 % difference between the minimum and maximum Nusselt numbers obtained for a Reynolds number of  $1 \times 10^6$  and a Prandtl number of 100. So, it is possible for a student to both choose an appropriate correlation and be marked as incorrect due to the numeric tolerance of the online learning management system. The discrepancy between these correlations under identical criteria is examined and presented.

## Introduction

To obtain a value for the convection heat transfer coefficient, students must select an appropriate Nusselt number correlation. Unfortunately, in undergraduate heat transfer textbooks, as many as six Nusselt number correlations are presented for fully developed turbulent flows in smooth tubes. Furthermore, the Nusselt number correlations often have the same, or similar, Reynolds and Prandtl number criteria. In an era of web-based assignment and assessment platforms, excess Nusselt number correlations can result in student answers outside of default numeric tolerances. Therefore, the most commonly utilized Nusselt number correlations are presented in the next section before addressing the correlations' discrepancies.

But first, it should be noted that while there have been several investigations about the effectiveness of online learning management platforms (e.g., [1-5]), there is limited literature regarding numeric tolerances for algorithmically generated numeric response questions. While Nicholls et al. [6] noted that numeric tolerances should be adjusted to account for rounding and significant figures, the answers were based on the *same equations* in the solution procedure. This study aims to comprehensively analyze the impact of numeric tolerances in online learning platforms, specifically focusing on the variability introduced by *different* Nusselt number correlations. The primary objectives of the research are as follows:

1. Systematically examine the various Nusselt number correlations presented in undergraduate heat transfer textbooks. This analysis will form the basis for understanding

the potential diversity in students' choices of correlations when solving heat transfer problems.

2. Investigate how default numeric tolerances impact the evaluation of student answers and explore scenarios where students may be marked as incorrect despite choosing an appropriate correlation.
3. Determine whether certain correlations are consistently utilized in textbook solutions, potentially influencing students to favor specific correlations. This analysis will shed light on instructor challenges when aligning textbook problems with online assessment platforms.

### Empirical Nusselt Number Correlations

An overview of the Nusselt number correlations presented in the undergraduate heat transfer textbooks examined [7-9] can be found in Table 1. The correlations are presented as shown in Çengel and Ghajar [7].

(Background Note: The empirical correlations assume a fully developed and turbulent flow in a smooth pipe and are valid for a range of Reynolds ( $Re$ ) and Prandtl ( $Pr$ ) numbers. More specifically, the flow is assumed to be fully developed if the hydrodynamic ( $L_h$ ) and thermal ( $L_t$ ) entrance lengths are much less than the total pipe length (typically,  $L/D > 60$ ). The hydrodynamic and thermal entrance lengths for turbulent flows are approximately equal to  $10D$ . In addition, the flow is assumed to be turbulent if the Reynolds number is greater than or equal to ten thousand ( $Re \geq 10,000$ ).)

Each Nusselt number correlation is a function of the Reynolds and Prandtl numbers. In addition, the Chilton-Colburn, second Petukhov, and Gnielinski equations are also functions of the friction factor. (For example, for smooth tubes, the friction factor in turbulent flow can be determined from the first Petukhov equation ( $f = 0.790 \ln(Re) - 1.64$ )<sup>2</sup>.) Finally, the Sieder and Tate equation is also a function of the fluid's dynamic viscosity ( $\mu$ ). It can be used when the difference between the fluid and wall surface temperatures is significant, and a considerable variation in properties is expected.

Table 1. Empirical Nusselt Number Correlations for Fully Developed Turbulent Flows in Smooth Tubes [7-9]

Equation	Correlation	Criteria	Textbooks
Chilton-Colburn analogy	$Nu = 0.125 f Re Pr^{1/3}$	$0.6 < Pr < 60$	Çengel and Ghajar; Incropera and DeWitt
Colburn	$Nu = 0.023 Re^{0.8} Pr^{1/3}$	$(0.7 \leq Pr \leq 160)$ $(Re > 10,000)$	Çengel and Ghajar; Incropera and DeWitt
Dittus-Boelter	$Nu = 0.023 Re^{0.8} Pr^n$	$(0.7 \leq Pr \leq 160)$ $(Re > 10,000)$	Bergman et. al; Çengel and Ghajar; Incropera and DeWitt
Sieder-Tate	$Nu = 0.027 Re^{0.8} Pr^{1/3} \left(\frac{\mu_b}{\mu_s}\right)^{0.14}$	$(0.7 \leq Pr \leq 16,700)$ $(Re \geq 10,000)$	Bergman et. al; Çengel and Ghajar; Incropera and

			DeWitt
second Petukhov	$Nu = \frac{\left(\frac{f}{8}\right) Re Pr}{1.07 + 12.7 \left(\frac{f}{8}\right)^{0.5} (Pr^{\frac{2}{3}} - 1)}$	$\left( \begin{array}{l} 0.5 \leq Pr \leq 2000 \\ 10,000 < Re < 5 \times 10^6 \end{array} \right)$	Çengel and Ghajar; Incropera and DeWitt
Gnielinski	$Nu = \frac{\left(\frac{f}{8}\right) (Re - 1000) Pr}{1 + 12.7 \left(\frac{f}{8}\right)^{0.5} (Pr^{\frac{2}{3}} - 1)}$	$\left( \begin{array}{l} 0.5 \leq Pr \leq 2000 \\ 3,000 < Re < 5 \times 10^6 \end{array} \right)$	Bergman et. al; Çengel and Ghajar; Incropera and DeWitt

### The Discrepancy in Calculated Nusselt Numbers

The Nusselt numbers calculated from the correlations in Table 1 for Prandtl numbers of 1, 10, 100, and 1000 and Reynolds numbers of  $10^4$ ,  $10^5$ , and  $10^6$  can be found in the Appendix. In addition, the Nusselt numbers calculated from the Chilton-Colburn, Dittus-Boelter, and Gnielinski equations as a function of the Reynolds number and for a Prandtl number of 10 can be found in Figure 1. The percent difference between the minimum and maximum Nusselt numbers calculated for the same Reynolds and Prandtl number criteria is of particular interest. Generally, the percent difference between the minimum and maximum Nusselt numbers increases with Reynolds and Prandtl numbers. For example, for a Reynolds number of  $10^6$  and Prandtl number of 10, the Colburn equation results in a Nusselt number of 3126.5 while the Gnielinski equation results in a Nusselt number of 5254.4, a 68.1% difference.

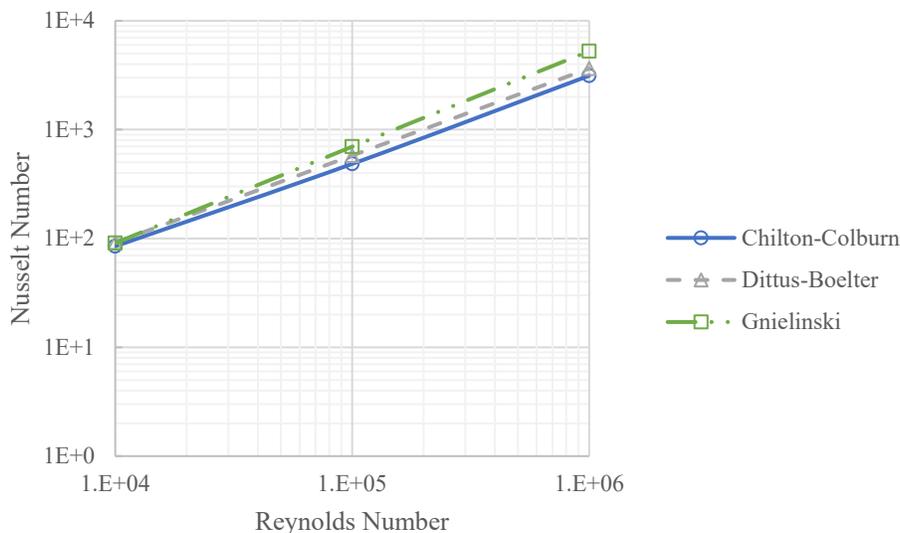


Figure 1. Nusselt Number as a Function of Reynolds Number (Pr = 10)

Unfortunately, each correlation is correct, assuming the Reynolds and Prandtl number criteria are satisfied. Therefore, two students may solve a problem correctly but arrive at significantly different answers. Furthermore, in an era of web-based assignment and assessment platforms, using an alternative, although correct, Nusselt number correlation may result in student answers outside of the assessment platform's default numeric tolerance (typically,  $\pm 2\%$ ). In addition, it is

unreasonable for instructors to modify the numeric tolerance to accommodate Nusselt number correlations that may differ by nearly 70%.

A homework problem was assigned in an online learning management problem with a default numeric tolerance of  $\pm 2\%$  to illustrate why this is so problematic. The platform arbitrarily chose to use the Dittus-Boelter equation to solve for the Nusselt number. However, all six of the Nusselt number correlations presented in Table 1 were equally valid choices for the conditions in the problem. Unfortunately, only two of the six Nusselt number correlations (Colburn and Dittus-Boelter) could be used for the platform to mark the answer as correct. If a student chose one of the other four Nusselt number correlations, their answer would be marked as incorrect – even though their choice was technically correct. In fact, the second Petukhov and Gnielinski equations are well-known to be more accurate. In summary, a student can solve a problem correctly but be marked as incorrect, which can be stressful for the student and require time-consuming error diagnosing from the instructor.

### Correlations Utilized by Textbooks in End-of-Chapter Problems

Another complicating factor for instructors is the choice of Nusselt number correlation that various textbooks utilize to solve end-of-chapter problems. Again, more than one Nusselt number correlation may be appropriate for the same criteria. For example, in Çengel and Ghajar’s undergraduate heat transfer textbook [7], six different Nusselt number correlations are presented for fully developed turbulent flows in smooth tubes. However, over 90% of the solutions to the end-of-chapter problems are based on just two of the five Nusselt number correlations – the Dittus-Boelter and Gnielinski equations (see Figure 2). Finally, suppose an instructor utilizes an online assessment platform. In that case, they need to be cognizant that a student may not receive credit for a problem they correctly solved because their choice of correlation resulted in an answer outside of the platform’s numeric tolerance.

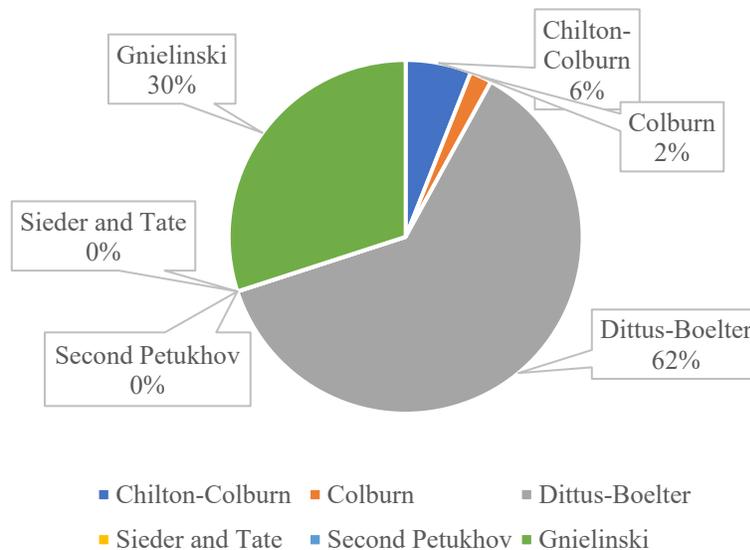


Figure 2. Percentage of Problems Utilizing a Specific Nusselt Number Correlation in Çengel and Ghajar’s text [7].

## Recommendations for Instructors

In light of the challenges posed by numeric tolerances in online learning management systems when assessing heat transfer problems involving Nusselt number correlations, instructors play a crucial role in mitigating potential issues and ensuring a fair evaluation of students' understanding. Here are three recommendations for course instructors:

1. Instructors could consider selectively assigning problems that align with the Nusselt number correlation of their preference. By focusing on specific correlations in the curriculum, instructors provide clarity for students and reduce the likelihood of answers that fall outside default numeric tolerances.
2. To address the inherent ambiguity arising from the multitude of Nusselt number correlations, instructors may opt to create custom problems tailored to the specific correlation they want to emphasize. By crafting problems that align with a chosen correlation, instructors can align assessments with their teaching objectives and reduce the potential for discrepancies caused by the diversity of valid correlations. This approach enhances the coherence between instructional content and assessment criteria.
3. Instructors may find it beneficial to recommend a specific Nusselt number correlation to students, offering guidance on the correlation that aligns with the course content and assessments. By communicating this recommendation clearly, instructors empower students to focus on a particular correlation when solving problems. For example, in the reference handbook for the FE exam [10], only two Nusselt number correlations are presented for turbulent flows – the Dittus-Boelter and Sieder-Tate equations.

## Conclusions

In this study, investigating the impact of numeric tolerances in online learning management systems on assessing heat transfer problems has revealed significant challenges students and instructors face.

The plethora of Nusselt number correlations available for fully developed turbulent flows in smooth tubes introduces ambiguity in selecting the “correct” correlation for a given problem. This study has highlighted the potential for substantial variations in calculated Nusselt numbers, with differences exceeding 90% under certain conditions. The implications of such variations become particularly pronounced in web-based assignment and assessment platforms, where default numeric tolerances often fall short of accommodating the diversity of valid correlations.

The investigation has underscored a critical issue – the possibility of students arriving at technically accurate solutions but facing penalties due to the limitations of numeric tolerances in online platforms. This induces stress among students and burdens instructors who must navigate these discrepancies and engage in time-consuming error diagnosis.

Furthermore, the study sheds light on the prevalence of specific Nusselt number correlations in end-of-chapter problems in textbooks, potentially influencing students to favor specific correlations. Instructors using online assessment platforms should be mindful of this bias, as

students may employ valid correlations that differ from those predominantly utilized in textbook solutions, resulting in marked discrepancies in assessment outcomes.

In conclusion, addressing the challenges posed by numeric tolerances in online learning management systems requires a nuanced approach. It is imperative for both educators and platform developers to recognize the inherent diversity in valid Nusselt number correlations and consider adjustments to numeric tolerances that align with the intricacies of heat transfer problems. This study advocates for a more comprehensive understanding of the implications of correlation choices on student assessments, emphasizing the need for adaptive assessment frameworks that accommodate the inherent variability in valid solutions.

## References

- [1] C. Y. Yan, "Online Homework Assignments: Instructor's Perspective and Students' Responses," presented at 2016 ASEE Annual Conference & Exposition, New Orleans, Louisiana, June 2016. DOI: 10.18260/p.25830
- [2] S. C. Maroo, "Positive Statistical Impact of Online Homework Assignments on Exam and Overall Course Grades," presented at 2019 ASEE Annual Conference & Exposition, Tampa, Florida, June 2019. DOI: 10.18260/1-2--33178
- [3] L. Reis, K. A. Evans, and D. Cahoy, "WIP: An Ongoing Analysis of the Impact of Assigning Online Thermodynamic Homework in Place of Traditional Homework," presented at 2017 ASEE Annual Conference & Exposition, Columbus, Ohio, June 2017. DOI: 10.18260/1-2—29131
- [4] K. Evans, P. Hummel, and M. Gates, "Assessing the Effect of Online Homework on Student Learning in a First Circuits Course," presented at 2015 ASEE Annual Conference & Exposition, Seattle, Washington, June 2015. DOI: 10.18260/p.23586
- [5] D. J. Broderick, "Qualitative and Quantitative Analysis of Use of Online Homework for Circuit Analysis," presented at 2019 ASEE Annual Conference & Exposition, Tampa, Florida, June 2019. DOI: 10.18260/1-2--33219
- [6] G. M. Nicholls, W. J. Schell, and N. A. Lewis, "Best Practices for Using Algorithmic Calculated Questions via a Course Learning Management System," presented at 2016 ASEE Annual Conference & Exposition, New Orleans, Louisiana, June 2016. DOI: 10.18260/p.26377
- [7] Y. Çengel and A. Ghajar, *Heat and Mass Transfer: Fundamentals & Applications*, McGraw-Hill Education, New York, 2020, pp. 514-515.
- [8] T. L. Bergman, A. S. Lavine, F. P. Incropera, and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 8th ed. John Wiley & Sons, ISBN: 978-1-119-35388-1, December 2018.
- [9] F. P. Incropera and D. P. DeWitt, *Introduction to Heat Transfer*, 2nd ed. Wiley, 1985. ISBN: 0471801267, 9780471801269.
- [10] *FE Reference Handbook*, Version 10.3, National Council of Examiners for Engineering and Surveying (NCEES), 2023. ISBN: 978-1-947801-11-0

## Appendix

Table 2. Nusselt Numbers as a Function of Reynolds Number, Prandtl Number, and Friction Factor

		Nu								
	Re	f	Chilton- Colburn	Colburn	Dittus- Boelter	Second Petukhov	Gnielinski	$Nu_{min}$	$Nu_{max}$	Difference (%)
Pr = 1	$10^4$	0.0315	39.3	36.5	36.5	36.8	35.4	35.4	39.3	11.1
	$10^5$	0.0180	224.9	230.0	230.0	210.2	222.7	210.2	230.0	9.4
	$10^6$	0.0116	1453.3	1451.2	1451.2	1358.2	1451.8	1358.2	1453.3	7.0
Pr = 10	$10^4$	0.0315	84.8	78.5	91.6	99.1	90.8	78.5	99.1	26.2
	$10^5$	0.0180	484.5	495.5	577.7	689.2	697.3	484.5	697.3	43.9
	$10^6$	0.0116	3131.0	3126.5	3645.3	5129.7	5254.4	3126.5	5254.4	68.1
Pr = 100	$10^4$	0.0315		169.2	230.0	225.7	203.9	169.2	230.0	35.9
	$10^5$	0.0180		1067.6	1451.2	1672.9	1664.9	1067.6	1672.9	56.7
	$10^6$	0.0116		6735.9	9156.5	13191.9	13263.0	6735.9	13263.0	96.9
Pr = 1000	$10^4$	0.0315				492.2	443.4	443.4	492.2	11.0
	$10^5$	0.0180				3705.4	3672.6	3672.6	3705.4	0.9
	$10^6$	0.0116				29658.5	29671.2	29658.5	29671.2	0.0