Session 2268

Observations of Improved Student Comprehension of Fatigue Analysis using a Novel Fatigue Pedagogy

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Abstract

This paper focuses on the author's observations of two pedagogical techniques for teaching fatigue related material in an upper division engineering machine elements course, where a working understanding in fatigue analysis factors heavily in the success of a student. Initially, a non-generalized method was utilized, where every new application area would require slightly different student learning. Using student feedback, the described method was devised by the author and presented to the students during lectures. The author observed that this generalized methodology lectures improved student comprehension after this methodology was introduced after one mid-semester break. After integrating the methodology at mid-semester break, student comments were favorable and supported utilization of the methodology during the entire semester.

The author's efforts to generalize teaching fatigue analysis enabled a much larger number of students to have the confidence to successfully attack complex homework and examination problems than when fatigue was taught in a non-generalized manner. The described structured methodology has been successfully applied in the classroom at three universities. The non-generalized approach for teaching fatigue analysis at the sophomore through senior levels has traditionally been incremental, starting with simple, alternating loading cases and then progressively working through more complex loading cases. The simple loading cases include alternating stresses compared with the endurance strength. More complexity is typically introduced through the addition of mean, midrange, or steady-state stresses to the model. As still more complexity is introduced through the addition of pre-load stresses, the students typically begin questioning when the incremental increases in complexity will taper off.

This paper presents an instructional methodology that enables students to use relatively simple analytical techniques to resolve constant-amplitude, sinusoidal loading patterns into maximum, minimum, steady-state, pre-load, and alternating components. Once these five loading components are determined, standard fatigue analysis techniques can be used to determine the steady-state, pre-load, and alternating stresses caused by these loads. Coupled with the ultimate strength and the determined endurance strength, a factor of safety with respect to fatigue can be determined using the described methodology for a large number of cases that are subsets of the generalized methodology.

While this paper presents an improved pedagogical approach to teaching fatigue, it doesn't present any new theoretical results from the author's fatigue research. The pedagogical method builds on the material in two well-accepted engineering textbooks[1,2], and demonstrates an improvement in student comprehension of fatigue principals.

I. High Cycle Life, Stress-Life Fatigue Analysis Background.

There are excellent, pre-existing texts that cover the subject of fatigue analysis theory incredibly well for a practicing engineer, including Mechanical Engineering Design, 5th Edition by Shigley/Mischke[1] and Machine Design, an Integrated Approach, by Norton[2]. Ductile and brittle materials are both analyzed using this methodology with a variety of loading patterns. The general approach for the factor of safety method with both texts is to determine the ultimate strength, Sut, the endurance limit, Se, a variety of stresses, and then determine a factor of safety with respect to fatigue, n_f, using a Modified Goodman Diagram depicted below in Figure 1.

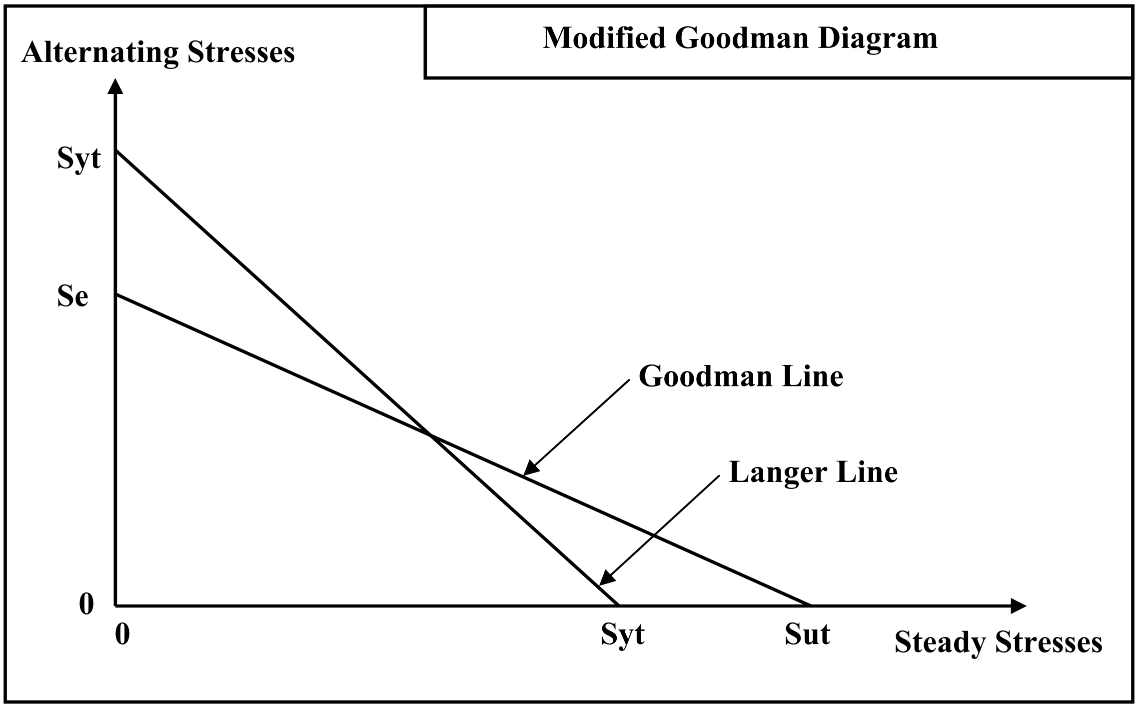

Figure 1: Typical Modified Goodman Diagram with Langer Line

Determination of stress concentration factors for application to the alternating stress component in the case of ductile materials are described well. Both texts also describe the addition of the Langer line to the Modified Goodman Diagram to determine the factor of safety with respect to first cycle yielding, n_y, and describe the concept of infinite machine element life if the combined stresses produce an operating point below the Goodman line with a resulting $n_f > 1$ and a resulting expected life of the machine element being greater than 10^6 cycles.

For finite life cases where $n_f < 1$ and, thus, the expected life of the machine element is less than 10^6 cycles, both texts describe a method to determine the expected number of cycles to failure down to a minimum of 10^3 cycles where the low cycle fatigue region of the Strength-Number of Cycles (S-N) diagram is encountered. Figure 2 depicts an S-N diagram that was developed experimentally by students during a series of fatigue experiments in the Mechanical Engineering Technology department at Purdue University during Fall semester 2000 in MET 211[3].

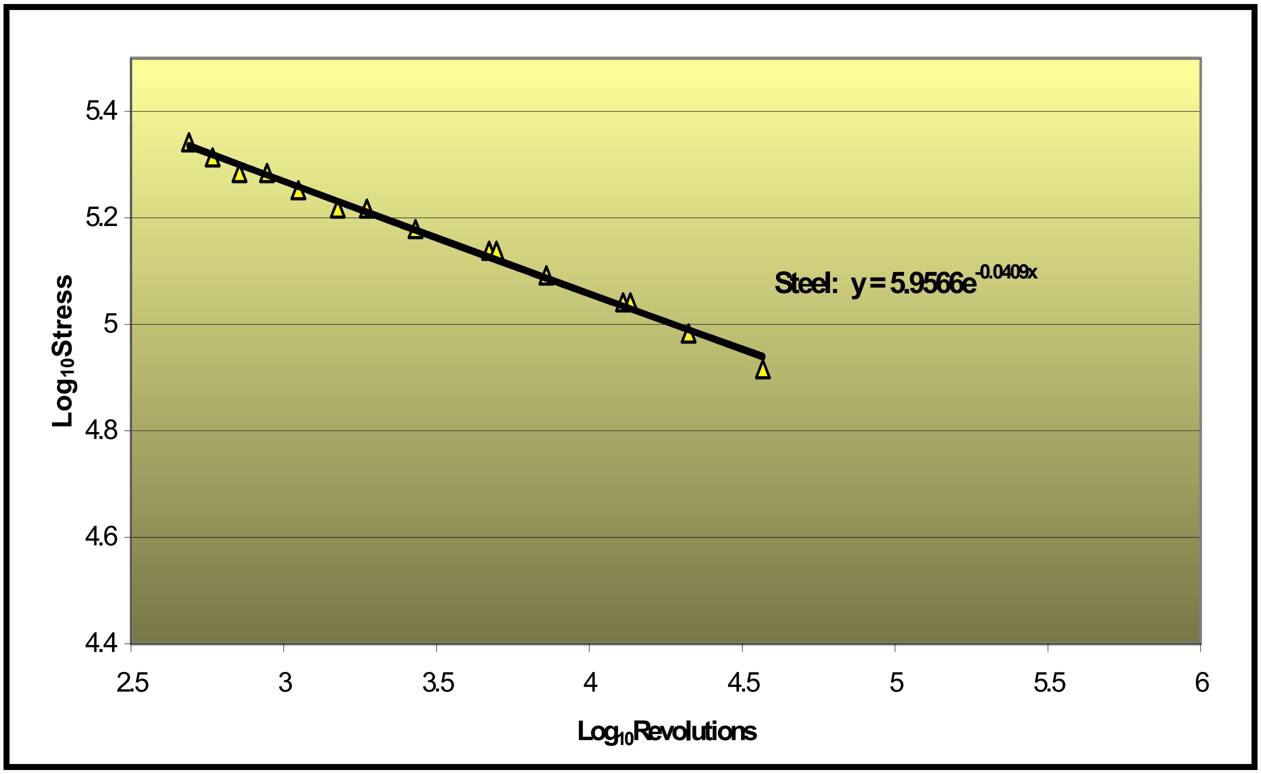

Figure 2: Example of Experimentally Derived S-N Diagram for Steel with a Alternating Bending Loading Mode

The relatively simple fatigue-loading mode of completely reversed bending is represented on the Modified Goodman diagram with an operating point and load line as shown in Figure 3 below. Since the operating point is below the Goodman Line, for this example, the machine element will have an infinite, expected fatigue life. For the data points depicted above in Figure 3, the respective operating points will be above the Goodman Line, indicating finite life. Of course, the knee in the S-N and Modified Goodman diagrams is represented by $Log_{10}Se$ or Se, respectively, and represents the demarcation between finite and infinite life. An observation about the above experimental results is that student interest diminishes with the longer cycle life results, even though a prediction was made analytically. A more in-depth discussion of this fatigue experiment will be given in a future paper.

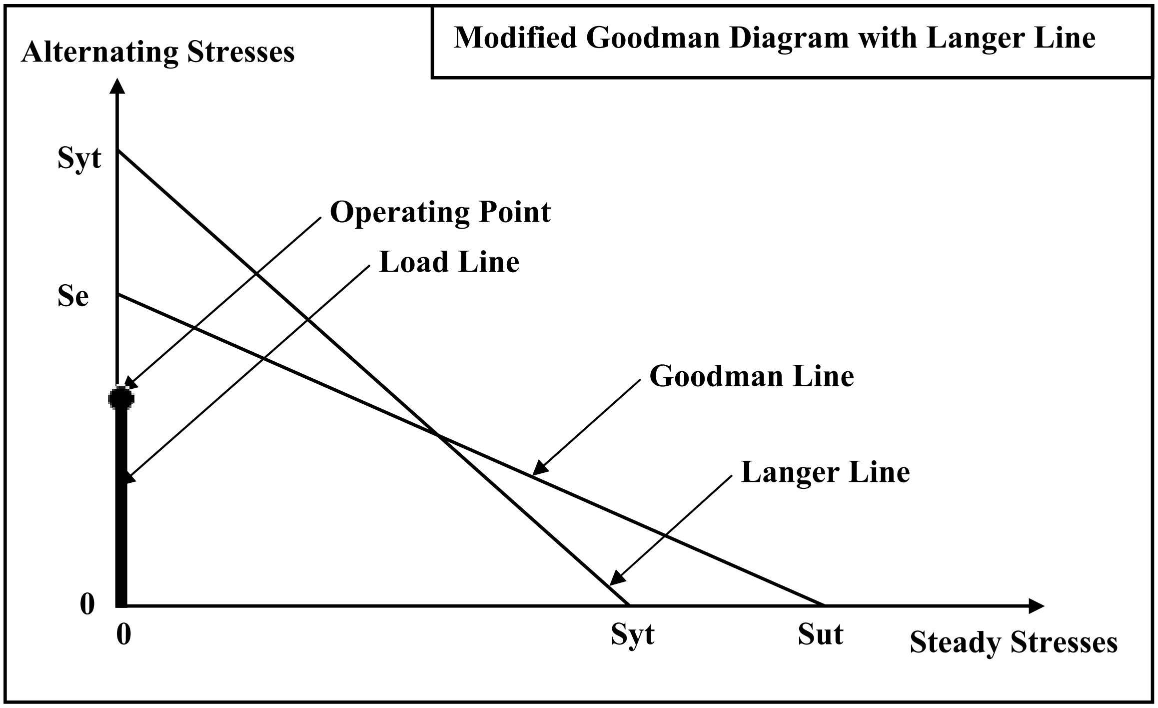

Figure 3: Typical Modified Goodman Diagram with Langer Line for Alternating Stresses and Infinite Life

II. Endurance Limit Calculation

In both Shigley/Mischke and Norton, one must first calculate a property of the specimen under investigation called the endurance limit. Both discuss determining a rotating beam endurance limit that is a function of the ultimate strength and correct that ideal strength for various correction factors. In Shigley/Mischke, the rotating beam endurance limit, Se', is determined as a function of the material's ultimate strength. Given the presence of strength modifying factors, Se' is reduced by Marin factors with account for surface finish with ka, specimen size with kb, specimen loading with kc, temperature with kd, and miscellaneous-effects with ke. Here, the endurance limit, Se is calculated as shown in the equation below:

Equation 1: $Se = ka * kb * kc * kd * ke * Se'$

Similarly in Norton, the rotating beam endurance limit is reduced with correction factors, Cload, Csize, Csurf, Ctemp, and Creliability giving a corrected endurance limit for the material, Se, as shown in the equation below:

Equation 2: $Se = Cload * Csize * Csurf * Ctemp * Creliab * Se'$

Students can generally quickly master the calculation of the endurance strength as presented in both texts, though the author has teaching experience with only the Shigley/Mischke text.

III. Loading Component Calculation

Both Shigley/Mischke and Norton begin their fatigue analysis with a discussion on calculating the force, moment, and torque components of the various stresses. They present equations to determine the alternating force, Falt, the alternating moment, Malt, and the alternating torque, Talt if there is a loading present that harmonically varies between a minimum and maximum value, as shown in the equation below in Equation 3.

Equation 3: Falt = (Fmax – Fmin)/2; Malt = (Mmax – Mmin)/2; and Talt = (Tmax – Tmin)/2

Figure 4 below depicts the Repeated Force Pattern used during lecture to support determination of Falt, Fss, and Fpre from Fmax and Fmin for the case of alternating loading.

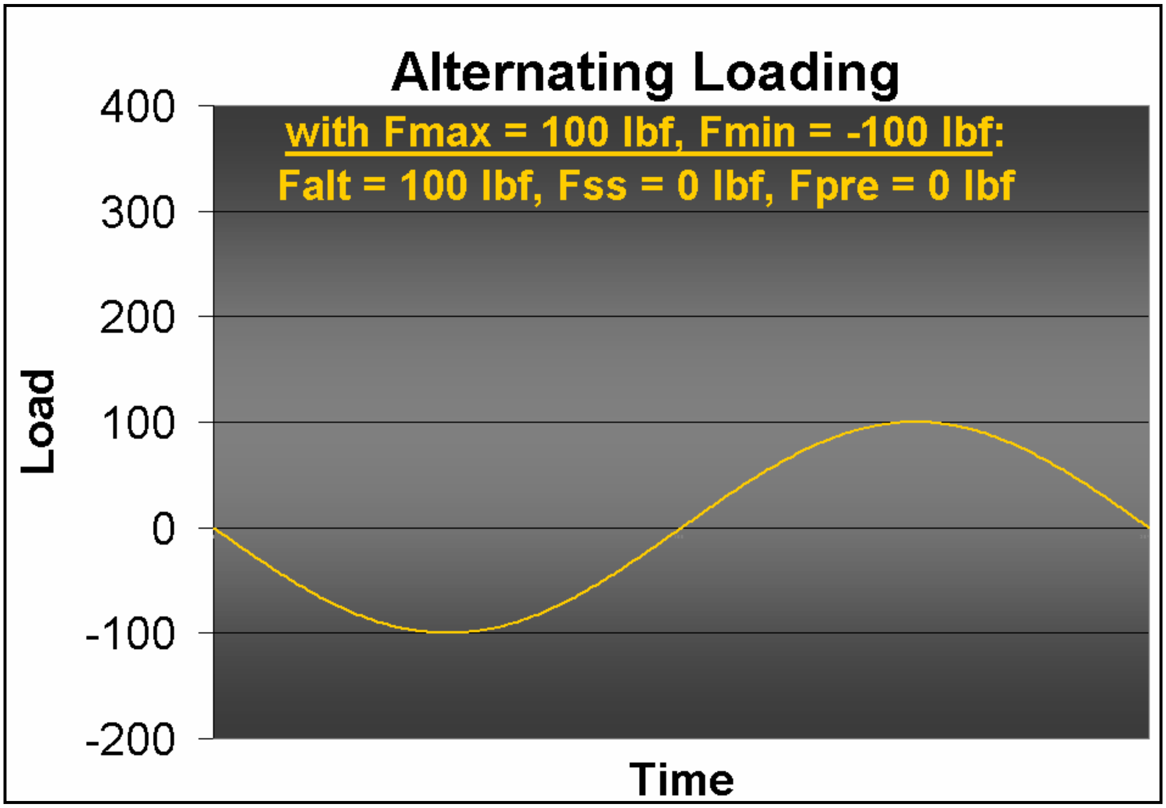

Figure 4: Repeated Force Pattern, F1(t) used to determine Falt, Fss, & Fpre from Fmax and Fmin

Both authors subsequently present the equation to derive the steady state, mid-range, or mean force, Fss, moment, Mss, and torque, Tss as shown in Equation 4 below:

Equation 4: Fss = (Fmax + Fmin)/2; Mss = (Mmax + Mmin)/2; and Ts = (Tmax + Tmin)/2

Figure 5 below depicts the Repeated Force Pattern used during lecture to support determination of Falt, Fss, and Fpre from Fmax and Fmin for the case of alternating and steady state loading.

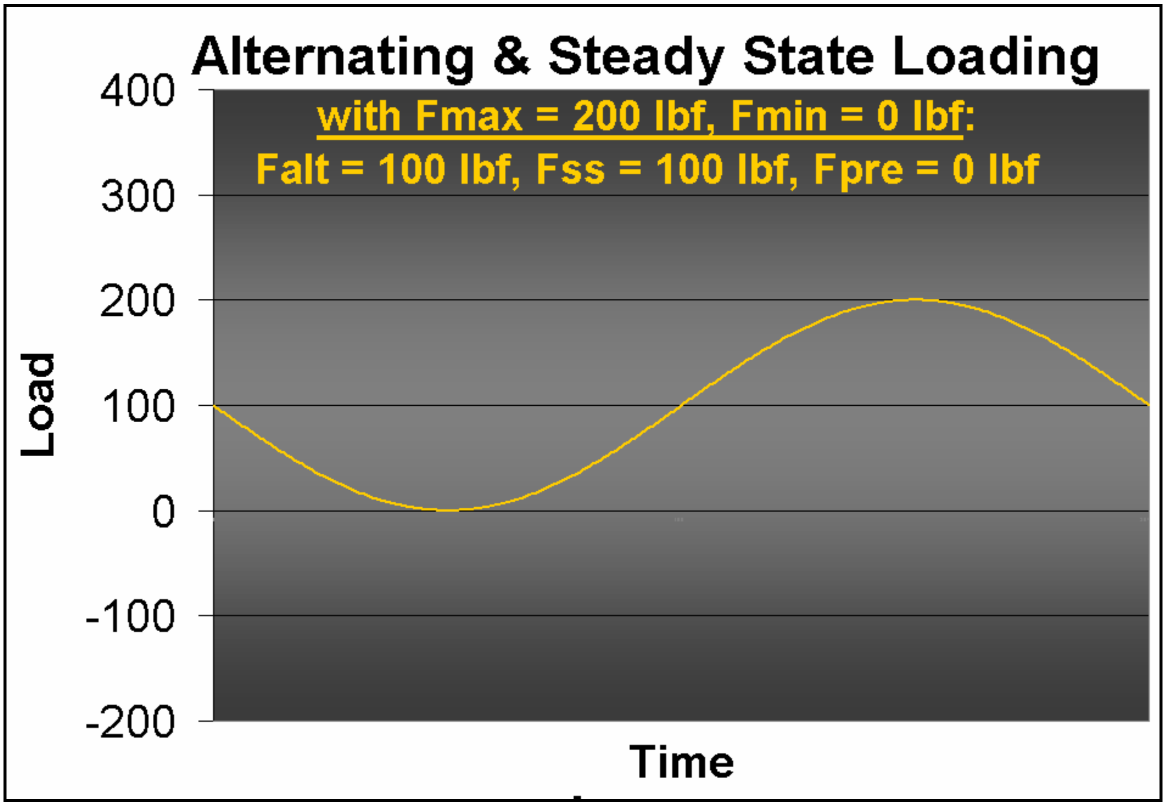

Figure 5: Force Pattern F2(t) used to determine Falt, Fss, & Fpre from Fmax and Fmin

For the case where Fmin, Mmin, or Tmin is greater than zero, a pre-load force, Fpre, pre-load moment, Mpre, or pre-load torque, Tpre are present and equal to Fmin, Mmin, or Tmin, respectively, as shown using Equation 5 below:

Equation 5: If Fmin > 0, then Fpre = Fmin, else Fpre = 0
If Mmin > 0, Mpre = Mmin, else Mpre = 0
If Tmin > 0, Tpre = Tmin, else Tpre = 0

Figure 6 below depicts the Repeated Force Pattern used during lecture to support determination of Falt, Fss, and Fpre from Fmax and Fmin for the case of alternating, steady state, and preload loading.

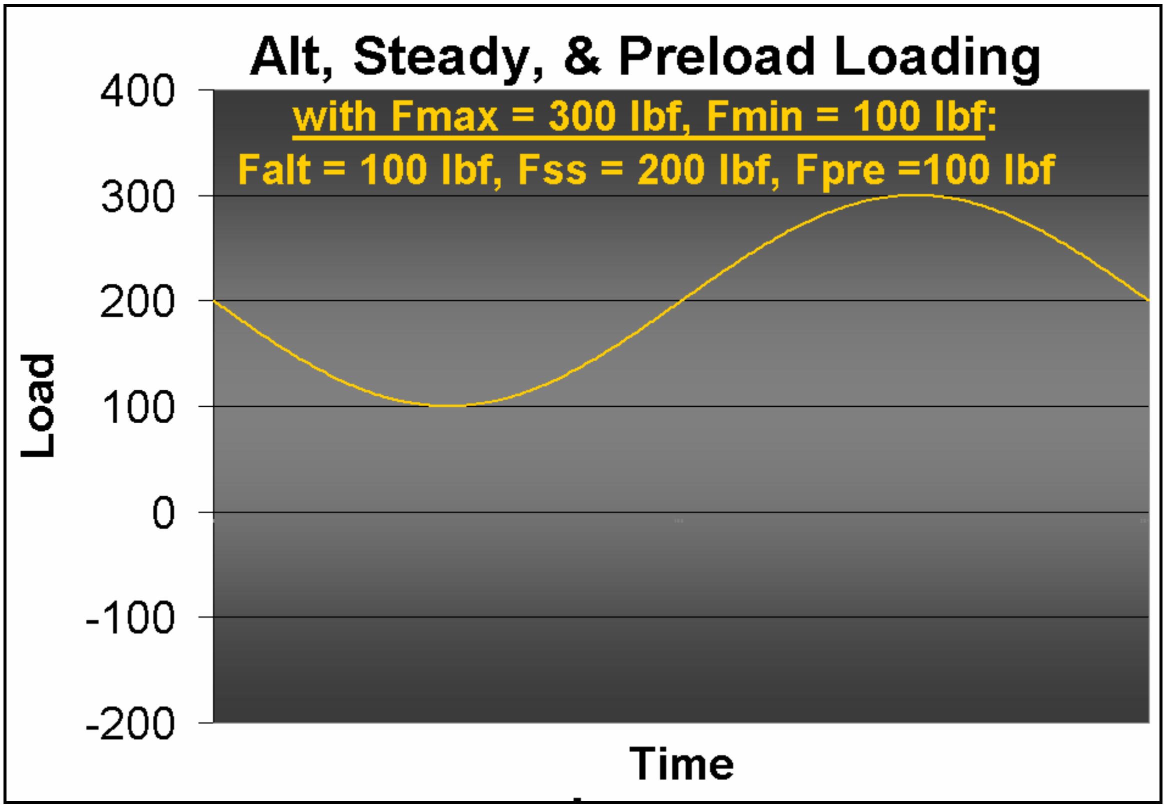

Figure 6: Force Pattern, F3(t) used to determine Falt, Fss, & Fpre from Fmax and Fmin

The author utilizes many machine element types with the loading patterns from Figures 4, 5 and/or 6 to build the fatigue analysis methodology. Typically, machine elements that have previously been analyzed earlier in the course for static loading are utilized, since this enables the lecture to focus more directly on the material at hand: fatigue. There is richness in fatigue analysis that can be more fully understood by the students when a well-known machine element is subjected to the varying load patterns as described above. A typical machine element used to illustrate the fatigue methodology to students is the basic bent rod with a fillet at the wall (for stress concentration) that is subjected to a forcing function, F(t) at the rod tip as shown in Figure 7 below:

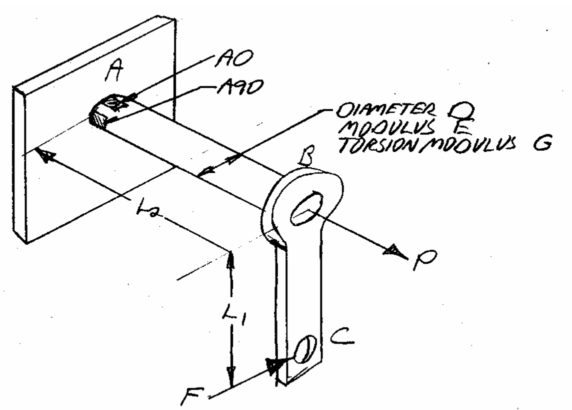

Figure 7: Generic loading configuration used to illustrate fatigue analysis methodology at stress element A90

The architecture of this type of problem allows the instructor to present the student a framework in which they can determine the values of what the author calls "the Big 5" forces: Fmax, Fmin, Falt, Fss, and Fpre, where Fmax is the maximum force, Fmin is the minimum force, Falt is the alternating force component, Fss is the steady state force component, and Fpre is the preload force component. A thorough grounding in finding "the Big 5" was found to be crucial to student success with subsequent analysis. Figures 4, 5, and 6 highlight forcing functions that are described below in words. With a forcing function, F1(t), that fluctuates regularly from –F to +F, it is clear from above equations that Fmax = F, Fmin = -F, Falt = F, Fss = 0, and Fpre = 0. With a forcing function, F2(t), that fluctuates regularly from 0 to 2F, Fmax = 2F, Fmin = 0, Falt = F, Fss = F, and Fpre = 0. With a forcing function, F3(t), that fluctuates regularly from F to 3F, Fmax = 3F, Fmin = F, Falt = F, Fss = 2F, and Fpre = F.

Utilizing F1(t), F2(t), and F3(t) in this manner prepares the students for fatigue analysis using the methodology will become clear during the stress analysis explanation. On examinations, a harmonic forcing function can be drawn with any maximum and minimum value and the students can easily get started accurately. It should be noted that originally, the author explored utililizing a fourth forcing function, F4(t), that fluctuated regularly from –2F to 0, but since compression is much less interesting in fatigue analysis, this function is mentioned during lecture with a homework problem or two.

IV. Stress Element Utilization

The basis for the pedagogical improvement presented is tied to the utilization of the stress element concept in a novel manner. By the time this topic is introduced, students are familiar with how to develop the stress elements for selected locations on a machine element. Mohr's circle analysis is performed to allow calculation of the maximum shear stress at the sophomore course level, allowing a factor of safety with respect to yielding to be determined for ductile materials with a defined yield strength, Syt. Von Mises stresses are introduced in the junior and senior level course, to more accurately represent the net stress state at any stress element.

Since in the example of the bent rod, any moments or torques at the top surface are a function of Fmax and Fmin as described above. The student is encouraged to list Max, Min, Alt, SS, and Pre from top to bottom on the left side of their solution sheet. Next, a series of five stress element are drawn in a stacked configuration corresponding to the forces, moments, and/or torques listed just to the left. On those elements, the corresponding forces, moments, and/or torques are shown, labeled according to the particular element they are drawn on (e.g. Fmax, Mmax and Tmax appear on the Max stress element, etc). The loading of Figure 6 is mapped onto this solution format below in Figure 8 for the case of axial loading only.

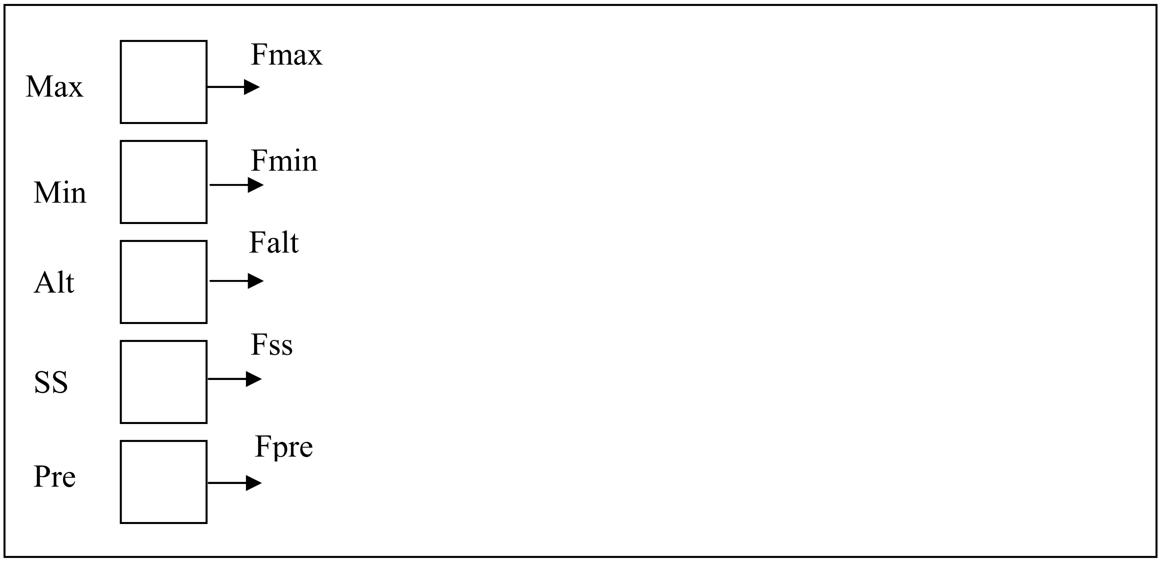

Figure 8: Initial stages of fatigue analysis methodology developed during lecture using Figure 6 loading pattern.

The above figure shows only the simple case of a machine element subjected to axial loading. For the more complex loading developed in the bent rod machine element, each element above would have the loading shown below in Figure 9. Typically, the bent rod is introduced after the simpler axial loaded machine element is introduced and analyzed with only direct normal stresses due to the axial force components. After this example is well understood, the bent rod is introduced with the loading shown below. Each of these loading vectors generates a stress component on one of the five stress elements as shown below.

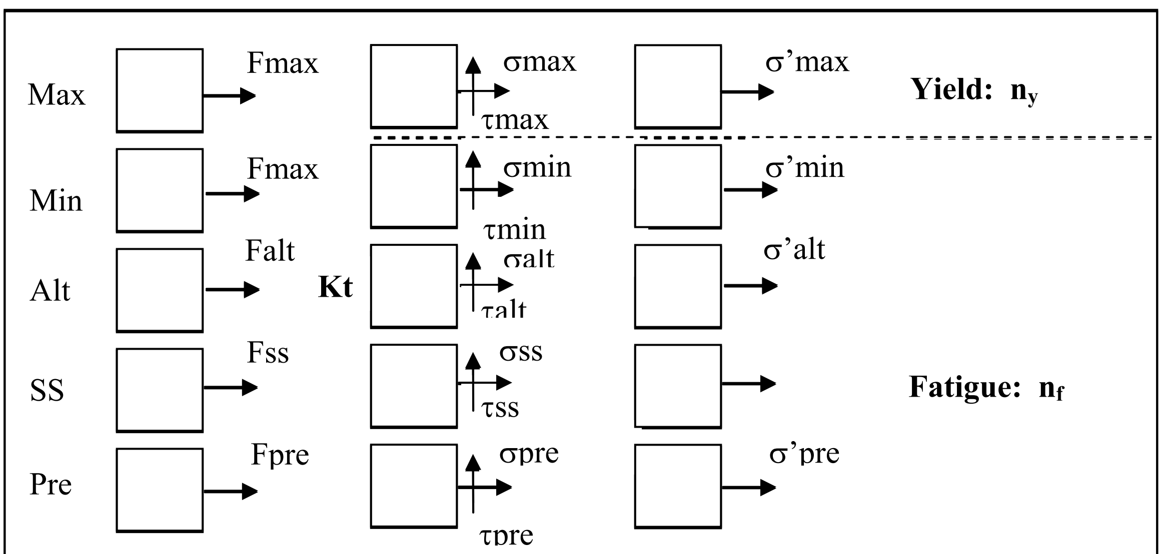

Figure 9: More Complex Loading Cases with Forces, Torques, and Moments due to Bent Rod.

V. Stress Concentration

At this point, the concept of stress concentration is reviewed. For ductile materials, the stress concentration factor, Kf, is utilized as an alternating stress multiplier only. Since Kf for bending is different (usually) from Kf for torsion, Kft is used to designate the stress concentration for torsion. (For brittle materials that experience no localized yielding at the root of a stress concentration location, Kf is applied to all of the stresses.) Both texts are very adept at providing the student the means to calculate stress concentration factors using notch sensitivity, q, and the Peterson charts to calculate Kf and Kft. The bent rod example provides the case where the alternating bending stress will be multiplied by Kf and the alternating torsional stress will be multiplied by Kft. As an aside, this example also provides justification for generally not using Kf or Kft as a strength reduction factor in the endurance strength calculation, since it is unclear how Kf and Kft interact and what is the net Kf that should be utilized to reduce Se. Since there is an alternating force present, the alternating stress must be multiplied by Kt as shown below in Figure 10.

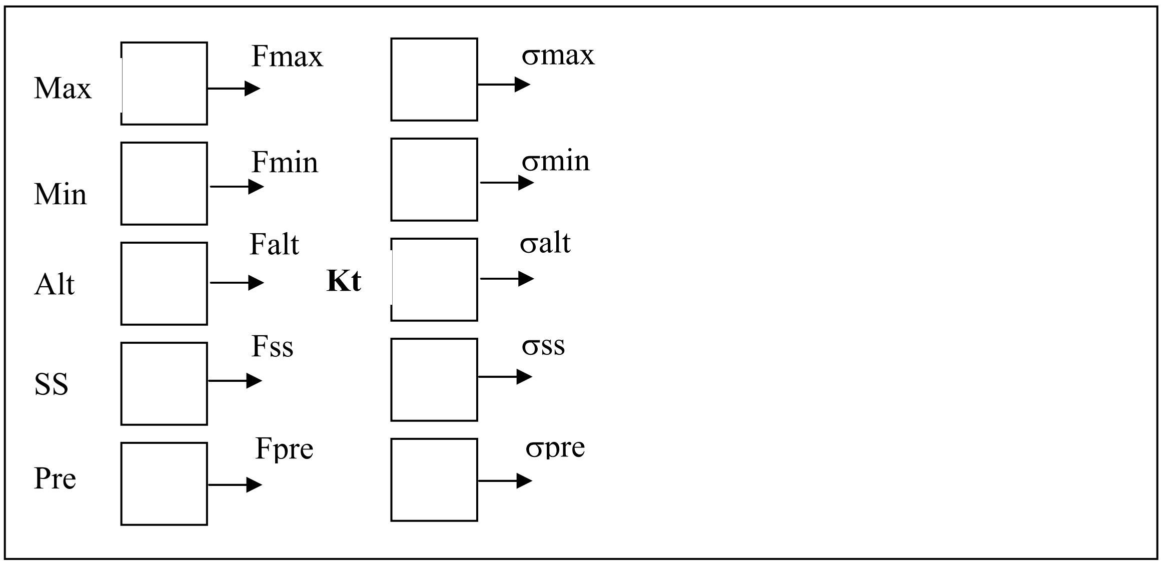

Figure 10: Intermediate stages of fatigue analysis methodology developed during lecture using Figure 6 loading pattern. In this stage, Kt is determined and the "Big 5" stresses are computed.

VI. Fatigue Stress Analysis

Both Shigley/Mischke and Norton begin their fatigue analysis with an explanation of calculating completely alternating stresses, σ'alt, from the alternating load components. For the bent rod example with loading cases F1(t), F2(t), and F3(t), σ'max, σ'ss and σ'pre can be easily determined from the values of Fmax, Fmin, Fss, and Fpre, respectively. The value of σ'alt can be determined, less easily, from the values of Kf, Malt, Kft, and Talt. Five more stress elements are drawn corresponding to the five stress elements just to the left, showing just a single von Mises stress. This effort reinforces to the student that a loading case can be reduced to five von Mises stresses for subsequent determination of the factor of safety, nf.

The factor of safety with respect to fatigue, nf for loading case F1(t) where σ'ss and σ'pre are both zero is shown in Equation 6 below:

$$\text{Equation 6:}\quad nf = Se / \sigma'alt$$

The above case corresponds to a Modified Goodman diagram that has no static stresses present so the operating point moves solely up and down the vertical axis as the magnitude of σ'alt varies up and down. Typically students question the need for the entire Modified Goodman diagram after this introduction to fluctuating stresses.

Subsequently, both texts go on to discuss loading cases that produce both alternating and steady von Mises stresses, σ'alt and σ'ss respectively caused in the bent rod example by F2(t). Interestingly, σ'alt is the same for loading F2(t) as it was for F1(t), saving the students from having to recalculate the Kf and Kft values, allowing them to focus better on the effect F2(t) has on fatigue life. Here both texts show that the factor of safety with respect to fatigue, nf, as shown in the equation below:

$$\text{Equation 7:}\quad nf = (Se * Sut)/(\sigma'alt * Sut + \sigma'ss * Se)$$

Later in both texts, cases where there exists a pre-load force in addition to alternating and steady-state forces are described and correspond to loading F3(t) as described above. Again σ'alt is the same a loading cases F1(t) and F2(t), but now σ'pre is non-zero and must be accounted for. After the author solved the equations for this loading case to find nf, it was pointed out to the author that Norton had done a derivation that was identical. The factor of safety with respect to fatigue, nf, is shown in the equation below:

$$\text{Equation 8:}\quad nf = (Se * (Sut - \sigma'pre)/(\sigma'alt * Sut + (\sigma'ss - \sigma'pre) * Se)$$

Equation 8 is the heart of the fatigue analysis methodology, as it provides students a generalized framework within which they can find nf for most simplified loading cases. For load case F1(t), equation 8 reduces to equation 6 properly. For loading case F2(t), equation 8 reduces to equation 7 properly. Thus, students only have to memorize equation 8 and they can handle any of the given loading cases, F1(t), F2(t), or F3(t) easily. Figure 11 below depicts the final stages.

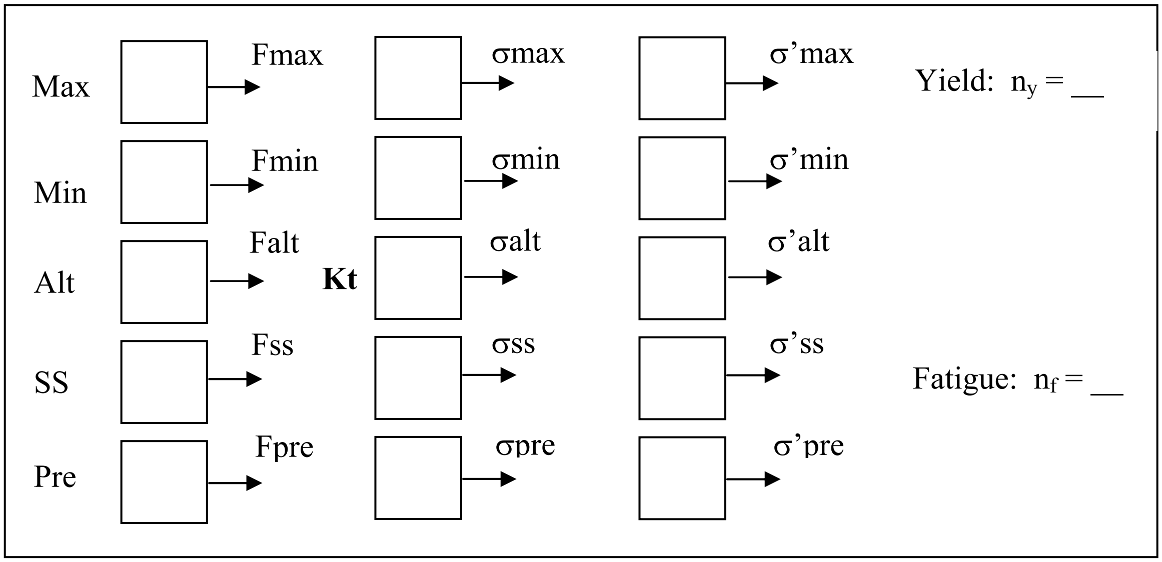

Figure 11: Final stages of fatigue analysis methodology developed during lecture using Figure 6 loading pattern. In this stage, n_y and n_f are both determined from the resulting von Mises stresses, Se, Syt, and Sut.

VII. Yield Analysis in Ductile Materials

Both texts recommend utilization of the Langer line, which on the Modified Goodman diagram, connects the yield point on the Static and Alternating axis. The author attempted this under the non-generalized framework, and observed large numbers of students experiencing confusion. To attempt to rectify this confusion, the author had the students plot σ'max on the Static axis (X axis) and mark the location of Syt on that same axis. Since σ'max is integral to the analysis methodology, static yielding and fatigue analysis were set up to be performed side-by-side using either the Modified Goodman diagram shown below in Figure 12 or the solution format shown above in Figure 11.

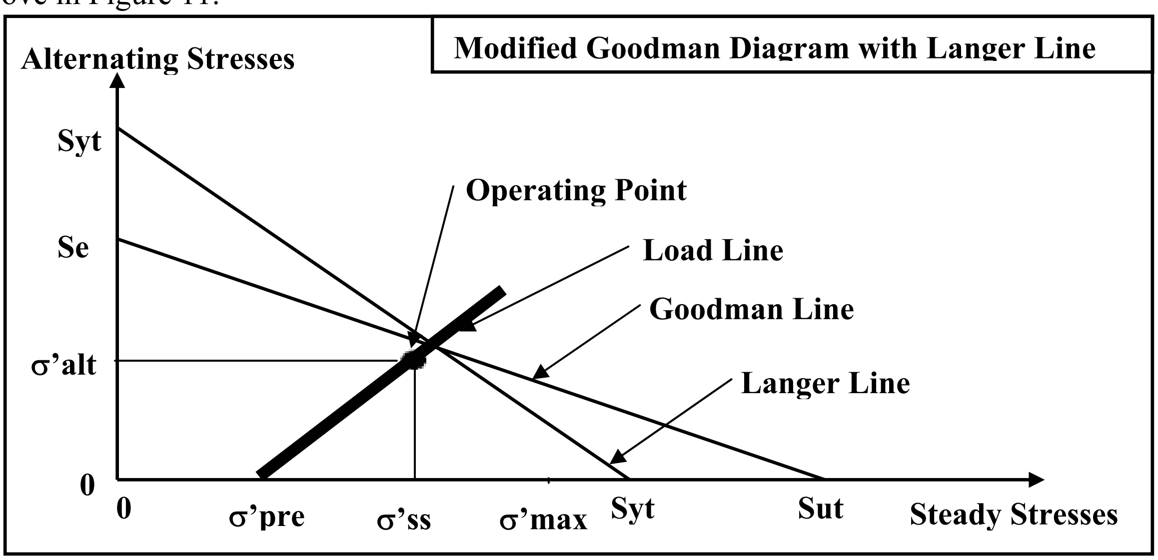

Figure 12: Plotting the Operating Point, Load Line, and Maximum Stress lines on the Modified Goodman Diagram based on loading pattern of Figure 6. Note that only σ'alt is affected by Kt for ductile materials.

VIII. Fatigue Analysis Methodology Summary

Using this fatigue analysis methodology, simple to relatively complex loading cases can be differentiated and easily solved. This range of loading cases includes beams with fluctuating bending stresses, beams with fluctuating and static bending stresses, beams with fluctuating, static, and preload bending stresses, shafts with alternating bending stresses and steady torsional shear stresses, compression springs with an installation pre-load stress, bolted connections subject to fluctuating loads, etc.

The author observed that while about one out of twenty of the students would "get it" in the non-generalized fatigue teaching framework, well over 50% of the students "got it" using the presented methodology. These numbers are derived from a comparison of the first and final examination scores. On student evaluations, comments were made that the material was made very clear and easy to understand.

Even though there are many load cases that Shigley/Mischke and Norton point out that are not covered specifically by the methodology, the author has found the class to be better prepared to explore these real-world loading cases in subsequent classes. These loading case sometimes have different operating points and load lines. Students were able to quickly perform static yielding analyses using the same Modified Goodman diagram, a useful convenience on both quizzes and examinations.

In summary, this fatigue analysis methodology has allowed the author to streamline fatigue instruction to both engineering and engineering technology students at the same time increasing comprehension.

VIII. Future Applied Research

Future applied research will focus on the cumulative fatigue damage first using Miner's Rule analysis techniques and then using artificial neural networks to model the varying loading patterns to enable predictions of fatigue life to be compared with experimental data. The goals here will be to give the students more hands-on validation of fatigue theory and analysis and, in particular, the extension of lecture material via experimental verification. The fatigue test equipment to be utilized for this research is depicted below in Figure 13.

Figure 13: Fatigue Testing Equipment for Alternating Loading

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Biography

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Professor Szaroletta is an Assistant Professor of Mechanical Engineering Technology at Purdue University where he presently teaches solid mechanics courses. He is a member of ASEE and ASME. He has 18 years industry experience in engineering and project management positions, with 12 awarded U.S. patents, and 6 years university teaching experience. He received his B.S. Degree in Mechanical Engineering from University of Michigan, Ann Arbor in 1977, M.S. Degree in Engineering (Product Design) from Stanford University in 1984, and a Master of Applied Mathematical Sciences Degree (Computer Science) from University of Georgia in 2000. His current applied research interests are product design, experimental mechanics, lab automation, and optimal design using genetic algorithms.