

Ocean Circulation in a Rotating Tank: An Outreach Project in Fluid Dynamics

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Abstract—This work presents outreach curriculum designed to expose high school students to research intensive fields such ocean modeling (or traditional fluid dynamics). The curriculum is for a five day learning experience and research project that presents the different ways to study fluid dynamics: theory, observations, experiments, and computations. Curriculum, resources, and results are all provided.

Index Terms—Fluid Dynamics, Outreach, Women and Minorities in Engineering

I. MOTIVATION

HERE is no shortage of evidence that we need to encourage United States (US) youth to enter the Science Technology Engineering and Mathematics (STEM) careers [1]. According to the Programme for International Student Assessment Report (PISA) [2] and the National Academies’ “Rising Above the Gathering Storm” [1], the US is behind with respect to STEM education and research and development. Only 50% of students in the US agreed or strongly agreed that they were interested in learning mathematics [2]. Compared to other nations, students receiving their undergraduate degrees in the natural sciences or engineering from US undergraduate institutions represent 16% of total enrollment of those institutions (compared to 47% in China, 38% in South Korea, and 27% in France) [3]. It was observed that over two-thirds of the engineers who receive PhD’s from US universities are not US citizens [4]. Furthermore, the higher up on academic STEM ladder (i.e. middle school student, high school student, college student, graduate student, post-doc, professor, etc.) , the more the percentages drop in both domestic representation [5, 6] and underrepresented groups (minorities, women) [7].

While there is clearly a need to promote the advancement of all the STEM fields, there is a particular lack of interest in the fields of engineering, physical science, and mathematics [3]. The representation of women among those receiving bachelor’s degrees in all fields from US universities exceeds

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57%, however, less than 20% of the degrees in engineering are awarded to women [3]. Black and Hispanic representation is also low. Physical sciences and mathematics have a slightly more balanced demographic than engineering, but those areas are still unbalanced [1]. It is possible that these underrepresented groups are interested in engineering, even mechanical engineering. However, they might not be interested in the traditional industries, which is why it is crucial to provide young people with many perspectives of the engineering fields.

II. INTRODUCTION

It was found that 89% of middle school students would rather do chores than math homework [8]. While that may be great for their parents’ dirty dishes, it calls for educators to cultivate passion in the fields of mathematics and science through innovative and provocative outreach. There has been much effort put forth to improve K-12 education, to implement hands on activities in the classroom, to provide better pay to retain good teachers, and to set up outreach activities for youth who are otherwise not exposed to quality math and science initiatives. However, research has shown that the number one reason students are not joining the Science, Technology, Engineering, and Mathematics (STEM) fields is because of personal interest [9, 10, 12]. Personal interest refers to students not having a genuine interest in the applications, technologies, or theory relating to a particular field. The outreach project presented here aims to provide underrepresented groups with a short term project that allows them to explore the field of fluid dynamics in a deep and meaningful way in hopes of fostering that interest.

III. THE OUTREACH SETTING

Fairfield University hosts a two-week residential summer camp for female high school students from Bridgeport, CT. The camp is called Broadening Access to Science and Education (BASE) camp and was developed by Dr. Shelley Phelan, Professor of Biology at Fairfield University. BASE Camp was designed to engage students in hands-on, research-based experiences in the natural sciences and mathematics. The goal is to inform the students of STEM research topics and excite them about the process of science.

The Participants

BASE Camp is open to female students who have

completed their sophomore or junior year at one of the Bridgeport high schools. Students apply to the program and are accepted based on their academic standing and their interest in science. Priority is given to first-time applicants. The camp is free of charge to the admitted students, who stay in dorms on the Fairfield campus. Once accepted into the program, students are assigned to one of six research projects led by STEM faculty at Fairfield University. The assignments are based off of student's prioritized interest in the projects, which they indicate in their application.

Camp Format

The camp runs for two weeks. The first week is spent working on the research project with the faculty advisor and two undergraduate student counselors. The second week is spent doing other activities and preparing for their research talks. Each individual faculty develops the curriculum for their project. That curriculum is being presented in this paper.

IV. THE PROJECT CURRICULUM

The goal of this project was to introduce the students to an application of fluid mechanics and, thus, was themed around ocean modeling. The project was explained within the context of Mechanical Engineering because the field of fluid mechanics is a foundation of the classical mechanics studied most traditionally in mechanical engineering.



Fig. 1. Clockwise from the top left: a photo of the students on “Theory Day”, a photo of the students at the beach on “Observation Day”, a photo of the students in the lab on “Experiment Day”, and a photo of the students in the computer lab on “Computation Day”.

Day One: Theory Day

The first day of camp was spent providing a theoretical framework for understanding ocean modeling. Given the backgrounds of the high school students, this theory was limited to non-mathematical conceptual ideas and motivations. The week was kicked off with some short films and discussions on climate change. A brief lecture was given on the Intergovernmental Panel on Climate Change (IPCC) and its most recent findings were reported [12] (note this camp was prior to the September, 2013 IPCC Report). In order to

TABLE I
PHENOMENA FOR THEORY DAY

Phenomena	Examples	Method of Study
Small Scale	-Micro-turbulence -Gravity Waves -Tides	Observations at the beach
Medium Scale	-Eddies -Gyres -Major Currents	Tank Experiment, Satellite Observations, Computations
Large Scale	-Thermohaline Circulation	Demo Experiments, Satellite Observations

*Shaded area indicated the concepts of greatest focus.

engage these young students, clips of popular YouTube science channels were shown, such as Hank Green’s SciShow episode on “Climate Change” [13].

After motivating the need for ocean modeling, the students then worked at understanding how ocean modeling fits within the earth system. Students were all given a copy of *Essentials of Oceanography* by Trujillo and Thurman [14], which is an introductory textbook geared towards students with little or no math and science background. The book was referenced during “Theory Day” often, but also used for different interactive activities and discussions. For example, each student was given a specific topic to research and then gave a 5-minute informal presentation of what they learned. Topics included: origins of the oceans, what is ocean bathymetry, voyages to inner space, 4+1 oceans, etc.

The first question that was directed to the students was, “What moves the ocean?” The answer was simplified to: density/stratification, rotation of the earth, wind, and continents. Each of these concepts was explained through diagrams, photos, and simple hands-on experiments. The week schedule is summarized in Table 1. The ocean phenomenon was categorized into different sizes. Then, the method of studying those phenomena was highlighted in order for the students to put into context the following days of camp. Appendix A provides direct references to some materials to explain specific concepts relating to “What moves the ocean?”



Fig. 2. Left: photo of students doing stratification demonstration. Right: photo of students doing thermohaline demonstration.

and these categories of phenomena.

Day Two: Observation Day

The second day was spent on observations, both in the form of visual observations at the beach and exploring satellite data on the computer lab. Since “Theory Day” categorized ocean

TABLE II
SMALL SCALE PHENOMENA OBSERVED AT THE BEACH

Phenomena	Examples	Student Photos
Micro-turbulence	Waves splashing, fish swimming, eddies forming around rocks, waves crashing, dog splashing, etc.	
Surface Gravity Waves	Induced by throwing rocks, waves coming into shore, wakes behind boats, etc.	
Tides	Direction of flow coming in and out of estuaries, visualizing whether current conditions were low or high tide based on water marks, etc.	

*Sample photos from top to bottom: a dog splashing in the water, surface gravity waves approaching the shore, and flow coming into estuary due to tide.

phenomena into small, medium, and large-scale features, the students were able to conceptualize which of the features would be visible at the beach. The small-scale phenomena was all observable from the Ash Creek location where the field trip took place. Table 2 gives example of different observable small-scale features and also shows an example of a student photograph taken for that feature. Appendix B shows a sample worksheet that students filled out during their beach observations. In the afternoon, students returned back to the lab to explore satellite data through guided activities. Resources for these activities are provided in Appendix C. Fig. 3 shows a snap shot of one of the available resources provided by NASA. Students completed an activity where they looked at data from western boundary currents (both the Gulf Stream and Kurishio) to observe the seasonal changes due to these slow moving, large currents.

Day Three: Experiment Day

The third day of the camp was focused on the scientific method and conducting an experiment, which focused on western boundary currents. The students took what they learned from “Theory Day” and “Observation Day” to form a hypothesis on what aspects of the ocean are most important to the formation and circulation of western boundary currents. As a group, the advisor guided the students to form the following hypothesis, “Western boundary currents are formed as a result of (1) winds, (2) rotation of the earth, and (3) continental boundaries.” Then, an experimental setup mimicking the western boundary currents was presented. This experimental tank was designed and built by senior design mechanical engineering students two years prior. The rotating tank is a square acrylic tank that sits on a rotating platform on top of a cart. The tank rotates to mimic the rotation of the earth. There is a square piece of acrylic that is placed at the bottom of the tank, but tilted so that bottom is

sloped. This accounts for the spherical effects not accounted for in the one-dimensional rotation. Two fans are clipped to the sidewalls to mimic the shearing effect of the wind stresses, which force the circulation pattern in a clockwise direction. Finally, the tank walls approximate the continental boundaries that exist inside ocean basins where western boundary currents form. The students designed three experiments that would test their hypothesis:

1. Run the experiment with sloping bottom and fans.
2. Run the experiment with sloping bottom and no fans.



Fig. 4. An image of the experimental setup.

3. Run the experiment with fans and no sloping bottom.

If their hypothesis was right, Case 1 would show the correct flow pattern observed in the satellite observations. The students first learned how to operate the equipment. They did one test run, and then they conducted all three test cases. A GoPro camera was attached to an arm, which rotates with the tank, so that the flow pattern video could be recorded. Fig. 5 shows snapshots from all three test cases. It is clear that Case 1 is most similar to observations, thus, the hypothesis is correct.

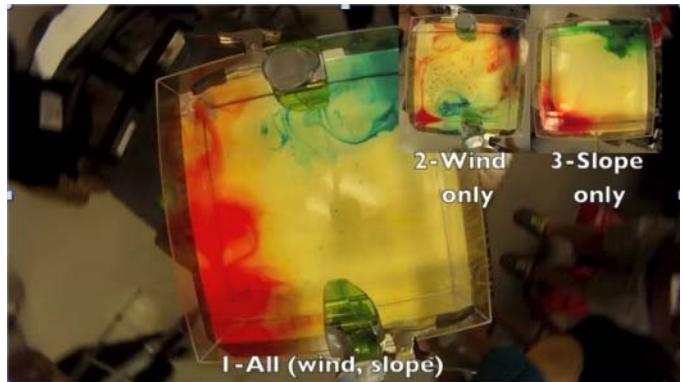


Fig. 5. Results from all three experiments conducted. It is clear that Case 1, which includes both effects due to wind and spherical effects, is most representative of what is seen in observations.

Day Four: Computation Day

Computation day was the day ocean modeling was introduced to the students. The difference between satellite data (viewable from computers) and ocean modeling (approximations calculated on computers) was stressed. Even though the students did not have the mathematical background needed to understand non-linear, coupled sets of partial differential equations that are solved in order to model the ocean, the equations were not ignored and were not feared. To start of computation day, the concept of vorticity was introduced in a purely conceptual way. It was explained that in order to represent vorticity using physical laws and mathematical equations, it is necessary to design a symbol. With no knowledge of greek symbols or symbols commonly used in mathematics or engineering, each student designed a symbol and then the group voted our their favorite. The five finalists are shown in Fig. 6. The winning symbol was Kat's, so that was the symbol used for the remainder of camp. With

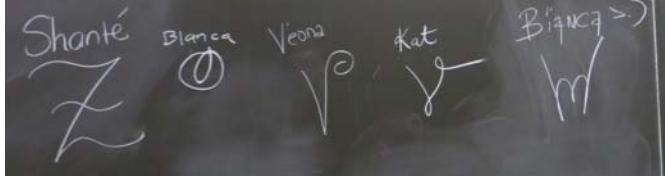


Fig. 6. A photograph of each of the symbols designed by the students to represent vorticity. The winning symbol was Kat's.

even a basic understand of vorticity, it was interesting that the students designed symbols with curvy, rotational features.

The barotropic vorticity equation and toy model problem was solved to give the students experience with the concept of ocean modeling. The unique aspect of this component of the camp is that the real equations were presented to the students. They were prefaced with an explanation that although they



Fig. 7. The barotropic vorticity equation in continuous and discrete form were presented as a beautiful mathematical representation of western boudnary currents.

may not understand the equations, they can appreciate the beautiful symbolic nature of the equations and how amazing that they can represent such a wonderfully complex fluid dynamics system, like a western boundary current. The

students absolutely loved writing out the equations. When presented with the confidence that they were important and beautiful, they were not feared. After the equations were written, they were also discretized, and labeled to represent what they were modeling (local changes, convection, rotation

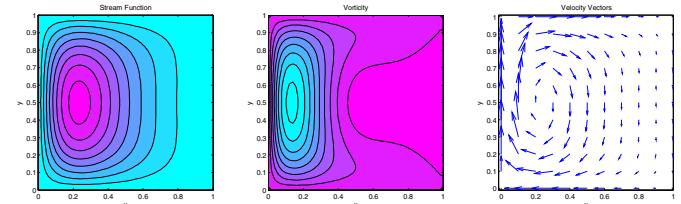


Fig. 8. Computational results of solving the barotropic vorticity equatation.

of the earth, wind, and viscous effects).

Finally, a MATLAB script was provided so that students could run the script and visualize the results from solving those equations. Similar to the experiment day, students went through the scientific process to determine a hypothesis and then creative computational experiments that could test that hypothesis. Lines were commented in such a way that they could easily comment out lines that included terms that represented the various aspects of the problem (wind, boundaries, rotation of the earth). They could comment out one term at a time to see the affect of those aspects. Fig. 8 shows an example of the results they were viewing when



Fig. 9. A photograph of the graffiti wall designed by the students during Reflection Day, which summarizes thing we can do to take action on what was learned at camp about ocean modeling and climate change.

solving the full equation with all terms.

Day Five: Reflection Day

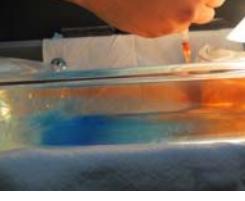
The final day of the first week was spent doing various things, but mainly reflecting on what was learned. Students were asked to reflect on which aspect of learning fluid dynamics they enjoyed the most (theory, observations, experiments, or computations). Interestingly, each student picked a different method of learning as their favorite method. Finally, the students designed a graffiti board filled with ideas they had to take action with respect what they learned at camp about ocean modeling and climate change. This was also a time that the students were able to reflect on mechanical engineering, fluid mechanics, ocean modeling, minorities and women in the field, etc.

V. RESULTS AND REFLECTIONS

Students left the camp more educated about academia and scientific research. It was clear from the feedback and general attitudes, that they were not intimidated but actually fired up

and inspired. They all reported having a new perspective on the field of engineering, fluid mechanics, and ocean modeling. There were various pieces of specific feedback that were especially encouraging and reflect on the success of the curriculum. This feedback is from the campers who were doing the ocean curriculum only. One of the students decided after the camp that she wanted to not only major in Mechanical Engineering, but wanted to attend Fairfield University. One student said, “I never knew fluids were so cool”. When asked if they enjoyed working with STEM females at an all female camp, they all reported that they found the environment less intimidating and more inspiring. They all reported that the experience gave them more freedom to be themselves than their traditional science and math classroom experience. Two of the students are going to high school at vocational high schools and have chosen technically

TABLE III
THEORY DAY CURRICULUM REFERENCES

Concept	Explanation/Reference	Example
Stratification	Students layer fluids with different densities using salty, room temperature, warm waters).(http://www.youtube.com/watch?v=I8kToTROCHA&list=PLNp9eDphQtUQP8lw70g_aMT5Qs_oASipr	
Thermohaline Circulation	Students set up a Thermohaline circulation using hot and cold dyed water forced by ice and heat lamps .(http://www.youtube.com/watch?v=uLuD_K0NgeE)	

orientated vocational tracks like shop and architecture. Those students were especially appreciative of getting to learn in a female only environment because they reported being one of only a few females in their high school programs. Lastly, the students were asked if they were intimidated by high level mathematics involved in their project. They all agreed that it was not scary and that they were proud to be apart of such important science. This was especially evident as they described their project to the campers on other projects and their families.

APPENDIX

Appendix A

Table III displays some references and examples of different activities pertaining to “Theory Day”.

Appendix B

Below is a sample worksheet for observations at the beach during “Observation Day”.

OBSERVATION DAY
BASE Camp 2013 - Team Ocean
Faculty Advisor: Sharon Recklinger
Undergraduate Counselors: Katherine Pita & Blanca Aca

Ash Creek/Jennings Beach Exploration
In your notebook, please complete these tasks and take notes on the questions below.

Small Scale Ocean Phenomena:

1. Micro-Turbulence/Tiny eddies
2. Gravity Waves
3. Tides

For each of the three small scale ocean phenomena above,

- I. Capture it in a photo or video (the more the better).
- II. What is making it possible to “viewing” the feature?
- III. Describe what you are seeing.
- IV. What is causing the feature?
- V. What is the time scale associated with the feature (in seconds)?
- VI. What is the length scale associated with the feature (in metric)?
- VII. What happens for the eventual end of the feature’s existence?

Additional Questions/Notes to make:

- a. Is it high or low tide? How do you know?
- b. Is the tide increasing or decreasing? How do you know?
- c. How many different kinds of gravity waves did you spot?
- d. Where was turbulence most prevalent?
- e. Could you induce turbulence? How? Did you?
- f. Could you induce gravity waves? How? Did you?
- g. Could you induce tides? How? Did you?

Appendix C

Below is a collection of references for activities designed for students to explore satellite data.

Gulf Stream:

<http://rads.tudelft.nl/gulfstream/>

Western Boundary Currents/Climate Data:

<http://studentclimatedata.unh.edu>

Perpetual Ocean:

<http://svs.gsfc.nasa.gov/vis/a000000/a003800/a003827/index.html>

Ocean Bathymetry:

<http://maps.ngdc.noaa.gov/viewers/bathymetry/>

Appendix D

A portion of the MATLAB script, which solved the barotropic vorticity equation, modeling western boundary currents. The programming comments explain conceptually the contribution of each mathematical term.

```

while t_norun<=final
    i=1;
    %extract_t_current = delta_x;
    display_t_current;
    %time step
    dt=(Tend-t_norun)/1000;
    %initial values
    u(1)=0;
    v(1)=0;
    %velocity components
    u(2)=0;
    v(2)=0;
    %vorticity
    w(1)=0;
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