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On the Presentation of the Physical and Mathematical Solutions Process of Problems in Physics to Engineering Students

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The solution process to problems is both a techne and a technique. To teach and train engineering students (our future engineers and colleagues) on how to solve problems (engineers solve problems) two similar but distinct approaches to the solution process of problems were presented to them. One characteristic of both approaches is that they include all the steps (including the "missing steps" in textbooks) of the solution process so a student concentrate on understanding the application of physical principles than trying to guess how the various stated results come into existence. This last part includes both physical thinking and mathematical implementation using appropriate mathematical methods and computer tools. To distinguish the two approaches, are called the Mathematical and the Physical-Mathematical.

Both approaches were presented to the students during the past two years. The students embraced the physical-mathematical more because it suits self-study and provides the explanations needed for the first study. But they suggested that the Mathematical approach is beneficial for review.

Introduction

The current approach in literature (textbooks) is to present many examples but skip the intermediate steps in the solution of solved problems. The solution is brief and leaves many steps missing and unexplained. Furthermore, there is no structured approach to the solution steps. This results in questions like "Why do we solve the problem this way?" (Physics) and "How do you go from one step to the next in the operations?" (Mathematics). The result is that a student never sees the physical thinking from the basic physical principles to the meaning of the final answer. Additionally, never fully sees the connection between the mathematics learned and their use in the solution of problems in Physics.

The proposed two approaches start from basic physical principles and provide all the steps in the solution process, so the student will learn how to solve problems starting from basic principles (identify the appropriate topic and laws) and let the initial formula guide to the following required steps. Both methods were developed out of the questions of the students "Why do we do this?", "How do you get that?" and the author's belief in the need to present the material to the students providing all the missing steps. The ideas were developed and worked fine during the time before the pandemic when the presentation was on a whiteboard and the missing steps were the author's oral explanations and writings on the whiteboard (the students will take photographs of the detailed solutions); during the pandemic when the Lecture/Socratic discussion, Recitation was synchronous and recorded and the presentation was on the computer screen annotating the text and using Microsoft OneNote[1]; and after the pandemic when the presentations were offered using the new interactive screens [2] and recorded using Blackboard's "Collaborate" [3].

The problem arose when the material was offered to the students as typing notes with an emphasis on the mathematical sequence of events. This was not adequate to explain the physical process of thinking, the audio recording was not there. So, the two approaches were developed: One in writing a presentation of the sequence of all the mathematical steps, the other in writing the sequence of all the mathematical steps including full narrative explanations step by step. The students liked both.

Using as an example a course of University Physics II (Electricity and Magnetism) the basics of the coordinate system are necessary because it helps to see the geometrical operations and their results during the solution process [4]. A review of the mathematical topics used in any mathematical-oriented course is necessary for the successful completion of the course.

A typical textbook solution is shown below [5]. Similar are the solutions to other currently used other textbooks.

The magnitude of the electric field $\vec{\mathbf{E}}'$ must be the same everywhere on a spherical Gaussian surface concentric with the distribution. For a spherical surface of radius r,

$$\Phi = \oint_S \vec{\mathbf{E}}_P \cdot \hat{\mathbf{n}}\, dA = E_P \oint_S dA = E_P\, 4\pi r^2.$$

Using Gauss's law

According to Gauss's law, the flux through a closed surface is equal to the total charge enclosed within the closed surface divided by the permittivity of vacuum ε_0. Let q_{enc} be the total charge enclosed inside the distance r from the origin, which is the space inside the Gaussian spherical surface of radius r. This gives the following relation for Gauss's law:

$$4\pi r^2 E = \frac{q_{\text{enc}}}{\varepsilon_0}.$$

Hence, the electric field at point P that is a distance r from the center of a spherically symmetrical charge distribution has the following magnitude and direction:

$$\text{Magnitude: } E(r) = \frac{1}{4\pi\varepsilon_0}\frac{q_{\text{enc}}}{r^2} \tag{6.8}$$

The Mathematical Approach

In the newly developed Mathematical approach, the mathematical steps of the solution process are organized in a diagrammatic approach. All steps of the mathematical solution process are presented logically, clearly indicating the necessary derivations and substitutions, until to get a final formula that to the left of the equality sign has the unknown quantity and to the right all the known quantities. The use of appropriate mathematics is introduced. This approach when presented as part of an audiovisual format has the advantage of including interactive Socratic dialogs between the instructor and the students (questions and answers, comments) which take place during class time. At the same time if presented alone without the audio-visual part will require the student to be familiar with the content. The physical explanation of every step was provided during the presentation (lecture/recitation) using a Touch Screen Monitor by the instructor. The presentation (video and audio) is recorded and made available to the students in the Collaborate part of Blackboard. It is ideal for class discussion.

In this approach, the various mathematical steps are applied one after the other such that at the end we have a symbolic formula that to the left has the unknown quantity, and to the right, everything is known. Typical steps include:

- Identifying the Physical phenomenon.
- Writing a formula or a system of formulas (equations) that can be solved such that to the left has the required unknown quantity and to the right must eventually have everything known.
- Performing mathematical operations.
- Getting the answer to the question of the problem.

A characteristic of this approach is compact presentation.

The Physical-Mathematical Approach

Some problems were presented using the Physical-Mathematical approach that includes both a narrative of the physical explanation of the various steps and their mathematical realization. The steps for the solution to the problem are identical as in the Mathematical approach. But now the inserted text explains the mathematical steps and it is followed by the corresponding equation. In the end, the final formula is derived in which to the left of the equality sign has the unknown quantity and to the right are all the known quantities. This approach has the advantage that does not require the presence of the instructor. It is ideal for self-study.

Comparison of the two methods: An Example

The two methods are compared side by side using a simple example from University Physics II (Electricity and Magnetism). The compactness of the Mathematical and the full narrative full explanations of the Physical - Mathematical are clearly emphasized.

Example: Let us consider a point charge Q.

Determine an expression for the electric field E at every point of the space.

Mathematical approach	**Physical - Mathematical approach**
Solution Process:	Solution Process: Strategy: Due to the symmetry of the system, we shall use a spherical coordinate system. To find the electric field intensity at a field point P around a source point charge Q, we shall start with the definition of the electric field:

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$$\vec{E} \triangleq \frac{\vec{F}}{q_+}$$
$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Qq_+}{R^2}\hat{R}$$
$$Q = +2e$$
$$R = r$$
$$\hat{R} = \hat{r}$$
</td>
<td>
$$\begin{aligned}\vec{E} &= \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{2e}{r^2}\hat{r}\\ &= \vec{E}_r\\ &= E_r\hat{r}\\ &= \left(\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{2e}{r^2}\right)\hat{r} \quad r > 0\end{aligned}$$
</td>
<td>
$$\vec{E} \triangleq \frac{\vec{F}}{q_+}$$
We shall position the source charge Q at the origin of the coordinate system. The force on the test charge q_+ located at the field point P due to the source charge Q located at the origin is given by Coulomb's law:
$$\vec{F} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{Qq_+}{R^2}\hat{R}$$
The charge $Q = +2e$ is the source of the electric field.
The distance R from the source point to the field point coincides with the radial coordinate r of the spherical coordinate system and it is denoted by r because we are using the spherical coordinate system we are using to solve this problem.
The unit vector in the radial orientation from the source point to the field point is $\hat{R} = \hat{r}$, in the positive the outward sense.
Substituting the force equation into the electric field equation, we get:
$$\begin{aligned}\vec{E} &= \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{2e}{r^2}\hat{r}\\ &= E_r\hat{r}\\ &= \vec{E}_r\\ &= \left(\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{2e}{r^2}\right)\hat{r} \quad r > 0\end{aligned}$$
We conclude that the electric field has only a r radial component.
Answer:
$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{2e}{r^2}\right)\hat{r} \quad r > 0$$
Comment:
When the source charge is positive and the test charge is, by definition, positive, the force will have orientation along the line connecting the two points and sense away from the charge Q. (Why?) (Elaborate).
Questions:
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	What will be the orientation and sense of the field if the source field is negative? (Elaborate) What will be the orientation and sense of the field lines if the source field is positive and not located in the origin? (Elaborate) What will be the orientation and sense of the field if the source field is negative and not located in the origin? (Elaborate) Summary: $\left.\begin{array}{l} \vec{E} \triangleq \dfrac{\vec{F}}{q_+} \\ \vec{F} = \dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\dfrac{Qq_+}{R^2}\hat{R} \\ Q = +2e \\ R = r \\ \hat{R} = \hat{r} \end{array}\right\} \begin{aligned} \vec{E} &= \dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\dfrac{2e}{r^2}\hat{r} \\ &= \vec{E}_r \\ &= E_r\hat{r} \\ &= \left(\dfrac{1}{4\pi\varepsilon_0\varepsilon_r}\dfrac{2e}{r^2}\right)\hat{r} \quad r > 0 \end{aligned}$

Conclusion:

Two methods for the solution of problems were presented. The Mathematical and the Physical-Mathematical. Both approaches were presented to the students during the past year. In class, discussions took place during the semester on the preferable and possibly more beneficial approach to them. They supported the Physical-Mathematical more because it suits self-study by explaining the thinking process and providing all the missing steps. Furthermore, it does not require the need to watch the recorded lecture/recitation which takes considerably more time; it repeats the class time.

But they suggested that the Mathematical approach is beneficial for fast review (i.e., preparation for examination) after they have mastered the Physical-Mathematical approach of the solution.

Further Work

To assist the student in their study an effort is underway to prepare problems along with their solution processes that include both the above-discussed approaches; the physical/mathematical solution process is presented first while the mathematical is presented under “Summary” at the end of the problem.

References:

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