

Parametric Analysis of a Stirling Engine Using Engineering Equation Solver

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The Stirling engine is a clean energy source that converts heat into work efficiently with a theoretical thermal efficiency equal to that of the Carnot engine. As an external combustion engine, there is no limitation on the type of fuel source chosen to power the Stirling engine—so a wide range of unusual fuel sources may be used. This paper describes a program created in Engineering Equation Solver (EES) that is designed to create a power output matrix and a parametric analysis for any Stirling engine—whether alpha, beta, or gamma configuration—and is developed as a tool for use in a Senior Design project at SUNY New Paltz. Upon successful completion of this code, EES will compute a resulting power output matrix and P-v and T-s diagrams. The resulting power output matrix is two-dimensional: one dimension is an array of Revolutions Per Minute (RPM) values from 100 rpm to 1,000 rpm; the second dimension is an array of compression ratios. This power output matrix is created by utilizing the Parametric tables in EES. The Nasa Ideal Gas Library within the EES program contains 1,262 ideal gases to be chosen as the working fluid. The thermodynamic properties of these gases were recorded in 2002 at the Glenn Research Center. The thermophysical property functions in EES calculate the thermophysical property values at each state using the ideal gas assumption. The purpose of this program is that the user will input the length and bore of any chosen cylinder, as well as the hot and cold temperature values, to determine a power output matrix that varies with compression ratio and RPM, and then perform a parametric analysis for the Stirling engine. To prove the effectiveness of EES in designing and analyzing thermal systems, there is survey data completed by undergraduate mechanical engineering students who are using EES in their Thermal System Design course at SUNY New Paltz.

Keywords— Stirling Engine, Engineering Equation Solver, Compression Ratio, Swept Path

I. INTRODUCTION

The Stirling engine is a high thermal efficient external combustion engine that can be powered by a wide range of unusual external heat sources, such as agricultural waste, biomass, and solar radiation [1]. Choosing a renewable fuel source means that the Stirling engine will be a clean source of energy. Solar energy can be used as the heat source for a Stirling engine by collecting light energy from the sun and then converting this light energy into the thermal energy inside the engine [2].

The Stirling engine was invented in 1816 by Robert Stirling, before the Diesel engine [3]. The Stirling engine is

much quieter than both the Otto and Diesel engines, but the quick development of the internal combustion engines during the 19th century hampered the development of the Stirling engine. Because the Stirling engine is an external combustion engine, this engine can be less harmful to the environment than the Otto or Diesel engines, depending on the chosen heat source. Besides heating the hot side, the temperature difference can be created by chilling the cold side. Because of the Stirling's numerous advantages, there are many potential applications that will produce less exhaust emissions than an internal combustion engine [2].

There is promising research that Stirling engines can be used to power underwater vehicles for the military: due to the low noise and low vibration of the engine, underwater military vehicles will decrease their probability of being detected by the enemy [4]. The reason for low noise is because there is no combustion process inside the engine to exert gas pressure onto the piston; instead, the Stirling engine is powered by the temperature difference between the hot side and the cold side of the engine. A rhombic drive shaft Stirling engine is a good fit for underwater applications since the mechanism is dynamically balanced [4].

There are three different types of cylinder configurations for the Stirling engine, and each configuration differs in piston and cylinder arrangement. The alpha design has two power pistons, each in two different cylinders: one cylinder is heated, the other cylinder is cooled, and both cylinders are connected using a regenerative heat exchanger (or conduit) [1]. The beta design has a power piston and a displacer piston placed in one cylinder. The gamma design has a power piston and displacer piston each in distinct cylinders [1]. The purpose of a displacer piston is to move the working fluid from the hot side to the cold side of the engine. The purpose of the power piston is to expand and compress the working fluid [2].

The disadvantage of the alpha design is there will be an increase in friction between the power piston and the cylinder on the hot side of the engine due to the heat source causing thermal expansion of the power piston: for this reason, the beta and gamma designs are superior [2]. The downside of the beta configuration is that the kinematic design of the engine is often more complex than the alpha and gamma types.

All three Stirling engine configurations have two pistons with a 90-degree phase angle difference, but there are various drive mechanism configurations designed to couple the pistons. Two common forms of piston couplings are slider-crank and rhombic driveshaft. The rhombic driveshaft is dynamically balanced but will only work for the beta

configuration because this mechanism cannot function with two distinct cylinders.

The Stirling engine prototype designed for this project is the alpha configuration because the slider-crank mechanism is a simple design to create and manufacture. The working fluid of this Stirling engine prototype is air, but any compressible fluid can be used for the Stirling engine [2]. Furthermore, [5] completed a separate beta Stirling design project that equally utilized this EES program to determine the power output matrix of the beta Stirling engine.

The ideal isothermal thermodynamic cycle of the Stirling engine, invented by Robert Stirling, operates from cyclical expansion and compression of the working fluid at a temperature differential [3]. Process 1-2 is isothermal expansion of the working fluid. Process 2-3 is isochoric heat rejection. Process 3-4 is isothermal compression of the working fluid. Process 4-1 is isochoric heat addition. The maximum pressure of the working fluid is at state 1. This project compares the theoretical isothermal model created in EES to the experimental model.

The purpose of this alpha Stirling engine project is to generate DC current that will power a phone charger. A major question regarding the design process of this Stirling engine is what the power output of the engine will be depending on different compression ratios and RPMs. Due to cost restraints, the team needed to ensure that the bore and length of the cylinder purchased would produce a sufficient power output to the DC generator so that 1 Amps of current will be carried through the phone charger. To determine the critical values necessary to produce the required output, a code is designed in EES that results in a power output matrix and subsequent P-v and T-s diagrams.

This paper discusses and explains the benefits of using EES to solve the thermodynamics properties and power outputs of any type of Stirling engine without having a known swept path, compression ratio, and RPM. The next section will elaborate on the academic benefits of utilizing EES to understand and design thermal systems. The results will prove the effectiveness of the EES code in determining the power output of any type of Stirling engine.

In addition to the Stirling engine Senior Design project, there is an undergraduate course at SUNY New Paltz that utilizes EES to teach students how to design thermal systems. Students are required to design a Rankine system, a refrigeration system, and an internal combustion system using EES. There are optional EES tutor sessions held by an undergraduate mechanical engineering student that the students may attend if they struggle learning the EES software. A survey is conducted to determine if the students found EES to be an effective software for designing thermal systems and performing parametric analyses.

The following section further explains how the programs within EES were utilized to create the power output matrix and solve the isothermal Stirling engine cycle. Furthermore, there will be an explanation on the limitations of EES Academic and how purchasing EES Professional would simplify the Stirling engine program.

II. METHOD

Engineering Equation Solver is a program that can solve differential and non-linear algebraic equations. An impressive feature in EES is there are thermodynamic

functions included in the program that are highly accurate; EES included a list of the 1,262 ideal gases in the NASA Ideal Gas Library. After solving a thermodynamic cycle, such as the isothermal ideal Stirling cycle, P-v and T-s plots can easily be created in EES.

To produce a power matrix of the Stirling engine, as well as P-v and T-s diagrams, there are two programs designed in EES. The first program is titled “Swept Path Verse Compression Ratio”, where the user inputs the bore, length of cylinder, and volume of displacer piston to output a Parametric table that lists the compression ratio for any swept path length. Then this Parametric table is saved as a Lookup table: Lookup tables can be opened and utilized in different EES programs and can be regarded as preprocessing data, meanwhile Parametric tables cannot be opened by other EES programs. The second program, “Stirling Engine Code”, takes the data of compression ratio values from the Lookup table and runs each compression ratio value through the ideal isothermal Stirling cycle code to output the power output matrix and P-v and T-s diagrams.

First step is to open the “Swept Path Verse Compression Ratio” EES program. Input the bore, cylinder length, and volume of the displacer piston. If the alpha configuration is chosen, then the volume of the displacer piston is zero. This program will first calculate the clearance volume, swept volume, and volume of the working fluid to determine the compression ratio.

$$V_C = \frac{\pi(Bore^2)}{4} * (l - s) - V_{Disp} \quad [1]$$

Where V_C is the clearance volume in mm^3 , l is the length of the cylinder in millimeters, s is the swept path in millimeters, and V_{Disp} is the volume of the displacer piston in mm^3 .

$$V_S = \frac{\pi(Bore^2)}{4} * s \quad [2]$$

Where V_S is the swept volume in mm^3 .

$$V_{wf} = V_S + V_C \quad [3]$$

Where V_{wf} is the volume of the working fluid in mm^3 .

$$CR = \frac{V_{wf}}{V_C} \quad [4]$$

Where CR is the compression ratio, and the units are dimensionless.

A Parametric table is a matrix created as the output of the “Swept Path Verse Compression Ratio” code. The dimensions of this matrix is $(L - 1)$ by 2, where L represents the length of the cylinder. The first column is an array of swept path elements; the second column is an array of the subsequent compression ratio elements. For a cylinder with a length of 90 mm, the Parametric table has the dimension of 89 by 2.

An option in EES Academic is to save the Parametric table as an Excel spreadsheet. Figure 1 is the Excel version of the Parametric table calculated in the first EES program, “Swept Path Verse Compression Ratio”, and only shows the first ten runs.

The columns and elements in any Parametric table cannot directly be used as preprocess data; there is no EES function or directive that can extract data from a Parametric table. On the other hand, a Lookup table can be used as preprocessing

data. For this reason, EES Academic requires two different programs to solve for the power output matrix: The first program outputs a Parametric table of swept path and compression ratios. Then this Parametric table is saved as a Lookup table so that the tabulated data can be used as input variables in an EES program. Then, the second EES program is opened to extract data from the Lookup table of swept path and compression ratio values.

The next step to find the Power Output Matrix is to open the second EES program titled “Stirling Engine Code”. This program will extract data from the Lookup table previously created and saved so that the final power output matrix of the Stirling engine will be determined.

There are three input variables in the second EES program. One of the three input variables in the array of compression ratios from the Lookup table. This tabulated data is extracted by using the “lookup” function built into EES. The second of the three input variables is the volume of the working fluid, which was found in the first EES program.

Table 1. Table of Swept Path Verse Compression Ratio. Includes the first 10 rows of an 89 by 2 matrix.

| Swept Path [mm] | Compression Ratio |
|-----------------|-------------------|
| 1 | 1.011 |
| 2 | 1.023 |
| 3 | 1.034 |
| 4 | 1.047 |
| 5 | 1.059 |
| 6 | 1.071 |
| 7 | 1.084 |
| 8 | 1.098 |
| 9 | 1.111 |
| 10 | 1.125 |

The third input variable is the temperature of the hot side of the Stirling engine. This is found by heating the hot side cylinder with the heat source for 20 minutes (until the cylinder reached a steady-state temperature). Then a thermocouple is used to measure 5 different sections of the cylinder: the average temperature of these readings is chosen to be the temperature of the hot side. The temperature of the cold cylinder is assumed to be 20 degrees Celsius but can be determined by measuring the atmospheric temperature.

Table 2. Table of temperature hot cylinder. A thermocouple was used to measure the temperature at different locations of the hot cylinder to find the average temperature.

| Location | Temperature [°F] |
|----------|------------------|
| Top | 92 |
| Bottom | 200 |
| Left | 130 |
| Right | 130 |
| Center | 120 |
| avg | 134.4 |

The purpose of the first EES program was to find swept path and compression ratio values, but the purpose of the second EES program is to determine the thermodynamic cycle, power output, and thermal efficiency. To find the thermophysical property values at each state, the build in EES function called “idealgasthermoprops” is used to calculate the temperature, pressure, specific volume, specific enthalpy, specific entropy, and specific internal energy of a state. (Once two independent thermophysical properties are found.)

State 3 of the Stirling cycle has two known properties: the pressure is the atmospheric pressure of 101.3 kPa and the temperature is the temperature of the cold side of the engine (assumed to be 20 degrees Celsius). By calling the “idealgasthermoprops” function, the rest of the thermophysical values at state 3 are calculated.

$$P_3 = 101.3 \text{ kPa} \quad [5]$$

$$T_3 = T_C = 20^\circ\text{C} \quad [6]$$

Process 2 to 3 is isochoric heat rejection, so the specific volume at state 2 equals the specific volume at state 3, and the temperature at state 2 is the temperature of the hot side of the cylinder.

$$v_2 = v_3 \quad [7]$$

$$T_2 = T_H = 57^\circ\text{C} \quad [8]$$

State 3 to state 4 is isothermal compression, so the temperature at state 4 equals state 3 and state 4 has the minimum pressure of the cycle. The compression ratio is used to calculate the minimum pressure:

$$T_4 = T_3 \quad [9]$$

$$CR = \frac{V_{wf}}{V_c} = \frac{v_{max}}{v_{min}} = \frac{v_3}{v_4} \quad [10]$$

Where V_{wf} is the volume of the working fluid and V_c is the clearance volume. (V_{wf} and V_c were computed in the first EES program.)

Process 4 to 1 is isochoric heat addition, so the specific volume at state 1 equals the specific volume at state 4. Process 1 to 2 is isothermal expansion, so the temperature at state 1 equals the temperature at state 2.

$$v_1 = v_4 \quad [11]$$

$$T_1 = T_2 \quad [12]$$

The specific work done to the system is from state 1 to state 2; the specific work done by the system is from state 3 to state 4. The net work of the system is the difference between the work done by the system to the work done to the system:

$$w_{1-2} = R * T_H * \ln\left(\frac{v_2}{v_1}\right) \quad [13]$$

$$w_{3-4} = R * T_C * \ln\left(\frac{v_3}{v_4}\right) \quad [14]$$

$$w_{net} = w_{1-2} - w_{3-4} \quad [15]$$

Where w_{1-2} is the specific work done to the system, w_{3-4} is the specific work done by the system, and w_{net} is the net specific work of the Stirling engine: the unit for specific work is W/kg. R is the gas constant of the working fluid.

Because there is a NASA ideal gas library within EES, the working fluid is coded as a string variable throughout the

program, so the user can assign the working fluid variable to be any ideal gas in the NASA ideal gas library. There is a “molarmass” function embedded in EES that will compute the molar mass of the assigned working fluid, and then the gas constant of the chosen working fluid can be determined:

$$R = \frac{R\#}{MW} \quad [16]$$

Where MW is the variable that outputs the value of the “molarmass” function for the assigned working fluid, and $R\#$ is a built in EES constant of 8.314 kJ/kmol-K.

Table 3 shows the molar mass of different ideal gases that will be used as the working fluid in the Stirling cycle of this EES program.

Table 3. Table of molar mass of various working fluids. The data was found using the built in EES “molarmass” function.

| Working Fluid | EES Ideal Gas Variable | Molar Mass [kg/kmol] |
|---------------|------------------------|----------------------|
| Hydrogen | H2 | 2.016 |
| Helium | He | 4.003 |
| Air | Air | 28.97 |

The mass of the working fluid is a ratio of the total volume of the working fluid (V_{wf} is found in the first EES program) over the specific volume at atmospheric pressure (atmospheric pressure is at state 3):

$$m = \frac{V_{wf}}{v_3} \quad [17]$$

An RPM array vector is coded in the second EES program with an initial element of 100 RPM and a final element of 1000 RPM. There are 10 total elements in the RPM vector, so this vector has an increment of 100 RPMs.

The “Duplicate” command in EES is useful for duplicating algebraic equations with array variables [6]. Array vectors in EES, such as the RPM array, cannot be included into algebraic equations without the use of the “Duplicate” command. Using the following formula and the Duplicate command, an array of the piston power output is calculated:

$$\dot{W} = \frac{RPM}{60} * w_{net} * m \quad [18]$$

Where \dot{W} is the resulting piston power of the engine in Watts. This formula outputs a two-dimensional matrix when a new Parametric table is created with the compression ratio array as the independent variable. The output of the new Parametric table is the Power Output matrix: the row vectors vary with compression ratio and the column vectors vary with the RPM array.

The last section of code in the second EES program is to determine how the power output and thermal efficiency of the Stirling engine varies when the temperature of the hot side is altered. The temperature of the cold side is held constant at 20 degrees Celsius; the swept path is held constant at 61 mm. A new array vector in EES is created to represent the temperature of the working fluid inside the hot side of the engine: the initial element in the array is 25 degrees Celsius and the final element is 250 degrees Celsius,

with increments of 25 degrees Celsius. Equation 19 shows how the array vector is coded into EES.

$$T_{HOT}[1..10] = [25,50,75,100,150,175,200,225,250] \quad [19]$$

Because the ideal isothermal cycle is used to analyze the Stirling engine, the specific volume at each state is not determined by the temperature of the hot side. Thus, the specific net work done to the system, w_{3-4} , is constant when the hot temperature value is altered. The specific work done by the system, w_{1-2} , varies with each change in the hot temperature value, and thus, the specific net work, w_{net} , will change. The RPM array previously created is used again to solve for a new power output matrix that depends on the hot temperature and RPM (the swept path is held constant at 61 mm, and thus compression ratio is constant). The “Duplicate” command in EES is used to output a two-dimensional power output matrix is created varying with hot temperature and RPM. Equation 20 shows how the power output equation is coded into EES.

$$\dot{W}_{HOT}[i,j] = \frac{RPM[i]}{60} * w_{net,HOT}[j] * m \quad [20]$$

Where \dot{W}_{HOT} is the power output that varies with hot temperature and RPM with units of kilowatts, $w_{net,HOT}$ is the specific net work that varies with hot temperature with units of kW/kg, and m is the mass of the working fluid with units of kg. i is the index from 1 to 10 that chooses the element in the RPM array. j is the index that chooses the specific net work array (that was created from the hot temperature array).

Equation 21 is used to calculate the thermal efficiency of the Stirling engine when the hot temperature is varied:

$$\eta_{th} = 1 - \frac{T_C}{T_H} \quad [21]$$

Where η_{th} is the thermal efficiency, T_C is the temperature of the cold side, and T_H is the temperature of the hot side of the engine.

The main downside of using EES Academic is that there needed to be two separate EES programs to solve for power output matrices, because a Parametric table cannot be used as a Lookup Table. If EES Professional was available, the \$CopyToLookup directive could be used to automatically create a lookup table [7]. Then the \$SaveLookup directive will automatically save the Lookup table [8]. These extra commands in EES Professional would allow the power output matrices to be solved in a singular simpler EES program.

Another limitation of EES Academic is that the run number (which represents the number of rows) in the Parametric tables need to be manually adjusted any time a new cylinder length is added to the code. If EES Professional was available, there is a command called “start run” and “finish run” that would automatically set the number of rows in the parametric table.

Exporting Lookup, Parametric, or Array Tables can be done automatically in the code with the \$Export directive, but this directive is only for EES Professional [9]. So, the tables that were computed by EES Academic were manually exported to Excel.

III. RESULTS

The output from the first EES program, is the Parametric table where swept path is the independent input variable and compression ratio is the dependent output variable. This table is saved as a Lookup table for the second EES program. Figure 1 shows an X-Y plot of the tabulated swept path verse compression ratio Parametric table.

Figure 1 shows how the compression ratio directly increases as the swept path is incremented by 1 mm for each element. Once the swept path is greater than 82 mm, the compression ratio increases significantly relative to swept path. This is because, at 82 mm, the swept volume surpasses the clearance volume. The input variables to find this data were the 57.4 mm bore and 90 mm length of the engine cylinder, and this data is independent from the choice of the working fluid.

The rest of this paper's output data is from the second EES program, which uses the ideal isothermal Stirling cycle to determine thermophysical properties, specific energy values, power outputs, and thermal efficiencies of the engine.

Table 4 shows a piece of the complete "Power Output" matrix: the number of rows is 89, because the length of the cylinder is 90 mm; the first column is of the array of compression ratio values, and the subsequent columns range from 100 RPM to 1,000 RPM, with an increment of 100. The working fluid is air, the temperature of the hot side is 57 degrees Celsius and the temperature of the cold side is 20 degrees Celsius.

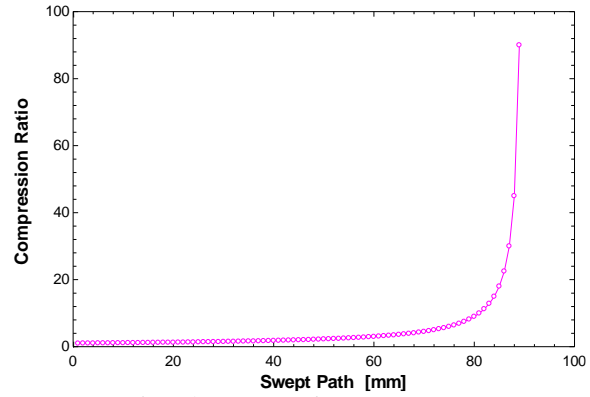


Figure 1. Plot of Swept Path Verse Compression Ratio. The data is taken from the output of the first EES program.

The goal of this program is to determine the RPM and swept path combination that would produce a power output of 10 Watts. The following table is data taken from the original 89 by 10 power output matrix. For a 57.4 mm bore engine with a length of 90 mm, if the temperature of the hot side is 57 degrees Celsius and the temperature of the cold side is 20 degrees Celsius, then the following table gives the swept path necessary to produce a minimum power output of 10 Watts.

The Tables menu in EES is used to create property plots that can be overlaid on top of each other, so that there is a visual to the processes of the Stirling cycle.

| CR | \dot{W} [1] [Watts] | \dot{W} [2] [Watts] | \dot{W} [3] [Watts] | \dot{W} [4] [Watts] | \dot{W} [5] [Watts] | \dot{W} [6] [Watts] | \dot{W} [7] [Watts] | \dot{W} [8] [Watts] | \dot{W} [9] [Watts] | \dot{W} [10] [Watts] |
|-------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|---------------------------|
| 1.011 | 0.1008 | 0.2017 | 0.3025 | 0.4033 | 0.5042 | 0.605 | 0.7058 | 0.8067 | 0.9075 | 1.008 |
| 1.023 | 0.2028 | 0.4056 | 0.6084 | 0.8112 | 1.014 | 1.217 | 1.42 | 1.622 | 1.825 | 2.028 |
| 1.034 | 0.3059 | 0.6119 | 0.9178 | 1.224 | 1.53 | 1.836 | 2.142 | 2.448 | 2.753 | 3.059 |
| 1.047 | 0.4103 | 0.8205 | 1.231 | 1.641 | 2.051 | 2.462 | 2.872 | 3.282 | 3.692 | 4.103 |
| 1.059 | 0.5158 | 1.032 | 1.547 | 2.063 | 2.579 | 3.095 | 3.611 | 4.127 | 4.642 | 5.158 |
| 1.071 | 0.6226 | 1.245 | 1.868 | 2.49 | 3.113 | 3.736 | 4.358 | 4.981 | 5.604 | 6.226 |
| 1.084 | 0.7307 | 1.461 | 2.192 | 2.923 | 3.653 | 4.384 | 5.115 | 5.846 | 6.576 | 7.307 |
| 1.098 | 0.8401 | 1.68 | 2.52 | 3.36 | 4.2 | 5.04 | 5.881 | 6.721 | 7.561 | 8.401 |
| 1.111 | 0.9508 | 1.902 | 2.852 | 3.803 | 4.754 | 5.705 | 6.656 | 7.606 | 8.557 | 9.508 |
| 1.125 | 1.063 | 2.126 | 3.189 | 4.252 | 5.315 | 6.378 | 7.44 | 8.503 | 9.566 | 10.63 |
| 1.139 | 1.176 | 2.353 | 3.529 | 4.706 | 5.882 | 7.059 | 8.235 | 9.411 | 10.59 | 11.76 |
| 1.154 | 1.291 | 2.583 | 3.874 | 5.166 | 6.457 | 7.748 | 9.04 | 10.33 | 11.62 | 12.91 |
| 1.169 | 1.408 | 2.816 | 4.224 | 5.631 | 7.039 | 8.447 | 9.855 | 11.26 | 12.67 | 14.08 |
| 1.184 | 1.526 | 3.052 | 4.577 | 6.103 | 7.629 | 9.155 | 10.68 | 12.21 | 13.73 | 15.26 |
| 1.2 | 1.645 | 3.291 | 4.936 | 6.581 | 8.227 | 9.872 | 11.52 | 13.16 | 14.81 | 16.45 |
| 1.216 | 1.766 | 3.533 | 5.299 | 7.066 | 8.832 | 10.6 | 12.37 | 14.13 | 15.9 | 17.66 |
| 1.233 | 1.889 | 3.779 | 5.668 | 7.557 | 9.446 | 11.34 | 13.22 | 15.11 | 17 | 18.89 |
| 1.25 | 2.014 | 4.027 | 6.041 | 8.055 | 10.07 | 12.08 | 14.1 | 16.11 | 18.12 | 20.14 |
| 1.268 | 2.14 | 4.28 | 6.42 | 8.56 | 10.7 | 12.84 | 14.98 | 17.12 | 19.26 | 21.4 |
| 1.286 | 2.268 | 4.536 | 6.804 | 9.072 | 11.34 | 13.61 | 15.88 | 18.14 | 20.41 | 22.68 |
| 1.304 | 2.398 | 4.796 | 7.193 | 9.591 | 11.99 | 14.39 | 16.78 | 19.18 | 21.58 | 23.98 |
| 1.324 | 2.53 | 5.059 | 7.589 | 10.12 | 12.65 | 15.18 | 17.71 | 20.24 | 22.77 | 25.3 |

Table 4. Power Output Matrix that varies with compression ratio, CR, and RPM The dimension of this table is a 21 by 10 matrix of power output values. The entire Power Output matrix for a Stirling engine with a length of 90 mm is an 89 by 10 matrix.

Table 5. 3 by 3 Power Output matrix extracted from the complete 89 by 10 Power Output matrix. Shows the compression ratio, CR, and RPM combination required for a minimum of 10 Watts.

| Swept Path [mm] | CR | \dot{W} [1] [Watts] | \dot{W} [2] [Watts] | \dot{W} [3] [Watts] |
|-----------------|-------|-----------------------|-----------------------|-----------------------|
| 61 | 3.103 | 10.22 | 20.44 | 30.66 |
| 39 | 1.765 | 5.126 | 10.25 | 15.38 |
| 28 | 1.452 | 3.363 | 6.727 | 10.09 |

Figure 2 is a P-v property plot; Figure 2 is a T-s property plot. Orange is the cycle when the swept path is 61 mm. Green is the cycle when the swept path is 39 mm. Purple is the cycle when swept path equals 28 mm.

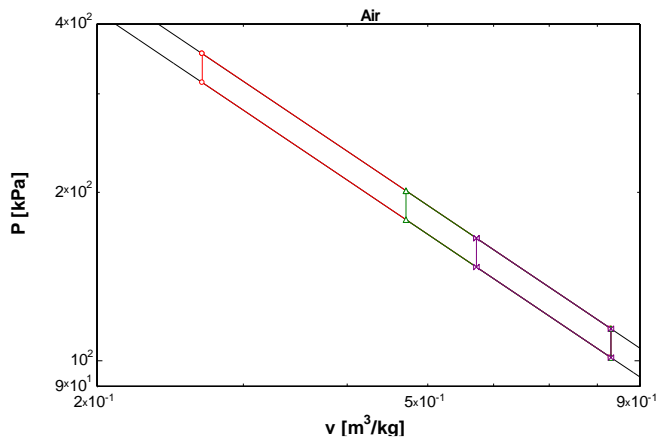


Figure 2. P-v Property Plot that shows the ideal isothermal thermodynamic process of a Stirling engine. Orange represents the Stirling cycle when the swept path is 61 mm. Green represents the Stirling cycle when the swept path is 39 mm. Purple represents the Stirling cycle when the swept path is 28 mm.

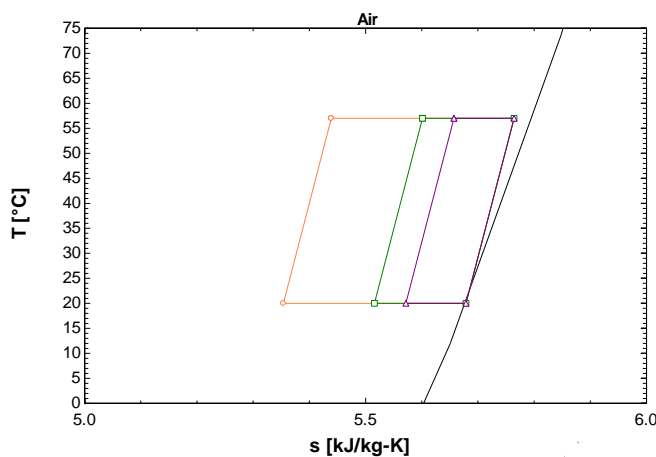


Figure 3. T-s Property Plot that shows the ideal isothermal thermodynamic process of a Stirling engine. Orange represents the Stirling cycle when the swept path is 61 mm. Green represents the Stirling cycle when the swept path is 39 mm. Purple represents the Stirling cycle when the swept path is 28 mm.

The power output of the 3 cycles chosen were around 10 Watts, but each cycle had a different compression ratio and RPM combination. This will affect the maximum pressure,

change in specific entropy, and specific volume of the isothermal Stirling cycle. When the RPM increased, the compression ratio and swept path decreased to reach a specific power output of 10 Watts. The maximum pressure of the cycle decreases as compression ratio decreases. This is useful knowledge for designing a Stirling engine, because the cylinder is a pressure vessel and can fail under the maximum pressure of the cycle. The difference in entropy decreases as the compression ratio decreases, which means that the cycle is less irreversible. A smaller irreversibility means that less energy will be lost due to friction, hence the Stirling cycle is more efficient.

Figure 4 and Figure 5 are X-Y plots for EES that elaborate on how the compression ratio and swept path relates to the maximum pressure of the cycle when air is the working fluid, the hot temperature is 57 degrees Celsius, and the cold temperature is 20 degrees Celsius.

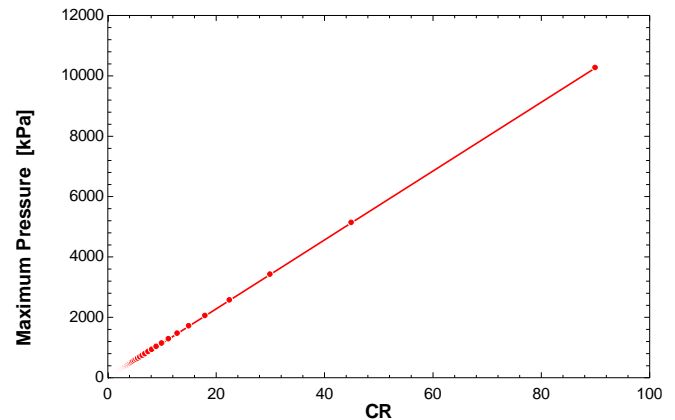


Figure 4. Maximum pressure of the Stirling cycle versus the compression ratio of the cycle.

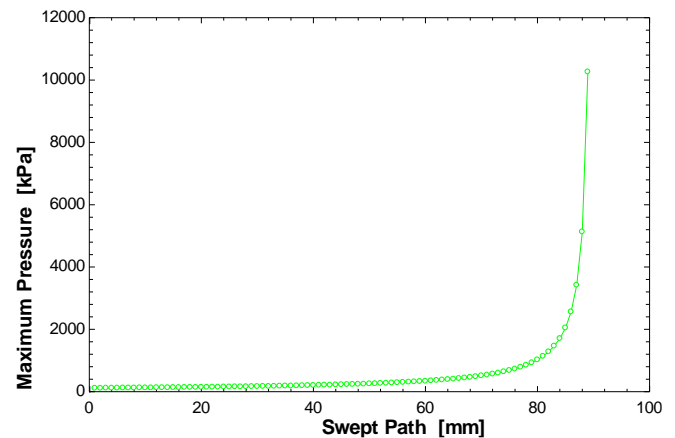


Figure 5. Maximum pressure of the Stirling cycle versus the swept path of the cycle.

The compression ratio has a linear relationship with the maximum pressure of the cycle, whereas the swept path has an exponential relationship to the maximum pressure. The maximum pressure of this cycle takes place at state 1, since state 3 is the lowest pressure of 101.3 kPa (atmospheric pressure).

Figure 6 and Figure 7 are X-Y plots from EES that show the specific net work of the Stirling cycle logarithmically increases with compression ratio, whereas the specific net work has an exponential relationship with swept path.

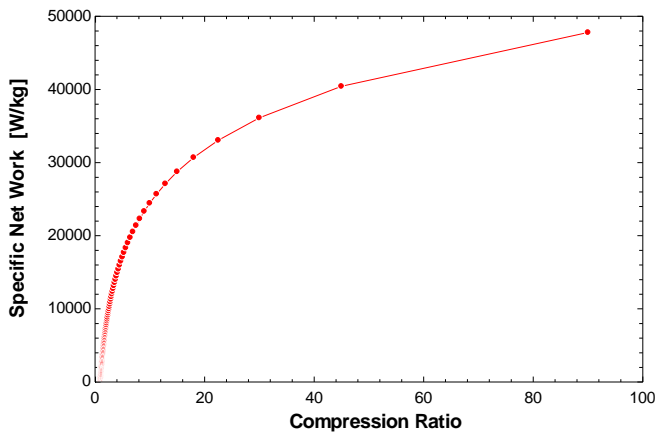


Figure 6. Specific net work of the Stirling engine verse the compression ratio.

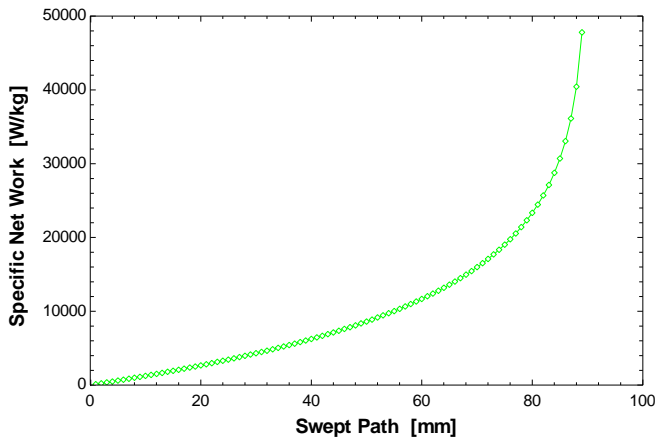


Figure 7. Specific net work of the Stirling engine verse the swept path.

Figure 8 and Figure 9 are X-Y plots that take all the data from the 89 by 10 power output matrix that varies with RPM and compression ratio:

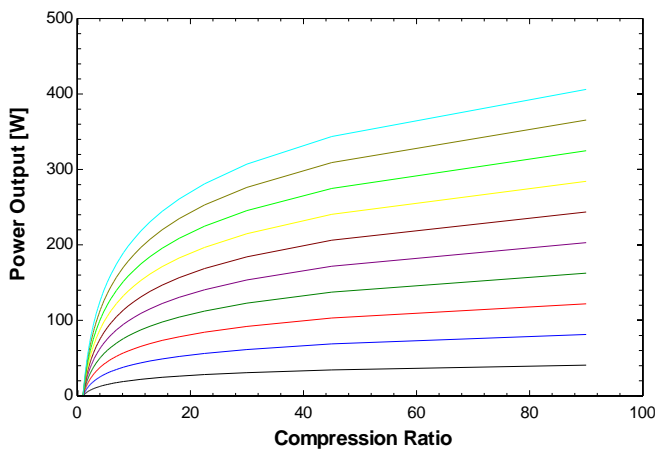


Figure 8. Power output of the Stirling engine verse the compression ratio. The power output function with the lowest wattage is with an RPM of 100. The power output function with the highest wattage is with an RPM of 1000.

The plots from Figure 8 and Figure 9 show that the Stirling engine power output linearly increases with RPM, logarithmically increases with compression ratio, and exponentially increases with swept path.

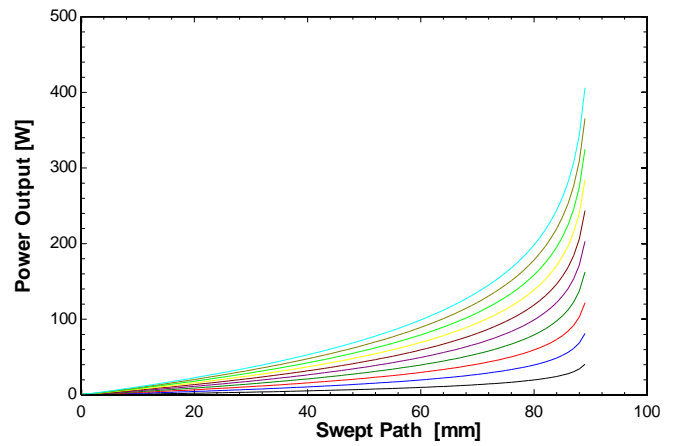


Figure 9. Power output of the Stirling engine verse the swept path. The power output function with the lowest wattage is with an RPM of 100. The power output function with the highest wattage is with an RPM of 1000.

Figure 10 and Figure 11 are X-Y plots that show the difference that the working fluid has on specific net work: red represents hydrogen, pink represents air, and green represents helium. Shows that a lower molar mass produces a higher net work, and thus great power.

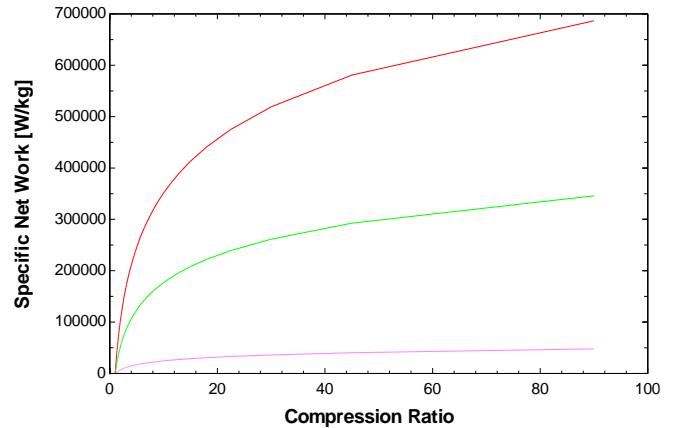


Figure 10. Specific net work of the Stirling engine verse the compression ratio.

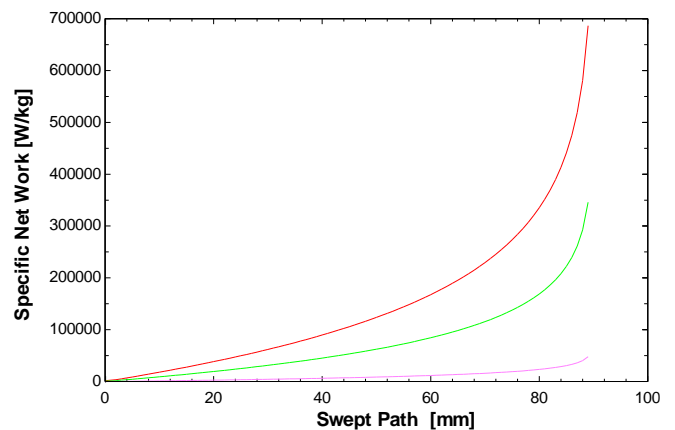


Figure 11. Specific net work of the Stirling engine verse the swept path.

Changing the working fluid does not affect P-v and T-s plots because changing the working fluid only changes the ideal gas constant and mass of the working fluid. Thus, changing the working fluid only changes power output and

energy values. Because the 28.97 kg/kmol molar mass of air is much larger than the molar mass of helium or hydrogen, the power output of air as the working fluid is less than that of hydrogen and helium.

The second EES program additionally solves for the power output matrix varying with the temperature of the hot side of the Stirling engine and RPM. Figure 12 is an X-Y plot that shows the linear relationship of the temperature of the hot side versus the power output.

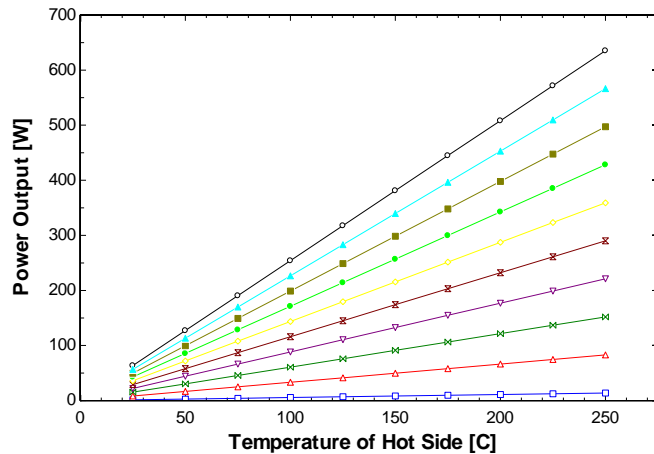


Figure 12. Power output of the Stirling engine versus the temperature of the working fluid in the hot cylinder. The blue function has an RPM of 100; the black function has an RPM of 1000.

The hot temperature and thermal efficiency have a direct relationship. As the hot temperature increases, the thermal efficiency improves. But the thermal efficiency can never reach 100%, because the temperature of the cold side would need to equal 0 Kelvin—this is impossible since no matter can equal 0 Kelvin.

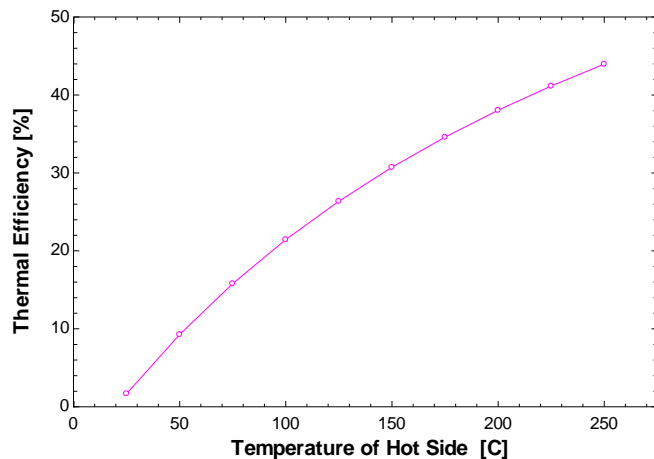


Figure 13. Thermal efficiency of the Stirling engine versus the temperature of the working fluid in the hot cylinder.

IV. STUDENT SURVEY AND INTERVIEW

Students taking the SUNY New Paltz Thermal System Design course during Spring 2022 were requested to complete a self-reported survey instrument regarding their experience with EES. There are a total of 50 undergraduate students taking the Spring 2022 Thermal System Design course: 37 are male and 13 are female. A total of 30 students completed the EES survey.

A Likert scale is used for the student survey to measure the students' opinions of EES. The purpose of a Likert scale is to transform an individual's subjective opinion into objective quantitative research [10]. There is a five-level scale for each question: *Strongly Disagree* (1), *Disagree* (2), *Neither Agree nor Disagree* (3), *Agree* (4), and *Strongly Agree* (5).

The first survey question asks the students, "EES is a more effective and efficient method for students to solve thermal system design problems than interpolating data from the thermodynamic property tables?" 25 students (83.3%) responded *Strongly Agree*, 4 student (10%) responded *Agree*, 1 student (3.3%) responded *Neither Agree nor Disagree*, and 1 student (3.3%) responded *Strongly Disagree*.

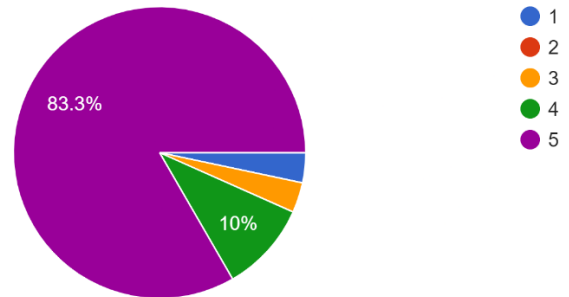


Figure 14. Result of Survey Question 1: "EES is a more effective and efficient method..." Five-level scale: *Strongly Disagree* (1), *Disagree* (2), *Neither Agree nor Disagree* (3), *Agree* (4), and *Strongly Agree* (5)

The second survey question asks the students, "A major component of EES is the parametric analysis, utilizing Parametric tables: you would choose EES as your software of choice for a parametric study?" 14 students (46.7%) responded *Strongly Agree*, 12 students (40%) responded *Agree*, 1 student (3.3%) responded *Neither Agree nor Disagree*, 2 students responded *Disagree*, and 1 student (3.3%) responded *Strongly Disagree*.

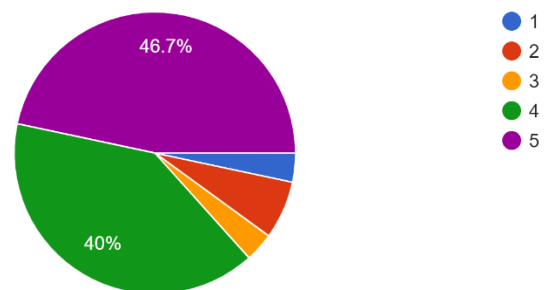


Figure 15. Result of Survey Question 2: "A major component of EES is the parametric analysis..." Five-level scale: *Strongly Disagree* (1), *Disagree* (2), *Neither Agree nor Disagree* (3), *Agree* (4), and *Strongly Agree* (5)

Many students who attended the EES tutor sessions expressed that they benefitted from their peer learning experience: They stated their professor focused more on the thermal system concepts during class than teaching the students how to code in EES. So, they attended the EES tutor

sessions to ask for clarification on EES concepts that they did not comprehend well during the professor's lecture.

An individual student who attended the EES tutor sessions was interviewed and asked the question "How would you compare learning EES code from the peer learning EES tutor Sessions to the professor's lectures?":

"...I would get more of an elaboration from my peers rather than the professor...peer learning has a more accurate representation of the expectations of what a college student knows."

V. SUMMARY

The thermodynamic analysis of the Stirling engine was completed using the ideal isothermal Stirling cycle, invented by Robert Stirling. Process 1-2 is isothermal expansion. Process 2-3 is isochoric heat rejection. Process 3-4 is isothermal compression. Process 4-1 is isochoric heat addition.

The purpose of the two EES programs coded is to learn how different input parameters affect the ideal isothermal Stirling cycle, the power output, and thermal efficiency. These input parameters are swept path, compression ratio, RPM, temperature, bore, cylinder length, and the working fluid.

These two EES programs were utilized in the design of both an alpha Stirling engine and a beta Stirling engine to assist the team members for what temperature, swept path, compression ratio, and RPM combination were needed to produce the required power output for the DC generator. More specifically, finding the required swept path helped in designing the length of the crank shaft and connecting rod linkages for the slider-crank mechanism chosen.

The benefit of using EES Academic as the chosen software instead of other programs is that EES has thermophysical functions already embedded into its system and a built in NASA Ideal Gas Library so that there is no need to interpolate any data. EES Academic had a few drawbacks that would have been solved if EES Professional was used instead.

- This "Stirling Engine Cycle" code in EES Academic is an effective method to learn how swept path, compression ratio, RPM, and temperature affect the Stirling cycle, power output, and efficiency.
- Most students think that EES Academic is an effective tool to teach students how to design thermal systems and perform parametric studies.
- EES Professional includes additional commands and directives that produce a simpler and more automatic code.

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Appendix 01: The first EES program

"!Find Compression Ratio of Stirling Engine"

"Input Cylinder Dimensions"

bore = 57.4 [mm]

l = 90 [mm]

V_disp = 0 [mm³]

{Bore of Cylinder}

{Length of cylinder}

{Value is 0 for Alpha Engine}

"Swept Path"

\$If Parametric Table

\$Else

s = 1 [mm]

\$End

"Clearance Volume of Engine"

$V_c = \pi \cdot (\text{bore}^2/4) \cdot (l-s) - V_{\text{disp}}$

"Swept Volume of Engnie"

$V_s = \pi \cdot (\text{bore}^2/4) \cdot s$

"Volume of Working FLuid"

$V_{\text{wf}} = V_s + V_c$

"Find Compression Ratio"

$CR = V_{\text{wf}} / V_c$

Appendix 02: The Second EES Program

"!Ideal Stirling Engine Cycle"

"Process 1-2: Isothermal Expansion"
"Process 2-3: Isochoric Heat Rejection"
"Process 3-4: Isothermal Compression"
"Process 4-1: Isochoric Heat Addition"

"Compression Ratio"

X\$ = 'Swept Path Verse CR 1st Engine' {!Input the name of the Lookup table}
\$If ParametricTable
CR = **lookup**(X\$,TableRun#,'CR')
s = **lookup**(X\$,TableRun#,'S')
\$Else
SweptPath = 61 {Units: mm}
CR = **lookup**(X\$,SweptPath,'CR')
\$EndIf

"Volume of Working Fluid"

V = **lookup**('Volume of Working Fluid',1,'Engine 1') {!Input the name of the Lookup table}

"Working Fluid--must be ideal gas for the code"

WF\$ = 'Air' {'Air' is ideal gas air; 'He' is ideal gas helium,
'H2' is ideal gas hydrogen}

"Gas Constant of Working Fluid"

MW=**molar**mass(WF\$)
R = R#/MW {R# is univeseral gas constant}

"Heat Source and Heat Sink Temperature Assumption"

T_H = 57 [C] {!Input the hot temperature value}
T_L = 20 [C] "Atmospheric temperature"

"State 3 Properties:"

P[3] = 101.3 [kPa] "Assume P3 is atmospheric pressure; P3 is
lowest pressure in cycle"
T[3] = T_L "Assume T3 is equal to heat sink"
Call idealgasthermoprops(WF\$, T=T[3],P=P[3] : T[3],P[3],v[3],h[3],s[3],u[3])

"State 2 Properties:"

v[2] = v[3] "Process 2-3 is isochoric, so v2 = v3"
T[2] = T_H "Assume T2 is equal to temp of heat source"
Call idealgasthermoprops(WF\$, T=T[2],v=v[2] : T[2],P[2],v[2],h[2],s[2],u[2])

"State 1 Properties:"

v_max = v[3] "Compression Ratio Formula is used to find v1"
CR = v_max / v_min
v[1] = v_min
T[1] = T[2] "Process 1-2 is isothermal, so T1 = T2"
Call idealgasthermoprops(WF\$, T=T[1],v=v[1] : T[1],P[1],v[1],h[1],s[1],u[1])

"State 4 Properties:"

T[4] = T[3] "Process 3-4 is isothermal, so T3 = T4"
v[4] = v[1] "Process 1-4 is isochoric, so v1 = v4"
Call idealgasthermoprops(WF\$, T=T[4],v=v[4] : T[4],P[4],v[4],h[4],s[4],u[4])

"State 5 Properties:"

P[5] = P[1]
T[5] = T[1]
s[5] = s[1]
h[5] = h[1]
v[5] = v[1]
u[5] = u[1]

"Air, Energy Values and Engine Efficiency"

w_12 = R*(T_H+273.15)*ln(v[2] / v[1])*1000
w_34 = R*(T_L+273.15)*ln(v[3] / v[4])*1000
w_net = w_12 - w_34
q_in = w_12
n_th = (1 - (T_L+273.15)/(T_H+273.15))*100

"Boundary Work done from Piston-Cylinder"
"Boundary Work done to Piston-Cylinder"
"Total boundary work of Piston-Cylinder"
"Total heat into system"
"Thermal Efficiency"

"Mass of piston-Cylinder at atmospheric pressure (State 3)"

m = V/v[3]

{Array RPM}

RPM[1..10] = [100,200,300,400,500,600,700,800,900,1000]

{Array RPM End}

"Power of Piston"

Duplicate i = 1,10

W_dot[i] = RPM[i]/60*w_net*m

End

"!Power Output and Thermal Efficiency Verse Hot Temperature"

{Array T_HOT}

T_HOT[1..10] = [25,50,75,100,125,150,175,200,225,250]

{Array T_HOT End}

Duplicate i=1,10

w_12_HOT[i] = R*(T_HOT[i]+273.15)*ln(v[2] / v[1])*1000 "Boundary Work done from Piston-Cylinder"
w_34_HOT[i] = R*(T_L+273.15)*ln(v[3] / v[4])*1000 "Boundary Work done to Piston-Cylinder"
w_net_HOT[i] = w_12_HOT[i] - w_34_HOT[i] "Total boundary work of Piston-Cylinder"
q_in_HOT[i] = w_12_HOT[i] "Total heat into system"
n_th_HOT[i] = (1 - (T_L+273.15)/(T_HOT[i]+273.15))*100 "Thermal Efficiency"

End

Duplicate i = 1,10

Duplicate j=1,10

W_dot_HOT[i,j]=RPM[i]/60*w_net_HOT[j]*m

End

End