Pediment Graduate Course in Transport Phenomena

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Abstract

The classical approach to teaching transport phenomena has been to develop the appropriate form of the equations-of-motion or conservation equations and then to explore various solutions to these equations. Transport phenomena textbooks typically focus on relatively straightforward problems that admit analytical or simple numerical solutions. However, with the rapid advance in packaged software for solving complex coupled nonlinear differential equations, there is a need to place more emphasis in a graduate-level transport course on model development rather than problem-solving techniques. This paper outlines the organization of a graduate-level course in fluid dynamics and heat transfer that addresses this need. Systematic scaling analysis is used to develop idealized models such as creeping, lubrication, and boundary-layer flows, quasi-steady-state transport, film and penetration theory, *etc*. This pediment course also provides an introduction to advanced topics such as porous and permeable media flows, free surface flows, moving boundary problems, 'jump' boundary conditions, non-material surfaces, and interfacial flows. The ABET 2000 outcomes addressed in this course are identified and the assessment of student achievement towards these outcomes is summarized.

1 Introduction

This paper discusses the pediment graduate course 'Transport Phenomena I' in the Department of Chemical and Materials Engineering at the University of Cincinnati (UC). It covers fluid dynamics and heat transfer during the Fall Quarter (UC is on a quarter rather than semester system). Transport Phenomena II covers mass transfer and is offered during the Winter Quarter. Transport Phenomena I is required for all graduate students in chemical engineering and is a technical elective for seniors. Typical enrollment is 20-30 graduate students and 2-5 seniors. The course involves 30 fifty-minute lecture periods, weekly homework assignments, and mid-term and final examinations. The required textbook, *Transport Phenomena* by Bird *et al.* [1], is supplemented by approximately 200 pages of word-processed notes. Weekly office hours are scheduled. The Blackboard® utility is used to provide a 24-hpd discussion forum and as a means to disseminate lecture notes, problem sets, and homework and exam solutions.

The course goals are: (1) to learn the language of continuum mechanics; (2) to develop an enhanced understanding of fluid dynamics and heat transfer; and (3) to learn the requisite skills to develop viable models. The course methodology involves: (1) motivating via example problems; (2) utilizing novel pedagogical tools to interrelate topics; and (3) assigning open-

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ended design-type homework problems. This course addresses the need for students to model the more complex problems that powerful software packages now permit us to solve. Problem-solving methods such as software packages are not covered in this course since they are the subject of a subsequent Mathematical Methods course. The specific ABET 2000 outcomes addressed by this course are the following: (a) ability to apply knowledge of mathematics, science, and engineering; (c) ability to design a system, component or process; (e) ability to identify, formulate and solve engineering problems; and (k) ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

This paper will discuss the content of this course with particular emphasis on novel pedagogical methods that are used. Typical homework problems will also be described. The assessment of student achievement in addressing the ABET 2000 outcomes is also summarized.

2 Discussion of Course Content

Table 1 summarizes the topical outline and lecture allocation for this course. In the following we will provide a brief summary of the content for each topic listed in Table 1.

Table 1: Topical Outline and Lecture Allocation for Transport Phenomena I

Topic	*Subtopics*	*Lectures*
Vector-tensor mathematics		3
Rheology of real fluids		2
Equations-of-motion and Energy-conservation equations		3
Scaling the equations-of-motion	*Low Reynolds number or creeping flows*	1
	Lubrication flows	1
	High Reynolds number or boundary layer flows	3
	Porous media flows	2
Scaling the energy-conservation equations	*Low Peclet number heat transfer*	1
	High Peclet number heat transfer	2
	Film theory for heat transfer	1
	Penetration theory for heat transfer	1
	Heat transfer with phase change	2
Special topics in fluid dynamics	*Transition to turbulence – stability theory*	2
	Turbulent flow – time-averaged equations	1
	Phenomenological theories for turbulent flow	1
Special topics in heat transfer	*Free convection heat transfer*	2
	Heat transfer with phase change	1
Midterm examination		1
Final examination		

2.1 Vector-Tensor Mathematics

This section focuses on the basic operations between scalars, vectors, and tensors and considers symbolic, index, and unit vector/dyad notation. The emphasis is on giving students a physical feel for the vector/tensor operations in the equations-of-motion and energy-conservation

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equations. For example, in the homework assignments, the students consider the solution to one-dimensional heat conduction into a semi-infinite solid to see that a positive Laplacian of the temperature at a point implies accumulation of thermal energy and that a negative volume integral of the divergence of the temperature implies heat transfer into the solid. They also consider solid body rotation to see that a nonzero area integral of the curl of the velocity implies circulation. This section also introduces generalized orthogonal curvilinear coordinates so that the students are aware of coordinate systems other than rectangular, cylindrical, and spherical.

2.2 Rheology of Real Fluids

Fluid behavior is broadly classified into time-independent and time-dependent. Students are given some insight into the molecular and particulate origins of non-Newtonian behavior. The Newtonian, Bingham plastic, Ostwald-de Waele, Ellis, and Sisko constitutive equations for time-independent and Maxwell model for time-dependent fluid behavior are considered in tensor form. Thixotropic, anti-thixotropic, and viscoelastic behavior is discussed. This topic also includes a discussion of the transport properties associated with the constitutive equations. Simple models for predicting the shear viscosity of Newtonian fluids are presented. The different way that gas and liquid viscosities respond to temperature is emphasized. Meaningful homework problems consider Bingham plastic fluid flows. Since the latter flow only when the critical shear stress is exceeded, these problems get the students to focus on applying Newton's law-of-motion at a plane in a fluid to determine proper boundary conditions. In other homework problems the students consider the Rivlin-Erickson constitutive equation (that involves the co-rotational time derivative) and whether the general constitutive equation for a purely viscous fluid (*i.e.*, a Reiner-Rivlin fluid) can admit viscous normal stresses in a simple shear flow.

2.3 Equations-of-Motion and Energy-Conservation Equations

We begin this topic by considering a shell balance for steady-state, fully developed, gravity-induced flow of two infinitely wide stratified liquid films with an upper planar liquid-gas interface. This problem introduces different types of boundary conditions, stratified flows, and free surface flows. We emphasize the importance of being able to carry out shell balances. The homework includes problems that cannot be modeled by directly simplifying the generalized equations-of-motion such as the draining of a two-dimensional liquid film down a vertical wall (problem 2D.2 on page 73 in Bird *et al*. [1]) and an axisymmetric liquid jet falling vertically owing to gravity. In these problems if the film or jet thickness is small compared to the vertical length scale, the average velocity can be used in a shell balance across the entire flow.

We consider the derivation of the equations-of-motion and energy-conservation equations within the same topic, which offers several advantages: (1) the students see that the fundamental laws are being applied to the same control volume; (2) the connection between the mechanical energy and momentum equations is apparent; (3) the students gain early exposure to the entropy-production equation that can be used in conjunction with variational methods to solve flow problems; and (4) the transport theorems (various forms of Stokes' theorem), Leibnitz' Rule, and vector-tensor identities are used in a similar way in all the derivations.

We define a fluid particle, which serves as the control volume for the derivations. Concepts such as stationary versus convected or Lagrangian reference frames are introduced. The various time derivatives are introduced and extended to include operations such as the co-rotational derivative

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that allows for time variation owing to temporal changes in the field variables at a fixed spatial point and due to both convection and rotation of a fluid particle. Students are cautioned that one takes a total time derivative of extensive properties but partial, substantial, and co-rotational derivatives of intensive properties (a rule that is occasionally violated in textbooks!).

The derivation of the continuity equation is done on a fluid particle and the physical significance of each term in the various forms of the continuity equation is emphasized. Homework problems involve carrying out this derivation in a fixed coordinate system and considering the implications of mass generation for a system involving interconversion of mass and energy.

The equations-of-motion are also derived for a fluid particle and the significance of each term in the various forms of these equations is stressed. Students are made aware that body forces also include electric and magnetic fields as well as acceleration fields of which gravity is a special case. Homework problems again involve considering the effect of mass generation, which enters the equations-of-motion indirectly through the continuity equation.

We then consider the conservation of total energy for a convected fluid particle. We distinguish between macroscopic kinetic energy associated with the fluid particle and molecular kinetic energy associated with the internal energy. We then derive the mechanical energy equation by taking the scalar product of the velocity with the equations-of-motion. By subtracting this from the total energy equation we obtain the thermal energy equation. The latter is then used to derive the entropy production and enthalpy equations. The physical significance of each term is emphasized; in particular, we show which terms contribute to the internal energy and kinetic energy, respectively. The entropy-production equation in combination with the Clausius inequality permits us to determine which terms in the energy equation contribute reversibly or irreversibly. An introduction to macroscopic irreversible thermodynamics is provided whereby it is assumed that the rate of entropy production will be a minimum for a quasi-steady process, which is the basis of the variational method for solving transport problems. An interesting homework problem involves showing that the less viscous fluid will be near the wall for the concentric laminar flow of two immiscible liquids in a cylindrical tube, since this configuration minimizes the drag and thereby the viscous dissipation and corresponding entropy production.

We end with a discussion of the boundary conditions in fluid dynamics and heat-transfer problems. Boundary conditions are considered that involve applying Newton's law-of-motion at an interface when surface-active molecules displaying surface rheological properties might be present. A general derivation of the kinematic surface condition that must be applied for free surface flows is also given. The students are introduced to 'jump' boundary conditions whereby a thin region is reduced to a mathematical plane across which discontinuities are allowed in the dependent variables and/or their derivatives. Homework problems involve specifying the kinematic surface and boundary conditions at a wavy interface and deriving the kinematic surface condition for flows in which mass enters or leaves the system.

2.4 Scaling the Equations-of-Motion

Scaling involves nondimensionalizing a set of describing equations so that the relevant dependent and independent variables are bounded of order one. Scaling is used to provide a systematic method for obtaining the minimum parametric representation of the describing

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equations for fluid dynamics, which thereby permits assessing simplifying assumptions that might be invoked. As such, it provides a valuable pedagogical tool for introducing subtle concepts such as creeping flow or boundary-layer theory in fluid dynamics. The use of scaling analysis as a tool for teaching transport phenomena has been discussed in prior papers of the author [2,3]. Recent papers discuss how scaling analysis can be used to correlate experimental data or the results of a numerical simulation [4 and to estimate the optimal design parameters for a process [5]. The following steps are involved in scaling analysis:

1. Write the complete set of describing equations including any relevant boundary, initial, and auxiliary conditions.
2. Define dimensionless variables for the relevant dependent and independent variables introducing arbitrary scale and reference factors.
3. Substitute the dimensionless variables into the describing equations.
4. Divide each equation through by the dimensional coefficient of a term that must be retained to assure a physically meaningful solution; this results in dimensionless groups containing the arbitrary scale and reference factors.
5. Determine the unknown scale and reference factors by setting the resulting dimensionless groups equal to one or zero in order to bound the relevant independent and dependent variables to be of order one; by this we mean that the magnitude of the dimensionless quantity ranges between zero and one.
6. Assess the relative importance of the various terms in the describing equations via the magnitude of the remaining dimensionless groups.

We consider an example problem (discussed in [2]) involving steady-state, two-dimensional, laminar flow between nonparallel plates. We scale the describing equations-of-motion and their boundary conditions to determine when the nonlinear inertia and axial viscous terms can be ignored so that this can be considered to be a ‘locally parallel flow’. This example provides a systematic means for introducing the concepts of ‘a creeping flow’ (low Reynolds number) and ‘a lubrication flow’ (low Reynolds number and small aspect ratio). Homework problems involve the design of hollow fiber membrane modules and fluid bearings.

A second example (discussed in [2]) involves a uniform laminar plug flow intercepting a semi-infinitely long, infinitely wide, horizontal, flat plate. Scaling is used to determine when the describing elliptic equations can be reduced to parabolic equations for which the solution to the axial momentum equation is decoupled from that of the transverse component of the momentum equation. This example provides a systematic means for introducing the concept of ‘a boundary-layer flow’ (high Reynolds number). Homework problems consider the implications of fluid injection or withdrawal and the use of scaling for analyzing entry region flows (discussed in [3]).

We then consider a problem (discussed in [3]) involving the steady-state flow of a liquid through a tube containing a porous medium. We scale the describing equations, in this case the Darcy equations with the Brinkman term that permits satisfying no-slip at the tube wall, to determine when the effect of the viscous drag at the wall can be neglected. We also determine the thickness of the region near the wall within which the effects of the wall drag cannot be ignored. This example introduces the students to flow through porous media. This example also illustrates that a boundary layer is a region-of-influence wherein some physical effect is confined; it need not be just a high Reynolds number boundary layer. In this case the region wherein the wall drag affects

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the flow constitutes a boundary layer whose thickness is determined by relative magnitude of the viscous shear at the wall and the viscous drag through the porous medium.

2.5 Scaling the Energy-Conservation Equations

Scaling analysis is also a valuable pedagogical tool for illustrating the analogies between fluid dynamics and heat transfer. It also provides a means for introducing subtle concepts such as thermal boundary layers, film theory, and penetration theory. We begin with an example involving steady-state, two-dimensional, heat transfer between two infinitely wide parallel plates with axial convection, axial and transverse heat conduction, and viscous dissipation (discussed in [2]). We determine when the axial convection and conduction of heat can be ignored relative to transverse conduction and heat generation via viscous dissipation. In this case scaling analysis indicates that this is a heat transfer analog to the lubrication flow approximation in fluid dynamics. Homework problems involve using scaling to estimate the maximum temperature and determining when the temperature-dependence of the viscosity can be ignored.

A second example considers steady-state, two-dimensional, laminar film flow down a vertical plate maintained at a constant temperature with axial convection and both axial and transverse heat conduction. We seek to determine when the convective heat transfer can be adequately described using a linear approximation to the parabolic velocity profile and when axial conduction can be ignored. Scaling indicates that this is a high Peclet number thermal boundary-layer analog to a high Reynolds number boundary-layer flow. Homework problems consider the same flow but with heat transfer occurring owing to an adjacent gas phase at a temperature different from that of the liquid film. In this case the thermal boundary layer is near the liquid-gas interface rather than near the solid surface.

A third example involves unsteady-state, one-dimensional heat conduction into a solid slab having a finite thickness across which a temperature difference is imposed. We estimate the time required to achieve steady-state conduction and also the time period during which the conduction can be assumed to be effectively into a semi-infinite medium. Scaling indicates that the former occurs when the contact time is long compared to the characteristic conduction time and introduces the students to the film-theory model for heat transfer. The latter occurs when the contact time is short compared to the characteristic conduction time and introduces the students to the penetration-theory model for heat transfer.

The last example (discussed in [3]) involves unsteady-state, one-dimensional heat conduction into a semi-infinite medium with phase change (melting of water-saturated frozen soil). We use scaling analysis to determine when quasi-steady-state melting can be assumed. This problem introduces the students to moving boundary problems for which they need to specify an auxiliary condition to locate the position of the moving front. Homework problems consider freezing rather than melting, the effect of an added snow layer, and the effect of different boundary conditions at the upper surface such as a constant solar heat flux or Newton's law-of-cooling to account for a prevailing wind.

2.6 Special Topics in Fluid Dynamics

Ttransition to turbulence is approached by considering stability theory. We consider the linear stability of a quiescent, initially planar, infinitely wide, liquid film suspended from a flat plate

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and subject to gravitational body forces. This is an ideal problem for illustrating how small perturbations can cause a fluid to depart from some steady-state condition that satisfies the equations-of-motion. Since the initial state is purely hydrostatic, the linear stability problem can be solved analytically. This is also a good example, since the students are familiar with manifestations of this instability; for example, the regular spacing of dripping water or icicles (formed when the former freeze) at the edge of a roof. For homework the students analyze the linear stability of a ball located atop a two-dimensional hill subject to gravitational body forces.

An introduction to turbulent flow is provided by first deriving the time-averaged equations-of-motion. We then attempt to solve the 'simplest' possible turbulent flow problem, namely fully developed flow between two infinitely wide, parallel, flat plates. The students quickly see that this cannot be done owing to the closure problem associated with the Reynolds stress terms. This provides the motivation for considering the phenomenological theories of turbulence. Mixing length theory is used to arrive at the universal semi-logarithmic velocity profile and friction factor correlation for fully developed turbulent flow in straight pipes and channels.

2.7 Special Topics in Heat Transfer

The important topic of free convection heat transfer is first considered. The Grashof and Rayleigh numbers are introduced and their physical interpretation is discussed. The prior treatment of hydrodynamic stability theory facilitates discussing the concept of a critical Rayleigh number required for the inception of free convection. Important correlations for the Nusselt number for free convection heat transfer are summarized. The students are referred to [4], which shows how scaling analysis can be used in lieu of the Pi Theorem to arrive at the dimensionless groups for describing heat transfer as well as other transport problems.

This course ends by introducing the students to heat transfer with phase change. We begin by discussing the benefits of latent relative to sensible heat effects for transferring heat. Filmwise and dropwise condensation are discussed and appropriate correlations for the Nusselt number are summarized. We then consider boiling by discussing what happens when the temperature of a heater placed in a fluid is progressively increased. The progressive change from pure conduction, free convection, nucleate boiling, and finally to film boiling and possibly burnout is discussed. Again, appropriate correlations for the Nusselt number in each of these regions are summarized.

3 Assessment of Achieving Learning Goals

Teaching a pediment graduate course on fluid dynamics and heat transfer in the 30 lectures available during one quarter is challenging. There is time to give only midterm and final examinations. Additional assessment comes through the weekly graded homework assignments. The open-ended nature of these homework problems has been alluded to in this article. Graduate students are also required to write a paper in which they apply the principles of this course to an original problem that is usually drawn either from their research or a recent journal article. Assessment of longer-term retention is provided by our oral Ph.D. qualifying examination that focuses on the pediment graduate courses. In this examination the students must apply the principles of this course to modeling open-ended problems. A question-and-answer exchange with the faculty examiners provides a very good indication of longer-term mastery of the skills that should have been learned in this course.

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4 Discussion and Recommendations

Most universities operate on a semester rather than a quarter system, in which case there are additional topics that might be included in this course. One topic to include might be a graduate-level treatment of the macroscopic balance equations wherein they are derived by integrating the differential forms of the equations-of-motion and energy-conservation equations. It is also desirable to expose students to generalized curvilinear coordinates and include examples that require constructing a coordinate system appropriate to the geometry of the problem.

Teaching this course in the quarter system requires that it be well-organized and reasonably fast-paced. For these reasons scaling analysis is particularly helpful. It provides an excellent pedagogical tool for introducing subtle concepts such as creeping flow, boundary-layer theory, film theory, *etc*., and also makes it easy to draw upon analogies between fluid dynamics and heat transfer. Moreover, scaling analysis provides the ideal tool for structuring a course in transport phenomena that focuses on model development such as described in this paper. Scaling analysis gives students a systematic method for assessing what assumptions they might invoke to develop a tractable model for a transport process. Moreover, as described in [5], scaling analysis can be used to estimate the optimal design parameters for a process. As such, it can be used to help students design experiments for their thesis research or future job assignments.

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