AC 2010-1914: POSTER: WINDMILLS IN TRIGONOMETRY CLASS

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Abstract

A set of windmill teaching aids has been developed for use by high school trigonometry teachers with the goal of bridging the gap between trigonometry and real world systems. The University of Saint Thomas created 16 mechanisms, and a teacher guides, that make up the Trigonometric Windmill teaching aids in response to a need expressed by the math curriculum coordinator in the Edina public school district. Working closely with the mathematics coordinator, an engineering student at the University of St. Thomas developed a variable set of systems that could assist students in understanding the equation $\theta(t) = A \sin(\omega t + \phi)$, with a specific focus on the terms $A$ (amplitude) and $\omega$ (angular velocity). There were four primary goals for this project: (1) to open lines of communication between the University Saint Thomas and local PK-12 teachers, (2) to aid in the teaching of trigonometry lessons through the use of hands-on activities created to satisfy MN Academic Standard #9.2.1., (3) to incorporate engineering content into math lessons, and (4) to hone students’ visual-to-mathematic conversion skills, which has been recognized as an increasingly important skill for students to possess. This paper discusses the collaboration process and presents a lesson plan that can be replicated by other schools.

Introduction

The engineering department at the University of Saint Thomas was approached by the mathematics curriculum coordinator for the Edina school district, with an idea for a collaborative project. The mathematics curriculum coordinator described the need for math students to have a hands-on learning device that could demonstrate both periodic motion and the relationships between the motion of the device and the mathematical equation which describes and predicts the motion. The challenge expressed here is an echo of a common concern among many mathematics and science teachers as well as the National Science Board. The National Council of Teachers of Mathematics claims that “[few curriculum materials] introduce real-world interdisciplinary problems and serve as advanced placement courses, school-to-work transition courses, or the challenges of a liberal arts college education.” This assertion, and others like it, strengthen the claim that math and science are related subjects which should be taught concurrently through hands-on experiences. We considered this pedagogical preference when we met to establish the basic requirements of teaching aids that could be used to satisfy MN Academic Standard #9.2.1.9.

MN Academic Standard #9.2.1.9 Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations. For example $(x) = |x - h| + k$.

To meet this standard, students should be able to write, graph and interpret relationships between an object with periodic motion and its corresponding trigonometric equation. Through discussions with educators, the following secondary criteria were established.
- The teaching aids need to be robust, as classroom teaching aids are often subjected to extreme wear and tear.
- The teaching aids should not employ batteries or electronics, as this would make them troublesome to maintain and batteries have strict disposal requirements. Instead, we gave preference to a mechanical power source.
- The teaching aids should be designed to be operated by students in small groups of two to three students, so as to allow students to work together while also being able to fully participate in the hands-on aspect of the lesson.

The developers recognized that in using examples like windmills, students are given the opportunity to make connections between trigonometric concepts and real world systems. It was hoped that the students would be able to draw parallels between the wind turbines used in the community and the concepts that they were learning in trigonometry class. According to Bransford, Brown, and Cocking’s How People Learn, these connections allow “students to see a need or the reason for their use in class—thus allowing for students to see the relevant uses of knowledge to make sense of what they are learning.”

**Collaborative Design Process**

This project began at a breakfast meeting where administrators and faculty from the University of St. Thomas met with teachers and administrators from local schools to discuss collaboration ideas. The Edina Public Schools (EPS) superintendent extended an invitation to the Edina Public School mathematics coordinator and two high school science teachers to attend this breakfast. The engineering faculty present offered to design and build materials or design projects that would integrate engineering concepts with Minnesota mathematics and science standards. The mathematics coordinator from EPS contacted the UST engineering department chair with an idea about using scaled down representations of windmills to aid in teaching trigonometry. This idea was forwarded to one of the UST engineering professors. The UST engineering professor then invited an undergraduate student to work on the project. The professor, student, and mathematics coordinator exchanged design ideas via email and then met at EPS to finalize details. Their first meeting included sketches and ideas for construction materials. The professor facilitated the meeting by asking clarifying questions and by providing parameters for the scope of the project. The professor had collected information from UST education professors about the nature of Minnesota standards, testing, and classroom practices about standards.

The strength of the UST/EPS collaboration model was the ongoing communication among the four-person team, consisting of the engineering professor, the engineering student, the curriculum coordinator, and the teacher. The professor facilitated the design process between the curriculum coordinator, and the engineering student. Part of the professor’s role was to let the curriculum coordinator know what design and engineering techniques were possible, and to let the engineering student know how to communicate effectively with the curriculum coordinator. The role of the engineering student was to listen to what parameters the curriculum coordinator needed, to provide ideas about how to design the windmills, and to build them. The role of the curriculum coordinator was to consult with the mathematics teacher, to provide information about what was needed to better implement good standards-based instruction, and to provide design specifications that were appropriate for classroom use. The role of the teacher was to
create a lesson plan to use the new product to improve instruction in accordance with the curriculum.

The process worked well in this case because the four person team was able to communicate without any notable problems. In future projects, the curriculum coordinator position could be eliminated, but only if the teacher has the time to invest in meeting with the student and professor to relay the requirements of the curriculum.

**Final Design**

The representatives from the University of Saint Thomas (UST), kept in frequent contact with the representatives from the Edina school district when determining key design factors of the windmills. The Edina representatives wanted to give their students a variety of options for the period and amplitude of the windmills. To allow for this variety it was decided that four sets of interchangeable base units and blades would be created. Each set is designed so that the output shafts rotate at different periods of revolution. In order allow for changes in amplitude, four different-sized sets of detachable windmill blades were designed. These blades needed to be able to withstand the many iterations of attachment to, and removal from, the base. The design also needed to allow students to easily switch out the blades on each base unit, without requiring assistance from their teachers. These interchangeable blades allow students to see how a graph of $f(x) = |x - h| + k$ reacts to changes in amplitude when the angular velocity is held constant, and conversely, how the graph changes when the angular velocity is made the variable.

Since the project is intended to be replicated by other schools which may have an interest in trigonometric teaching aids, the project budget was limited to approximately $150 in material expenses for the creation of 16 units. The limited expense and the desire to employ an eco-friendly philosophy in the design led the designers to gather a majority of the components from dollar stores, thrift shops and surplus stores. From these places we procured gear sets from inertia toys and old printers, and spring mechanisms were extracted from windup toys. Plywood, used for the housing, was purchased from a local home improvement center, and the gear shafts were ordered from an online parts supplier. Each of the 16 units required an average of 1.5-2 hours to build. Approximately 20% of the time was spent searching for suitable parts and extracting components, 50% of the time was spent cutting and shaping plywood plates, 25% was spent placing gears and trouble shooting, and the remaining 5% was spent cleaning and finishing the base units.

It took approximately two prototypes to determine how the final design should look, feel and operate. To establish the final prototype, the design team from UST and the Edina mathematics curriculum coordinator met to discuss the initial prototypes and to determine what needs were not being met and how to satisfy those design needs. It was during this meeting that the need to make the four distinct base sets and blade sets, as well as the need to attach the units magnetically to a white board surface were determined. The final solution is shown in Figure 1.
The team strove to build these tools in such a way that other educational institutions could easily reproduce a set of the Trigonometric Windmills. To expedite the reproduction process, we have created a set of detailed instructions for manufacturing and assembling the four sets of base units and blade sets. Interested individuals are encouraged to contact the authors to receive these instructions. Below we present the lesson plan that is currently used with the windmills.

**Lesson Plan**

This project is a way to introduce concepts from engineering and sustainability into a math classroom setting. We have included a short summary on wind power which features a brief history on the evolution of wind power harnessing devices, an overview of energy concepts, an explanation of the way energy is harnessed through wind turbines and the environmental impacts of wind energy harvesting. By including this literature with the Trigonometric Windmills a tertiary goal of increasing energy literacy is addressed. The concept of energy literacy includes a broad set of citizenship knowledge as well as attitudinal and behavioral aspects, which can enable people to embrace appropriate decisions and behaviors with respect to energy consumption in everyday life.\(^6\) Studies show that the general public possesses very low levels of knowledge and awareness with regards to energy literacy.\(^6\)\(^{13}\) The summary on wind power was provided with the Trigonometric Windmill teaching aids so that teachers could have basic knowledge of wind power and how it relates with history, math, science and environmental science. Below is the lesson plan which was used in the Edina high school trigonometry class. The student worksheets are presented in Figures 2 and 3.
1. Mark one blade with a piece of tape to serve as a reference point.
2. Measure the length of a blade in centimeters. This is the amplitude (b).
3. Sketch the rotor blades in the above circle graph on the left. Label the axes so the radius of the blades is accurately represented.
4. Wind up the motor and determine the period in seconds.
5. Label the x-axis (time) and the y-axis (position) on the graph above.
6. Determine the value of the period multiplier (a) using; \( \text{Period} = 2 \pi a \)
7. Use the equation of rotation for modeling the blade position as a function of time.

\[
y = b \sin \left( \frac{t}{a} \right)
\]

8. Sketch two periods of rotation on the above graph.
9. Use equation 1 to calculate the height of the blades tip at \( t = 8 \) seconds.
10. At what time(s) during the first two cycles is the tip of the blade at a height of 3 cm?
11. Use the square on p.2 to repeat steps 1-10 for an additional 2 rotor and motor combinations.

**Figure 2:** Page one of the student worksheet used with the windmills.
Wind Energy Supplemental Information

Windmills have been used by societies since 1000 B.C. The earliest windmills were created by the Persians to facilitate the irrigation of their fields. Windmill technology remained a tool of the mid-east until the middle dark ages when around 800 A.D. it began being utilized in Europe as well. Traditionally windmills have been used for irrigation, grinding grains and sawmills. Wind energy has also been used for centuries by mariners and more recently through kite-sports. The modern form of a windmill used for electrical power generation is commonly known as a wind turbine. The wind turbine is used as a means to harness the wind's energy for production of electricity. Harnessing of wind power on a utility scale (such as commercial wind farms) has only been done for approximately the last sixty years, and did not gain momentum until the energy scare of the early eighties. Figure 4 is a chart depicting the general progression of wind turbine output capacities since 1981.

Energy:

Energy is quite useful; as it is the ability to do work. As the energy in a system increases, so does the amount of work that the system can perform, such as how many lights can be powered or how fast a car can go from 0-60mph. Energy is a quantity which can neither be created nor destroyed; and consequently it is considered to be a conserved property. Energy comes in many forms. In a wind turbine, the kinetic energy of the wind is transformed into electrical energy.

The wind, by virtue of the fact that air has mass, is a source of kinetic energy. Wind comes from the warming of one air mass relative to the other cooler air mass. As the warm air rises, the cooler air flows in beneath to fill the void left behind. This motion is commonly known as wind. The amount of kinetic energy (K) associated with any moving mass is proportional to one half
the mass \( (m) \) times the wind velocity \( (v) \) squared; i.e. \( K=0.5mv^2 \). The kinetic energy in the wind is converted into electric energy by means of a generator. The wind turbine spins the generator which produces electricity that can be transmitted through power lines.

**Figure 4:** Wind turbine output capacities since 1981

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**How windmills work:**

Modern day wind turbines are used to generate electricity by means of a mechanical conversion of kinetic energy (wind) to electrical energy (electricity). This is accomplished by harnessing power from the flowing wind mass which is running into the rotor blades of the turbine. Since a moving mass has kinetic energy it is able to transfer some of this energy into a rotation of the rotor blades, called rotational kinetic energy. This rotation turns the generator’s shaft, i.e. the rotor, within a stator. The stator is a cylinder comprised of copper coils surrounding the rotor. As the rotor rotates a set of magnets connected to it also spins within the stator. The rotation of these magnets creates an alternating current in the stator coils, due to electro-magnetic laws. The current produced in these coils can be transformed into electricity which is then sent to locations where it can be used, like houses and schools.

**Electricity:**

The common unit of electrical measurement is the kilowatt (kW), or 1000 watts. The watt (W) is commonly used to describe the power output or requirement of modern electronics or European engines. An average U.S. household will consume 10,774 kW in one year. The average-utility size turbine can produce 2.5 MW annually. Such a wind turbine can provide approximately enough electricity for 220 American homes. Current wind turbine technologies allows power generation in winds as low as 7-8 mph.\(^{14}\)

**Power:**

Power is a measure of the amount of energy used or produced over a certain period of time. Evaluating a wind turbines generation capability is dependent on the following equation:\(^{16}\)
Power(W) = 0.5 x ρ x A x Cp x V³ x Ng x Nb

ρ = Air density
A = Area swept by the blades = π(D/2)²
Cp = Performance Coefficient (limit 0.59)
V = Velocity
Ng = Generator efficiency < 80%
Nb = Gearbox and bearing efficiency < 95%

Current technologies do not allow for 100 percent of the wind’s kinetic energy to be extracted. Aside from current technological deficiencies it is suggested that not all energy should or even could be harnessed from the air. If all the kinetic energy was converted into electricity by a turbine it would require that all velocity from the wind is removed. As a result air behind the turbine, downstream, would not be moving, a condition known as the doldrums. Such a state does not allow fresh air to flow in and replace the stale and potentially stagnant air. Furthermore; wind energy cannot be fully converted into electricity because of mechanical and natural inefficiencies. For example, anytime that heat is given off or a noise is made it requires some form of energy dispersion to exist. Hence, no wind turbine can be 100 percent efficient.

Size and Capacity:

There are two common types of wind turbines that are available for use in today’s market, the horizontal axis turbine, see (Figure 5) and the vertical axis turbine (Figure 6). The dominant type used throughout the world is a horizontal axis turbine. Regardless of what type of turbine is being used they all share a common rating system based upon their power output capacities.

Small turbines produce less than 10kW and are generally designated for rural houses or farms. Medium turbines produce up to 50kW and can be used to power schools or small towns when they are assisted by another type of generator. While utility scale turbines put out an excess of 500kW, currently the largest turbine in use is 7MW. The power generated is typically in a form which can be fed directly into the electrical grid for consumers to use.

In order to produce greater amounts of electricity, the rotor and stator must be made larger. In turn, this requires even larger blades to turn these new rotors. At present, typical rotor diameters range from 8 to 110 meters and can produce between 25kW to 5MW. As seen in figures 1 and 2, the turbine typically sits atop a tower. By using towers to elevate the turbine the blades are able to rotate unobstructed and be positioned within optimal wind zones. As a general rule of thumb the tower upon which a horizontal turbine is perched is typically three times taller than the rotor’s diameter.

The largest turbines in the world are generally built in the ocean because of favorable, consistent wind conditions and lack of obstruction by surrounding buildings. At present the largest turbine is in Germany. This turbine is capable of producing 7 MW by using a 207ft long blade (approx. 414 ft rotor diameter). The turbine weighs in at over 160 tons and it is capable of providing power for 5000 homes in Germany or a mere 2000 homes in the United States.
Site Selection:

“Most modern wind turbines can produce electricity 75-85% of the time or else whenever a minimum wind speed of 7 mph is reached.”\(^\text{14}\) For this reason the site’s topography is critical in determining whether or not a turbine can produce electricity efficiently. The most basic requirement for a site is a continuous undisturbed flow of wind. This typically requires the site to be flat land without many buildings or trees within close proximity. One type of exception to this rule is a location which acts as a wind tunnel, i.e. mountain passes, where in wind flow is generally stable and acute.

Once a location is deemed eligible it must be audited for at least one year prior to commencement of construction process. During this audit, a survey team collects data on wind speeds, number of days in a year that the wind blows, the wind’s direction and more. They use this data to construct gradient maps for each location and average wind maps for the year.\(^\text{17}\) This information helps the survey team determine whether or not the site is suitable for a turbine.

Environmental Preferences:

As you may have noticed, the wind does not blow continuously. Consequently, power derived from wind cannot be a 100 percent reliable source for electricity. Because of their dependency on steady wind, wind turbines are usually located in large open areas where wind patterns are consistent. The largest domestic wind warm in the US is in Texas. It consists of 421 turbines that provide power for approximately 220,000 homes.

Wind farms should be located in areas that are not exposed to severe climate swings, since mechanical objects tend to fail more frequently when exposed to these types of changes. The ideal locations in the US are the flats of the Dakotas and around the Texas prairies.\(^\text{14}\)

There are strict environmental requirements for the placement of wind turbines. Ideal sites tend to be a long way from cities, where the energy is required most. Therefore, in order to transmit
power, often new transmission lines must be run from the wind farms to the metropolises of the world.

**Some Advantages of Wind Power:**

Wind power is a clean and renewable energy option. There are no active carbon emissions released by a turbine, itself, through its operational life. The major pollutants associated with wind power are from the manufacturing of the wind turbine and the maintenance of the turbines. Since wind will be around for as long as the sun burns, the opportunity to extract energy from the wind will be available for a long time. Wind energy is one of the least expensive forms of electricity and generally costs less than 10 cents per KWh.\(^\text{14}\)

Turbines can be an additional source of income to ranchers and farmers due to a turbine’s requirement for open space and consistent wind patterns. To take advantage of this farmers and ranchers typically rent land to the turbine owners in order to supplement their yearly earnings.

**Some Disadvantages of Wind Power:**

Turbines are a noticeable mark on the land. If they are not laid out in a visually pleasing array they may be a nuisance to local populations. To help disguise wind turbines a light gray paint is used, which helps turbines blend into the background. The rotation of the blades and the operation of the generator produce a low humming noise, about 35-45dB at 300m away.\(^\text{19}\) Occasionally this amount of noise that can become distracting to people living near the windmills. On smaller turbines, the blades’ noise can be reduced by supporting the blade tips. Using these supports reduces blade vibration, and consequently decreases the strength of the low hum.

Since wind is not continuously blowing, there are times during which the turbine is sitting idle and not contributing to the energy grid. To reduced this inefficiency the turbines are designed and calibrated to their sites wind patterns, in doing so a turbine can run up to 80% of its life.\(^\text{14}\)

**Learner outcomes**

Before working with windmills, the students studied trigonometric functions. They learned how to write equations for sine and cosine graphs and their transformations including changes in period, amplitude, vertical shift and phase shift. The students were successful with the equation writing, but with the addition of the windmill activity, two additional learner outcomes were observed.

First, with the windmill attached to the whiteboard, the students initially struggled with how to represent the height of the windmill blade when it dipped below the center point of the blades. They solved the problem two different ways. One way was to orient the center of the blades at the origin of the graph and to use negative numbers to represent the height, and the other was to use a vertical shift to orient the center of the blades above the x axis far enough so negative height values were not needed. This basic problem-solving was not observed before the students were engaged with the windmills.
Second, with the initial graphing problems, the domain of the trigonometric functions is usually angle measure. When the students write equations for the motion of the windmill blades, they need to switch to a time domain. The change from an angle domain to a time domain is a problem-solving opportunity that leads students to realign their approach with the standard being studied. This discussion was not observed before the windmills were implemented.

Overall, through the discussions generated by the windmill activity, the instructor observed the students having a greater ability to apply the thinking behind transformations to a concrete problem. The increase was particularly evident regarding the period of a function. The windmills functioned as planned, and the learner outcomes were consistent with well-designed hands-on activities that engage students in ways which help them create more connections about the mathematics they are studying.

**Conclusion**

The staff at Edina had a clear idea of the problem they wanted to solve, the criteria this project needed to meet, and some basic forms they wanted to use in order to remedy this problem. By holding regular meetings, the design team was able to discuss and establish further criteria for the windmill systems and the advantages and disadvantages inherent in different materials options as the UST staff tried to make the systems affordable. It proved quite beneficial to keep the lines of communication open between all of the individuals involved. Frequent meetings helped the team to improve upon the initial prototypes and to discuss the issues which were encountered by UST’s designers.

According to their teacher, the trigonometry students were highly engaged during the windmill lesson. The hands-on nature of the experience provided a valuable learning opportunity for the students to learn about the relationships between trigonometric equations and the motion of the windmill teaching aids. Having several different combinations of motors and blades made it easy for the students to experiment with various amplitude and frequency combinations. This direct application of the trigonometry had been missing in previous lessons, and including it helped the students with a typically difficult unit of study. The windmill lesson will continue to be used in mathematics courses in the Edina school district. The bodies and blades of the devices were durable and easy to use, and the students could set them on their desks top or attach them magnetically to the whiteboards.

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Bibliography