AC 2011-1123: PREPARING ENGINEERING STUDENTS TO TAKE A CALCULUS COURSE: AN ENGINEERING-ORIENTED APPROACH

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Preparing Engineering Students to Take a Calculus Course: An Engineering-Oriented Approach

Several national reports have discussed the importance of increasing the quality and number of engineering graduates in the United States. One segment of the population in which substantial improvements are possible is students who enter college with a declared major or interest in engineering but who need to improve their mathematical preparation before it is likely they will succeed in the first calculus course. This segment includes a disproportionately high number of Blacks, Hispanics, and Native Americans, populations that are the focus of many efforts to diversify undergraduate engineering enrollment and the engineering workforce.

Results from other initiatives suggest that an engineering emphasis in mathematical preparation can improve performance and retention of engineering students. Most engineering curriculums list Engineering Calculus I as the first mathematics course. Students not prepared academically to enter this course can often feel neglected because they may not learn any engineering concepts for another semester or two. Also, they are out of step with many of their peers, possibly including friends with whom they came to college. Their course load their first semester includes courses, such as English and Chemistry, in addition to their pre-calculus course, and the word 'engineering' is non-existent.

Faculty members at Texas A&M University (TAMU) and Blinn College worked together to create an engineering approach to preparing students to take a calculus course. One of the instructors is a faculty member from the university who also teaches mathematics courses at the community college. Multiple sections of the course were taught on the TAMU campus, so the course was better synchronized with the students' schedules.

Their intent is to better prepare engineering students for their first engineering calculus and increase the percentage of students being retained in engineering.
This paper will describe:

- The approach taken by the instructors to design the course,
- The skills and topics determined to be needed for engineering calculus instead of the course simply providing a complete review of trigonometry and algebra,
- The instrument used to assess readiness of students for Calculus I,
- The use of engineering based activities to motivate the students.
- Results for students who have completed the course.

Introduction

Several recent national reports[1,2,3] have discussed the importance of increasing the quality and number of engineering graduates in the United States. One segment of the population in which substantial improvements are possible are students who enter college with a declared major or interest in engineering, but who need to improve their mathematical preparation before it is likely that they will succeed in the first calculus course. In 2002, 14% of first-year engineering students reported a need for remediation in mathematics[4]. The percentage of engineering students varies widely depending on the mission of the particular institution[5].

However, enrollment in a pre-calculus mathematics class is not working as well as anticipated. Students nationwide who start in pre-calculus persist in engineering at lower rates than students who start in calculus[6]. Statistics collected by personnel in the mathematics department at TAMU as well as in the engineering department state that 76% of engineering students who start in Engineering Mathematics I at TAMU are still in engineering one year later, as compared with 60% of students who start in Pre-calculus. In addition, the statistics gathered show that only 46% of all new under-prepared students who took developmental courses gained college-readiness (readiness to take the first required course in college) in their first year at TAMU.

Informal interviews with students suggested that part of the problem was that students were not taking the pre-calculus math course seriously—they felt that it was a form of "punishment" since they were enrolled in engineering but were not allowed to take any engineering courses. Furthermore, students felt that the pre-calculus class offered did not meet their needs in preparing students for Calculus I. In other words, rather than treating it as a preparatory course for Calculus I, (i.e, forward looking), the course, as currently structured, had a feeling of remedial math, (backward looking). In addition, informal interviews with engineering faculty highlighted the importance of pre-calculus mathematics in most engineering courses.

This led to the central premise of this work: Is it possible to develop an "engineering pre-calculus course" that is specifically tuned to the needs of engineering students that simultaneously (i) helps in the preparation for calculus in a "forward" looking manner and (ii) highlights vital roles played by pre-calculus mathematics in real-world engineering tasks?

The new engineering pre-calculus course developed was thus founded on three premises: (i) most problems asked in calculus are actually algebra problems; (ii) most calculus problems can be reformulated as algebra problems; and (iii) apart from their utility in calculus, problems in algebra have tremendous impact in engineering. This paper looks at an alpha version of a course to prepare engineering students to be successful in calculus through the use of focused topics, practice with feedback, and activities to promote engineering applications.

Background

Results from other initiatives suggest that an engineering emphasis in mathematical preparation can improve performance and retention of engineering students. At Wright State University, engineering faculty members have developed an engineering course that provides the required elements of mathematics for many core engineering courses[7]. In the Wright State Model, engineering students take this new engineering course, which is intended for calculus-ready students, during their first semester. Then, they can take several engineering courses while they concurrently complete a traditional four-course mathematics sequence in calculus and differential equations. In its first iteration, over 80% of the students successfully completed the new engineering course (earning a grade of 'A', 'B', or 'C'), compared with around 42% of the students who, based on performance in prior years, successfully completed the first-year calculus sequence at Wright State[7]. At Boise State University, engineering faculty members created a preparatory engineering course that students can take concurrently with their pre-calculus course. Their preliminary results indicate that students who take the engineering course concurrently with the pre-calculus course achieve higher success rates in pre-calculus than those who do not[8].

At Wayne State University, faculty members included a course on introduction to the engineering profession, together with courses in pre-calculus, chemistry, physics, and English, in a one-year bridge program[9]. These examples demonstrate that engineering students' success can be enhanced by helping them to build stronger connections between engineering and the study of mathematics, including pre-calculus.

In all these cases, the engineering content was either developed for calculus ready students or was a separate course that is taken concurrently with the pre-calculus course. On the other hand, given the credit restrictions at TAMU, the model proposed was a new "pre-calculus engineering class" that combines both pre-calculus math with engineering content. A question naturally arises as to how to integrate engineering content into pre-calculus classes? At the institution, Model eliciting activities (MEAs) were chosen as the main vehicle for such an integration.

Model-eliciting activities[10] represent an area of research that shows promise in improving preparation of students for their first calculus courses. In a model-eliciting activity (MEA), students are offered a description of a phenomenon and asked to propose a mathematical model to capture some aspect of the phenomenon. For example, students could be shown a shotgun-like pattern of points in a rectangular area and asked to construct a function that estimates the degree of scattering (or oppositely, cohesion) in the pattern. Students then apply their proposed function to several different patterns, show the value for scattering that their function gives, and evaluate the degree to which their proposed function measures scatter in the pattern. MEAs have been developed and used with first-year engineering students at Purdue University with good results[11,12]. MEAs offer students opportunities to construct mathematical entities that address a particular situation and for which there is no one correct answer. Students are encouraged to engage evaluative and synthetic reasoning skills in addition to the more traditional analytical skills that are developed in a mathematics course for engineering students. Thus MEAs provide a natural means for incorporating engineering activities. MEAs can also provide a way to organize the assessments, learning activities, and topics in an engineering approach to calculus preparation.

As shown in Table 1, Moore and Diefes-Dux[11] have proposed a systematic framework for the development of MEAs:

Table 1. Six Principles for Developing Effective Model-Eliciting Activities[11]

Principle	Description
Model-Construction	Ensures the activity requires the construction of an explicit description, explanation, or procedure for a mathematically significant situation
Reality	Requires the activity to be posed in a realistic engineering context and to be designed so that the students can interpret the activity meaningfully from their different levels of mathematical ability and general knowledge
Self-Assessment	Ensures that the activity contains criteria the students can identify and use to test and revise their current ways of thinking
Model-Documentation	Ensures that the students are required to create some form of documentation that will explicitly reveal how they are thinking about the problem situation
Construct Share-Ability and Re-Usability	Requires students to produce solutions that are shareable with others and modifiable for other engineering situations
Effective Prototype	Ensures that the model produced will be as simple as possible, yet still mathematically significant for engineering purposes

At first glance, the MEA seems identical to the commonly used "engineering project" where a student team is asked to develop a physical object to carry out a particular function. The core element of an MEA is that students are ***asked to develop a process*** that a client could apply to individual problems, instead of solving a specific single problem. In addition, students are asked to validate their process by trying it on individual problems and reporting the results.

Method / Implementation

The new engineering pre-calculus course developed was founded on three premises: (1) most problems asked in calculus are actually algebra problems; (2) most calculus problems can be reformulated as algebra problems; and (3) apart from their utility in calculus, problems in algebra have tremendous impact in engineering. One reason for the premises is due the actual content students need preparation for in follow-on courses. The mathematics department at TAMU has posted all the calculus exams for the past 10 years. These exams were carefully reviewed to identify the pre-calculus skills necessary to solve the problems. It became immediately apparent that the most difficult step in solving the calculus problem was related to some algebraic simplification rather than any specific calculus skill. If the classical limit problem is considered, one of the trickiest steps for students can be the factorization of the polynomial. Students must be able to find all common factors before they utilize skills learned in the calculus course. If problems related to maximizing or minimizing a function are considered, before students can perform this operation they must be able to adequately frame the function in the problem statement. Also, engineering-focused MEAs help address the notion that pre-calculus has an impact on engineering.

The course has been developed to improve the preparedness of engineering students who are not calculus ready for their first-year engineering studies (including calculus, calculus-based physics, and engineering). Retention in engineering has been shown to be increased if the student was matched appropriately to first year coursework levels. Lee, Harrison, Pell, and Robinson[13] looked at students' ability in mathematics to predict their performance in engineering. Evaluating mathematics ability allowed students to begin in the appropriate level course, which increased their success in engineering.

Engineering students at the Institution are required to take a 33 question Mathematics Placement Exam (MPE) before they register for their first semester of classes. Depending on the score they receive on the exam determines whether the student enrolls in pre-calculus or calculus their first semester. Six topics comprise the 33 question exam, including: exponential and logarithms, polynomials, graphing, functions, trigonometry, powers, and problem solving. While the SAT math score of the student is also taken into account, most students scoring only 24 questions correctly or below are advised to take pre-calculus. When a student enrolls in pre-calculus, this eliminates the option of taking engineering or physics courses as calculus is the prerequisite for these courses.

The new engineering course has been organized around a sequence of carefully selected and constructed tasks intended to improve foundational algebraic, geometric, and trigonometric knowledge and skills as well as higher-level mathematical reasoning skills that are essential to success in calculus and beyond. Only a maximum of 30 students are allowed to take each section of the engineering pre-calculus class. This is much smaller than the typical pre-calculus course taught at the Institution, which enrolls approximately 125 students into each lecture course. The smaller number of students allowed much more faculty interaction with the students. The course was taught two days a week and practice with feedback with emphasized. Students were grouped in teams of four students, depending on their choice of major and section of chemistry class. Many of the students complete their chemistry requirement while in pre-calculus because it does not have a perquisite of calculus like the engineering and physics courses. Therefore, clustering students in their pre-calculus teams and chemistry section allows interactions outside of the mathematics material and instant study teams.

MEAs have been developed and found to be useful to introduce engineering applications. Topics covered by the MEAs include polynomials, inequalities, logarithms, and vectors. The MEAs allow students to gain further insight into the material and see the connection to engineering. They help students use the engineering design process (problem identification, breaking down a problem into a sequence of tasks, proposal of alternative solutions, the iterative process of proceeding along a chosen alternative, evaluating progress, and making decisions as to proceed further or choose a different alternative) as a paradigm for problem solving. For example, the instructor presents a sequence of mathematical "word problems" and systematically works with the students through the four steps:

1. Identify the problem: What is the need? What are the inputs?
2. Develop three alternative ideas in order to solve the problem.
3. Pick one and proceed: after every step, evaluate progress.
4. Make a decision: Do we continue? Do we go back and try a different alternative?

This is a vital and challenging part of the class. Engineering students for the most part have been successful students in high school where they often became proficient at looking for a one-step solution to the problem (i.e., formula searching). Since people do not easily change behavioral patterns that have been successful for them, faculty repeatedly focus on helping students develop processes for approaching new problems.

One area that causes frustration for students is moving easily between verbal, symbolic, and graphical representations. This represents a major challenge to students, especially in their physics classes. A quick look at a typical physics books used at TAMU[14] reveals that almost all the problems are presented verbally, although pictures are used on occasion. This is also the case with optimization problems in calculus, which are often used as applications of the derivative. Often, students do not have any major difficulty in carrying out the mathematical manipulations. Rather, they struggle to convert the verbal description into a mathematical description. Students are unable to answer questions such as:

1. What is(are) the variable(s) of interest?
2. What is(are) the independent variable(s)?
3. How are they related?

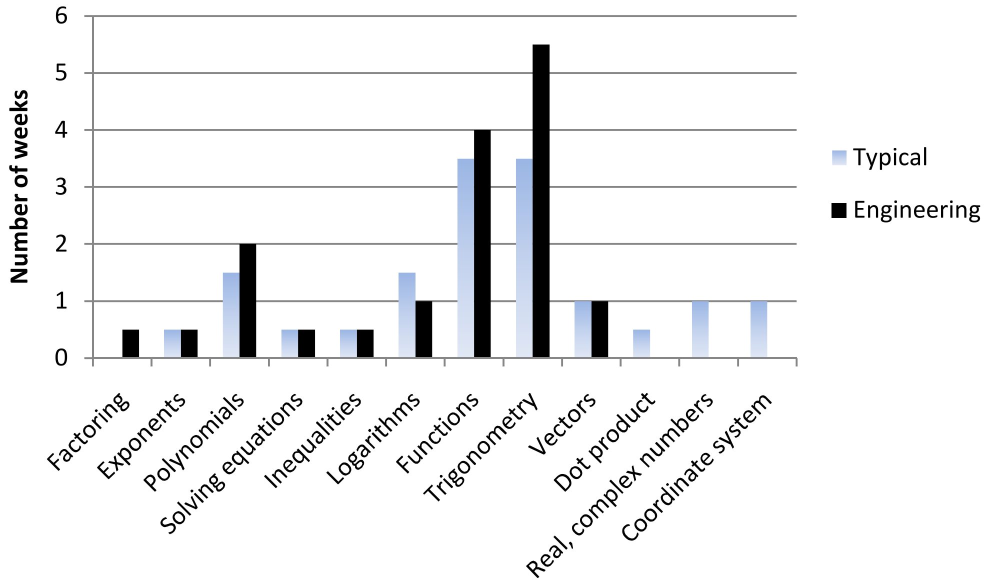
One of the MEAs developed assists students with moving more fluidly between representations. For example:

> You work as a production manager for a firm and must decide on the mix of products to produce for the coming week. Product A requires three minutes per unit for molding, two minutes per unit for painting, and one minute for packing. Product B requires two minutes per unit for molding, four minutes for painting, and three minutes per unit for packing. There will be 600 minutes available for molding, 600 minutes for painting, and 420 minutes for packing. Both products have profits of $1.50 per unit. Your supervisor suggests that you need to produce 50 Units of A and 30 Units of B. Is it possible, and if so, what is your profit?

This is a good example of systematically converting the problem from a verbal to a mathematical and then a graphical situation, and subsequently using the graph to answer a variety of questions.

While MEAs definitely help reinforce the engineering application of the skills in the course, there some differences in the skills taught between a typical pre-calculus course and the engineering pre-calculus course. Figure 1 compares the topics covered in a typical pre-calculus course at the Institution with those taught in the engineering pre-calculus course. As shown in the figure, several topics are not covered in the engineering pre-calculus course, such as real numbers, complex numbers, rectangular coordinate systems, etc. Evaluating the scores of most students entering the engineering pre-calculus course, students typically had low scores on the MPE, but no student typically received a score of zero. Therefore, many students lack the knowledge of higher level skills and practice of these skills, and they are just not used to applying them. These topics were intentionally left out of the curriculum of the engineering pre-calculus course to avoid the misconception of the course being classified as a remedial class.

Figure 1. Comparison of Topics Taught in Typical Pre-calculus Course versus New Engineering Pre-calculus Course



Results and Discussion

Overall, the central question to be addressed is to what extent participation in the new engineering pre-calculus course as preparation for the first-year engineering curriculum aided the performance and retention of students when compared to the performance and retention of other students who took a regular pre-calculus course in prior years. This new course has been offered each fall and spring semester since fall of 2008 with approximately 110 TAMU engineering students completing the course through fall of 2010. Overall, the students have shown a dramatic improvement in their initial mathematics placement exam when taken again at the end of the semester with scores rising from an average of 13% at the beginning of the course to 87% at the completion of the course. Results from 52 students completing the course in fall of 2010 have not been compiled yet. However, 50 students were followed through two years of pre-calculus and into the calculus sequence, and their performance was compared with that of a regular pre-calculus course taught at TAMU, for which 10 year's worth of data on 2,705 students was available. The results are quite remarkable. The percentage of students who took this course and then continued on to pass calculus jumped from 47% for regular pre-calculus students to 61% for engineering pre-calculus students. The grade distribution of those who passed the engineering pre-calculus was remarkably consistent with the usual pre-calculus courses except for those who got a grade of B. There was, among those who took the engineering pre-calculus course and earned a grade of C, a larger percentage (33%) who received a grade of B in the subsequent math class as compared to the regular pre-calculus classes (17%). This is a very encouraging sign since this is an indicator that the pre-calculus as taught by this new method might be helping students who earned a grade of C in the engineering pre-calculus course by possibly motivating them and enabling them to do better in calculus. Studies at TAMU for the last 10 years show that getting a grade of B or higher in the first calculus course is vital to subsequent performance in mathematics courses. Findings show that an approach based on a positive looking engineering pre-calculus course tuned to prepare students for calculus is making an impact. By properly aligning the curriculum, in addition to incorporating related activities, such as MEAs, significant gains in follow-on courses was achieved.

Targeted MEAs to help anchor key concepts in student's minds have been very successful. This is not an easy task to accomplish. One of the most successful MEAs was the one on signal processing and using characteristics of functions to differentiate between two signals, with application to voice recognition. This was particularly attractive to students and faculty alike and they were able to see the value of an MEA through this particular example.

Conclusions

Our findings have shown that an approach based on a positive looking engineering pre-calculus course tuned to get students ready for calculus is making an impact. The combination of incorporating engineering MEAs into the classroom curriculum, addressing topics necessary for engineering calculus is important, and having a smaller class size have shown to be effective. Many times, pre-calculus courses have a negative connotation by instructors stating they are trying to fix deficiencies in mathematics. Future work will include further investigating results from the pre- and post-MPE to see which specific skill categories show a more direct correlation to success in calculus. In addition, an active website is being developed to share objectives, approach, and outcomes for benefit to the engineering pre-calculus community.

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