PRINCIPLES OF 1st ORDER LINEAR SWITCHED DC CIRCUITS

by

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Abstract

In the study of transient signals, an important class of circuits is those containing one energy storing element, resistors in any combination, constant voltage or current sources and either a switch or a step signal source. This class is the simplest of the linear transient circuits and yet the principles may apply to higher order circuits. We will list the principles governing the behavior of such circuits and then apply the principles to describe and compute any of the response waveforms without solving a differential equation. It is assumed that the reader understands the principles of solving DC circuits and, in particular, the reader is capable of computing a Thevenin's equivalent resistance. Except for the waveform principle the other principles hold in circuits that contain more than one energy-storing element. We will also include meaningful definitions of common linear components.

Principles

of

1st ORDER SWITCHED LINEAR DC CIRCUITS

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A resistor is a two terminal device for which the voltage waveform is proportional to the current waveform.

$$v(t) = R\ i(t)$$

The constant of proportionality is the value of the resistance.

A capacitor is a two terminal device for which the voltage waveform is proportional to the integral of the current waveform.

$$v(t) = \frac{1}{C}\int i(t)dt$$

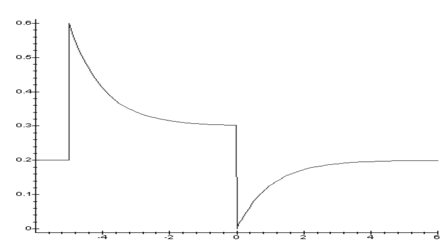
An inductor is a two terminal device for which the voltage waveform is proportional to the derivative of the current waveform.

$$v(t) = L\frac{di}{dt}$$

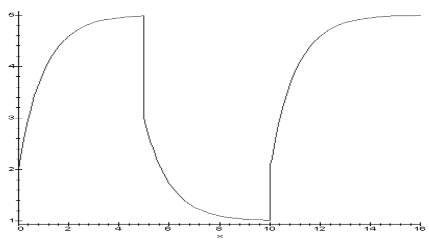
FIRST ORDER PRINCIPLE

- Δ In any first order linear circuit with DC sources, when you open or close a switch, any waveform in the circuit might have a jump and an ensuing exponential motion (rise or decay) to a settling value.

Jump Overshoots



Jump Undershoots



STEADY STATE PRINCIPLE

To compute the steady state values of any waveforms in these circuits, replace the capacitors with opens and replace the inductors with shorts. Then the steady state values can found by treating the resulting circuit as a D.C. problem.

TIME CONSTANT PRINCIPLES

- Δ In a capacitive circuit the time constant is obtained by the formula:

$$t = R_{th} C$$

- Δ In an inductive circuit the time constant is obtained by the formula:

$$t = L / R_{th}$$

MORE TIME CONSTANT PRINCIPLES

- Δ The Thevenin’s resistance, $R_{th,}$ is found by removing all sources and calculating the resistance that the energy-storing element sees.
- Δ The time constant, t , is a circuit property. The time constant is independent of the response location, that is, every waveform, which occurs following the opening or closing of a switch, in a 1st order linear DC circuit is an exponential with the same time constant, t .

JUMP PRINCIPLES

- Δ Capacitor voltage and inductor current cannot jump.
- Δ One can replace, at any instant, any capacitor with an equivalent DC voltage source or an inductor with an equivalent DC current source in order to compute any other instantaneous waveform value.

EXPONENTIAL WAVEFORM EQUATION

$$v(t) = v_f + (v_{in} - v_f) \exp(-t/\ t)$$

v_{in} = initial voltage

v_f = settling final voltage

v_{th} = threshold voltage

INVERSE EXPONENTIAL WAVE FORM EQUATION

(equation for time to cross a level)

$$t_D = t \ln\{(v_f - v_{in})/(v_f - v_{th})\}$$

v_{in} = initial voltage

v_f = settling final voltage

v_{th} = threshold voltage

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