AC 2008-2721: PROBLEM-SOLVING EXPERIENCE THROUGH LIGHT-DOSE COMPUTATIONAL MATHEMATICAL MODULES FOR ENGINEERING STUDENTS

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Problem-solving experience through light dose computational mathematical modules for undergraduate engineering students

Abstract

In this paper, the authors discuss the development of a framework for creating computational mathematics modules for engineering students. The purpose of the modules is to introduce mathematical concepts through modeling real-world applications and is intended to develop the students' ability to generalize a concept and to work with models of varying abstraction. The authors represent an interdisciplinary team contributing expertise from the fields of mathematics, computational science, and teacher education.

Introduction

Postmodern technology is characterized by great complexity and demands tremendous modeling and abstraction capabilities. For students to be successful in most engineering program, they should be able to apply the mathematics to model this complexity[1,2]. Problem-solving experiences have been advocated for decades in numerous textbooks, reference articles, and teaching modules[3,4,5,6,7,8,9,10,11]. The authors observed that many students enrolled in entry-level engineering and computer science courses tend to plug in data without considering the purpose of the analysis and demonstrate little ability to extend mathematical concepts beyond an algorithmic level. These observations motivated the authors to form an interdisciplinary team of university faculty to discuss the development of instructional mathematics and computer science modules that would enhance students' ability to apply complex mathematical reasoning when presented with novel real world problems. The authors met once a week for six months in roundtable discussions. In these discussions the authors identified the following challenges teachers face when teaching mathematics : 1) motivating students in the applications of the mathematical concepts that reflect realistic problems in their prospective engineering careers, 2) integrating these complicated applications into the tight schedule of engineering courses, 3) leveraging the overwhelming complexity so that the students are not intimidated, and 4) compensating for the lack of physical models required in most engineering applications The authors considered these challenges in the development of modules and agreed that a framework for module development was required. The authors began preliminary research to develop such a framework centered on the concept of abstraction.

The authors began by exploring resources developed by other colleagues, reviewing the existing digital library and organizations that provide free course materials relevant to engineering courses offered at the authors' home institutions. Researches also examined a past module developed by one of the authors that implemented light doses of mathematical modeling that accommodated the tight schedule of various mathematics courses taught by those authors. Based on polarized feedback from students who participated in these past modules, the authors began exploring ways to address the diverse learning needs of entry level computer science, and engineering students.

Examination of past modules

Past modules were designed to facilitate modeling behind differential equations and to enable students to overcome the complexity inherent in DEQ modeling,   One researcher had previously developed several modules, each of which targeted a variety of related problems with incremental complexity. Each module was made up of multiple models spanning from simple idealistic ones to complex realistic ones. For example, a module in spring-mass systems started with a linear spring model, transited to a nonlinear spring model, and finally ended with a coupled spring system model for the landing system of a spacecraft. A module about fluid mixing problems started with a one-tank model having equivalent entering and exiting rates to a multiple tank model with different entering and exiting rates. The strategy he used to teach these modules was to gradually increase the complexity of the models by asking "what if" and then relaxing some unrealistic assumptions. These modules clearly demonstrated to students that mathematics models are based on model assumptions and invariants derived from the domain knowledge of the modeler. Hence, idealistic assumptions result in a basic model. The students learned that when more and more realistic concerns are addressed, some idealistic assumptions must be removed, and then more and more complicated models have to be built. The students also realized that when the model changes, the differential equations must also change, resulting in a need to change the mathematical tools as well. Students saw that they could solve the linear spring-mass equation analytically, but had to use computers to approximate non-linear systems of equations for the landing system of a spacecraft. The incremental approach allowed the instructor to introduce software tools such as Maple, Stella, and Microsoft Excel as real world problem-solving tools. These tools not only helped students to get numerical solutions, but also provided intuitive simulations and visualizations to help students to understand the problems and solutions.

From observing the students' work and the feedback from course evaluations at the end of each semester it became apparent that there were two main groups of students, those who could handle the required level of abstraction and those who couldn't. The coauthor who taught these courses was concerned about the reasons behind the latter group's failure and subsequently tested his modules on a group of talented students in an honors course. The student work and survey results at the end of the honors course showed that 13 out of 14 students enjoyed the modules and 5 out of 6 objectives of the modules have been achieved. This result was significantly different from the bipolarized data obtained from the students in the previous semesters. These results indicated that the module approach could be effective. The challenge was to make the approach effective to groups of students with a wider range of capabilities. This provided the motivation for the development of the proposed framework.

Development of module framework

The authors considered a framework that would require modules arrangement according to levels of mastery. The different levels represent the different curriculum levels. The objectives for each level will be determined by the goals and objectives of the particular curriculum level. Each level will have a set of self-contained modules. Modules at a particular level will address one or more of these objectives. The levels themselves range from the introductory level for the freshmen engineering students in basic calculus to the expert level for students enrolled in advanced engineering mathematics courses. Each module will start with initial problems that introduce the concepts through basic ideal examples typically found in textbooks. Each subsequent module in that level will slowly relax unrealistic assumptions, thus increasing the number of related variables and ultimately resulting in a problem close to real world application. Thus, within a given level, module sets contain modules that vary in complexity and abstraction from simple and concrete to complex and highly abstract. The final module at the expert level will be comparable to a capstone course project requiring complex modeling for solving a real-world application.

One of the pedagogical requirements for module development is that the module be inquiry based and introduce problems, and sub problems, by posing questions. The module will then guide students gradually from concrete thinking with the use of visual aids and hands-on experiments to mathematical modeling and abstract thinking through its sequence of questions. Beginning modules at the lower level may be more concrete than beginning modules at a higher level that may start at a higher level of abstraction. Authors will contribute computational expertise to introduce applications through textual-graphic representations. The authors believe that the combination of mathematical rigor and visual intuitiveness will facilitate students' comprehension of complicated problems and retention of the underlying mathematical concepts.

Sample module lesson plan

Module 1: Given below is a teacher's lesson plan for a sample module at the algebra level following an inquiry based approach. This may be considered as an intermediate module at that level, that is, neither totally concrete nor highly abstract for that level.

Geosynchronous satellites

This module allows students in a beginning algebra class to apply their algebra and physics knowledge to solve a real-world application problem concerning orbiting satellites. The students get to make predictions, model the problem using an algebraic equation, solve the algebraic equation, and compare the results among themselves.

Pre-requisite Knowledge
To be able to effectively use the module students should be able to solve simple linear algebraic equations, and should be familiar with the concept of centripetal force and gravitational force. [Familiarity with the physics concepts is not absolutely necessary as they can be introduced gradually as part of the concrete cases.]

Objectives
By the end of the module the students should be able to

1. Identify the geosynchronous satellites from the NASA's satellite tracker website[12].
2. Understand that acceleration due to gravitational force is proportional to mass and inversely proportional to the square of the distance between the objects
3. Apply geometric properties and relationships to the solution of mathematical problems involving ratio, proportion and right triangle geometry

Introduction

1. Present J-Track – 3D from NASA's website that tracks satellites in real-time as shown in Fig. 1.1.

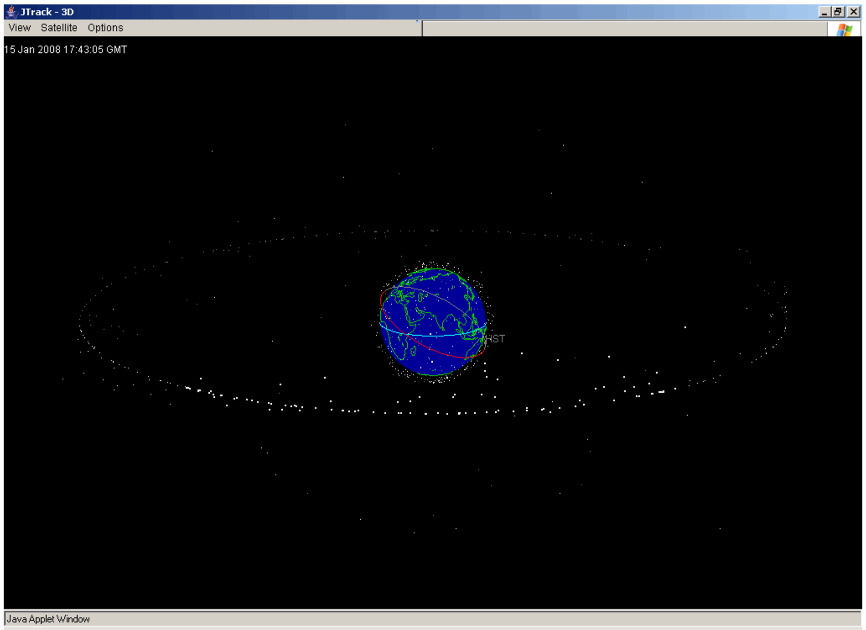


Fig. 1.1: Screen shot of NASA's J-Track 3D

2. **Pose part of the problem**

   Ask:

   What do you think those dots are around the earth in the image?

   Have the students write down their answers. Explain to them that those dots are satellites that are being tracked in real-time on NASA's website.

   Some seem closer to earth and some are farther, why do you think that is so?

   Is there any pattern to any of the sets of dots?

   Explain the term geosynchronous and ask if the picture shows satellites that exhibit that behavior.

   Ask:

   How far do you think are the geosynchronous satellites from earth's surface?

   Have the students write down their ideas and reasons for their beliefs.

3. **Present the problem to be solved**.
   a. Explain: Period, orbital period, and rotational period with the help of students acting as satellites around you, the teacher. Then explain that geosynchronous satellites are satellites whose orbital period around the Earth matches Earth's rotational period.
   b. Ask: Why doesn't a geosynchronous satellite drift off into space? Or why doesn't it crash into the earth?

      Help them understand about forces especially gravitational and centripetal forces and then show what happens when the two balance.
   c. Explain: The location of a geosynchronous orbit can be obtained through the application of the mathematical formulas involving these two forces.

d. For the sake of students who are unfamiliar with the mathematical formula for the gravitational force, illustrate that the gravitational force is directly proportional to the product of the masses of the objects (M and m) and inversely proportional to the square of the distance (r) between them, thereby getting

$$F \propto \frac{Mm}{r^2}, \text{ so } F = \frac{GMm}{r^2} \text{ where G = gravitational constant}$$

e. Introduce the equation

$$\text{(Centripetal)} \frac{m v^2}{r} = \frac{G M m}{r^2} \text{(Gravitational)}$$

f. Walk through the formula, ensuring that the students understand the following:
- m = mass of the satellite
- r = height of the geosynchronous orbit
- Formula for velocity is Distance / Time (d/t)
- Distance, d, equals Circumference of the geosynchronous orbit
- Time equals 1 day (need to convert to hours and then to seconds)
- G = gravitational constant ($6.67 * 10^{-11}$ $N*m^2/Kg^2$)
- M = Mass of Earth ($5.976 * 10^{24}$ Kg)

g. Ask: Study the equation and decide how this equation can be used to find the location of the geosynchronous orbit.

h. Confirm with the students that the task is to solve for *r*

i. Allow the students to work through the solving of *r* in small groups to determine the location of the geosynchronous orbit.

4. Discuss the results

Have the students share their results and their procedures.

<u>Exploration</u>

Clarify as needed for students who still have trouble with the activity. Walk through the following steps. With each step, wait for them to finish from that step before moving onwards. If they still are having trouble, show the next step and so on.

$$\frac{m}{r} v^2 = \frac{m}{r} \frac{GM}{r}$$

Canceling out $\frac{m}{r}$ on both sides, we get

$$v^2 = \frac{GM}{r}$$

Using $v = \frac{d}{t} = \frac{2\pi r}{1\,day} = \frac{2\pi r}{24*60*60\sec}$

$$v = \frac{\pi r}{43200} m/\sec$$

Substituting this in $v^2 = \frac{GM}{r}$ we get

$$\frac{\pi^2 r^2}{(43200)^2} = \frac{GM}{r}$$

$$\frac{\pi^2 r^3}{(43200)^2} = GM$$

$$r^3 = \frac{GM * (43200)^2}{\pi^2} = \frac{6.67 * 10^{-11} * 5.976 * 10^{24} * 43200 * 43200}{\pi^2}$$

$$r^3 = 75.3701 * 10^{21}$$

$$r = \sqrt[3]{75.3701 * 10^{21}} = 35,786\ km$$

Extension

Semi-synchronous satellites are satellites that have half the orbital period of geosynchronous satellites and rotate around the earth twice a day. Satellites in the Global Positioning System are semi-synchronous. Have the students form small groups and recalculate the height of the semi-synchronous orbit from the Earth's surface.

Summary

Discuss the results with the entire class. After the students have completed recalculating the height of the semi-synchronous orbit, have them share their results with others in class. Show them the satellite tracking website again and have them select a satellite at random and look at the satellite information from the pull-down menu as in Fig. 1.2. Have the students figure out if the satellite they selected is a geosynchronous satellite or not. Once the students select a

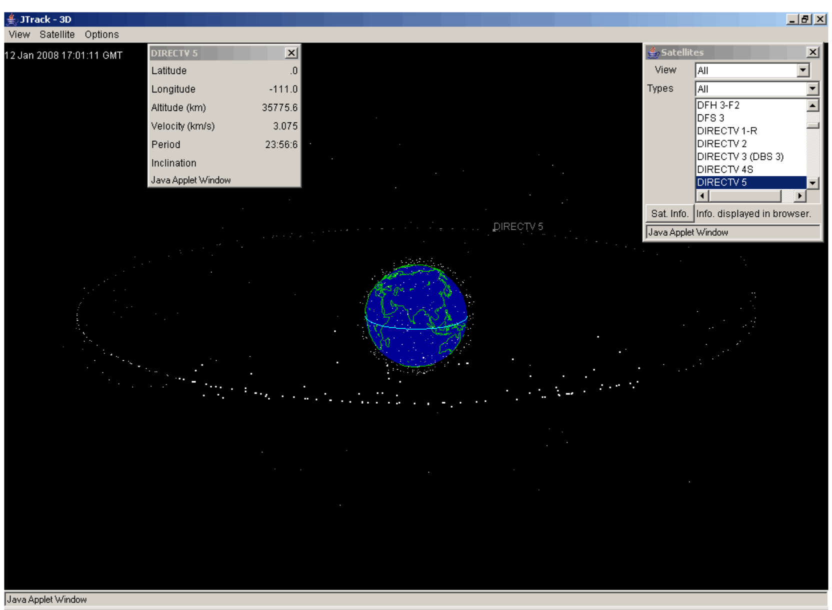


Fig. 1.2: Screen shot of NASA's J-Track 3D with satellite information for DirectTV 5

geosynchronous satellite, have them compare the height that they got for geosynchronous orbit from their calculations with the altitude of the orbit of the selected satellite. Their calculations may be a few kilometers off from what is listed in the satellite information. Ask them why they think it is different. Have them also look at the satellite's period listed. Help them understand that while they used 1 day = 24 hours for the Earth's rotation period, the actual rotation period is 23 hours 56 minutes and 4.0962 seconds.

If the teachers feel that the students need more practice with this topic, then have the students select their favorite satellite, a DirectTV, GPS or any telescope, and find the satellite information. Have them use the rotational period information to find the altitude or the altitude information to find the period and so on...

Module 2: Given below is the summary of a sample module entitled Ordinary Least Squares (OLS) and Error Analysis for GPS [13,14] at the differential equations level. This module can be considered as one of the capstone modules at this level. The summary below is provided, more from a content perspective than a teacher's perspective and shows the incremental transition from the naive concrete model to the abstract model which is based on more realistic assumptions.

Ordinary Least Squares (OLS) and Error Analysis for GPS

The challenges in navigation have driven human beings to achieve many groundbreaking mathematical theories and applications. Today, GPS is capable of locating an object on the earth to multi-foot accuracy by using signals from satellites approximately 25 thousand miles above. This module uses mathematics pertinent to GPS and guides students from naïve algebraic solutions to an iterative algorithm for solving least square problems in incremental complexities. Understanding these computational theories and techniques can assist students in adapting those techniques to the solution of problems in many other fields so that these solutions achieve high measuring and computing accuracy.

The module is designed to be worth one credit hour, which is approximately 15 class hours. Users may freely divide the module into three sub-modules. Each of these submodules may take 3 to 4 class hours. Since the module also hinges on a final team project, time for that project needs to be factored in.

The target students of the module are juniors in engineering or physical sciences majors. To be able to fully utilize the module, students should already have knowledge of the material normally covered in a Calculus II course together with partial differentiation. Knowledge of basic matrix operations is desirable, but not necessary. Students are expected to be familiar with either MATLAB® (preferably), Maple® or Mathematica®.

The module is organized according to the Guidebook for Authoring Modules, a document that can be downloaded from Capital University's web site.[15] There is an overall description of the problem followed by a brief problem statement, then major steps and questions that guide students to pursue solutions to subproblems, problems related to the module, a few similar projects that students need to use the OLS method to solve and, lastly, the pedagogical approach together with some observations and hints designed to help instructors deliver the module in the most effective way. Due to space constraints, we only provide for the problem statement and outline of the three sub-modules here.

1. Problem Statement: How do we determine the position of an object on earth, as accurately as possible, using satellite data from GPS? GPS often requires at least four satellites in detection range so that sufficient data are available to position the GPS users. The fundamental computational techniques applied in GPS are stochastic processes designed to estimate the random errors and regression methods used to minimize the measuring errors. In this module, we will learn how to define, mathematically, a model of the GPS and how to use OLS for estimating position over time.

2. Sub-Modules: The major steps are organized as three sub-modules. The first sub-module starts with an over view of the GPS such as system architecture, signal structure, receivers and measurements and ends up with a naïve algebraic solution under ideal assumptions (Fig. 2.1). The second sub-module starts considering time drift and introduces least square optimization. Adding time bias to three variables for a spatial position, we need to solve a system of four variables. Therefore, we need signals from 4 satellites. Students need to know partial differentiation for the linearization of distance formulas. An OLS method to minimize total error is used to find solutions. In the third sub-module, we finally reveal the whole truth –

1. The earth is rotating and communication satellites rotate around the earth twice a day.
2. More than 4 satellites are used in real GPS computation models because over specified systems result in improved accuracy,
3. Users often move and therefore, their velocities must be considered.

Therefore, students need to learn position transformation from a rotating frame to an inertial frame. Matrix representation and basic matrix operations have to be introduced to be able to implement a computationally efficient algorithm. This algorithm is referred to as a linear, minimum-error variance, sequential state estimation algorithm (see Fig. 2.3). A model that represents major error sources is illustrated in Fig. 2.4.

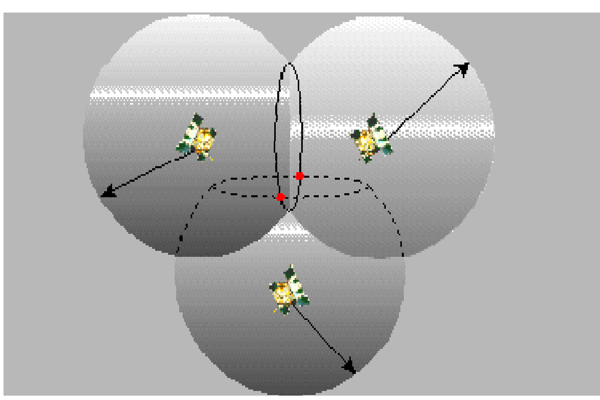


Fig. 2.1: Naïve solution

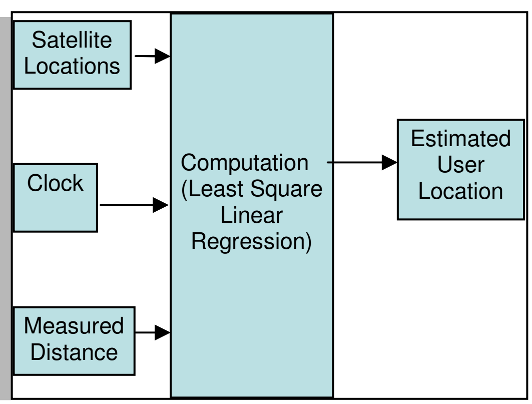
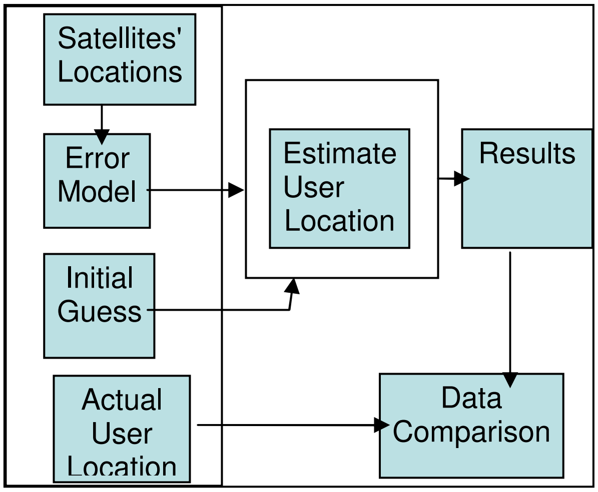


Fig.2.2: Solution to Inertia Frame



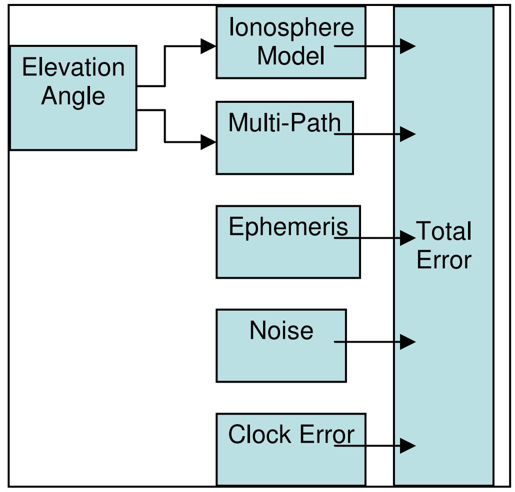


Fig. 2.3: Iterative Computation Model

Fig. 2.4: Error Model

For further investigation:

Students are now in a position to consider problems such as determining ionospherical and tropospherical delay, detecting multipath errors, seeking an algorithm to find the optimum geometrical configuration of satellites, etc.

Students may also find the following questions intriguing:

Since 4 satellites are sufficient to determine the location, is it possible to pick the best four in the sky to reduce the computational complexity and provide best estimation?

We can typically detect signals from 5 to 6 satellites simultaneously, how many of them should we use to attain the most accurate position estimate?

If theoretic answers to these questions are beyond our reach, can we use good simulations to find empirical answers to these problems?

Conclusion:

There is a tremendous amount of work still to be done. The framework has to be completed as it is missing the explicit definitions of levels as well as methods of assessment and promotion from one level to the next. Once the framework is complete the task of generating modules for each level begins. Specifications for each module have to be developed and implemented. Software engineering methodologies need to be employed so that requirements do not increase uncontrollably with time and that the project stays within the scope and budget. The modules will have to be tested and data collected concerning their effectiveness at the different levels. The modules will be made available to teachers both at the college level and at high school and we will document feedback and tally results. Like software, module sets will need to be maintained and kept relevant.

The authors believe that use of these modules will enhance student comprehension, abstraction, and problem solving skills in mathematics and its related fields.

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