AC 2009-807: PROBLEM SOLVING IN STATICS INVOLVES MENTAL SEARCH

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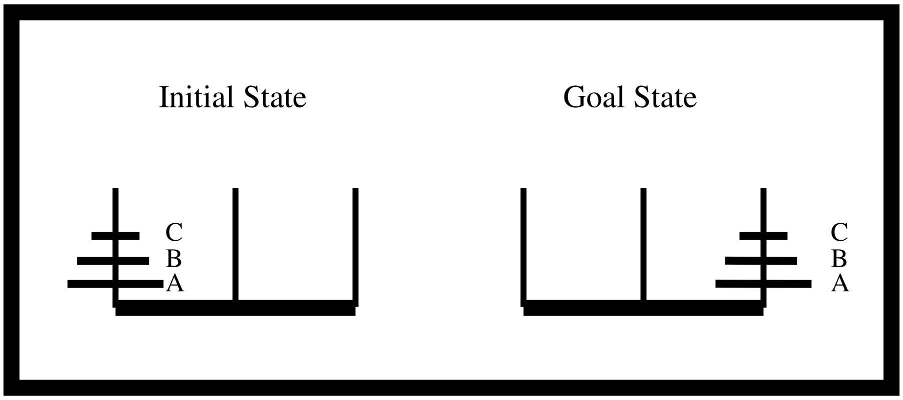
Problem-Solving In Statics Involves Mental Search

Introduction

Theory of Human Problem Solving. In their seminal work on human problem-solving, Newell and Simon[1] described the process of solving a problem as consisting of finding a path that leads from an initial state to the goal state in a problem space. A problem space consists of discrete problem states, which are simply explicit configurations of the problem elements. The initial state consists of a description of the problem elements at the outset of problem solving. Through the application of problem solving operators, a person is able to transform the current problem state into the next problem state. A classic example of the elements of this theory is the Tower of Hanoi puzzle, as shown in Figure 1, consisting of three moveable disks and three pegs. Beginning with an initial state and a given goal state, the initial state can be changed to the next state by applying the operator "Move disk C from peg 1 to peg 3," for example. Constraints on the operators include moving only one disk at a time, never placing a larger disk on a smaller disk, and never taking a disk out of play. The Newell and Simon theory has been applied to visual puzzles like the Tower of Hanoi, but applies readily to other types of representations and problems, including scheduling problems, decision-making, game-playing, language, and mathematics[2].

Figure 1. Tower of Hanoi Example



The size of the problem space for a typical game of chess[3] is 10^{117}. In spite of the immensity of the problem space for chess, even beginning players can play a respectable game. Humans, faced with the task of chess or other problems, rely on heuristic search of problem spaces. Heuristic methods often do not require a great deal of specific knowledge about the problem. Because they do not require specific knowledge, they are widely applicable and therefore very useful. The description of problem solving as heuristic search in a problem space has proven to be quite powerful in understanding humans' knack for solving simple and complex problems, and those that are well-defined and those that are ill-structured[4]. The operational parameters of this theory are clearly quite broad, applying as much to the astounding performance of Gary Kasparov against the massively parallel computer Deep Blue[5], at one extreme, and the performance of a child who grasps the essential operators (rules for moves) for chess and begins playing the game.

When problem solving succeeds, the person has discovered a path that leads to the goal state. Familiarity with a domain, and generally one's level of expertise in the area, lead to efficient search and relatively shorter solution paths. Somewhat surprisingly, even expert problem solvers in a particular domain experience a certain amount of "near-sightedness" when solving a problem, largely considering what a good "next step" would be, serially transforming the current state to the next state, creating a path of problem states through a vast possibility of possible paths.

Problem Solving Models in Engineering. Problem solving is a dominant activity in undergraduate engineering training[6]. The centrality of problem solving to engineering has led engineering faculty and researchers to propose didactic models of "best practices" for undergraduate engineering education[7-11]. These models include problem-solving processes that cover a wide range of activities. Using the Wankat and Oreovicz[7] model as an example, some elements relate to motivation (e.g., the problem solver feels confident). Other processes relate to analyzing the problem (e.g., the problem solver breaks the problem into parts; identifies parts/segments that are routine). Yet other processes are at a metacognitive level, consisting of evaluating, monitoring, and checking a solution as it develops. A comprehensive list of processes included in several published models of engineering problem solving, as well as an extension of those models, can be found in the ASEE 2008 Proceedings[12].

The present work adopts the Newell and Simon[1] model in a fresh look at engineering problem solving. In this work, we implement the idea of a problem space. The problem states, or nodes in the space, are specific instances of the equations that the solver develops. The beginning point in the problem space is the problem statement, and the end points are specific solutions. By examining students' problem worksheets, we identify a small number of processes that the solver applies in order to progress from the current equation in the solution to the next equation. By working directly from students' worksheets, we try to assess students' performance using the same information that would be used by an instructor. However, we attempt to go beyond traditional "grading" and use our depictions of the processes that solvers use to better understand the differences between highly-skilled and less-skilled problem solvers in the area of statics. These differences are used to consider specific weaknesses between skilled and less-skilled problem solvers, and the implications of these findings to curriculum practices.

Lower-Order and Higher-Order Problem Solving Processes. Bloom's[13] classic work on a hierarchy of cognitive processes has informed instructional research and practice over the decades. Bloom's work is complemented by research on comprehension, particularly the work of Pressley and colleagues[14]. Drawing on this knowledge base, recent research in engineering education has shown that students generally draw on lower cognitive levels of Bloom's taxonomy[13] (Level 2: Comprehension; Level 3: Application) than on higher levels (Level 4: Analysis; Level 6: Evaluation)[15, 16].

Higher-performing students, as defined by conventional grades, engage in significantly more higher-order processing[16], in the senses defined by Bloom, Pressley, and colleagues. As a framework for separating lower-order and higher-order processes for the present research, we assume that those aspects of problem solving that simply require individuals to retrieve and apply a fact or procedure that they already know are lower-order processes (consistent with Bloom's[13] Knowledge (Remembering), Comprehension (Understanding)). Those processes that are consistent with Bloom's higher levels (Analysis, Synthesis, Evaluation] and Pressley's[14] metacognitive strategies associated with monitoring processing, are higher-order processes.

Statement of Hypotheses. The course of problem solving is affected by a person's prior knowledge[10], representation of the problem, available operators or processes to transform one problem state into the next, and grasp of the underlying problem. With these considerations, we expected skilled problem solvers to be more efficient in discovering a path from initial state to goal state, to make fewer representational and computational errors, and to engage in more monitoring and control of the problem solving process. Using the problem in Figure 2 as an example, Table 1 provides instances of knowledge and operations that could apply to solving the problem.

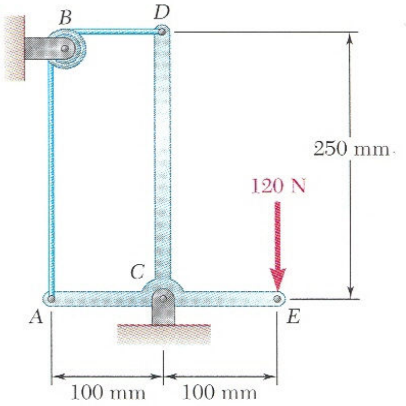


Figure 2. Problem: *Neglecting friction, determine the tension in* cable ABD *and the reaction at support* C.[17]

Table 1. Knowledge and Equations Related to Figure 2.

Written Assumption	$T_X = T_Y$
Key Relation	$\sum M_C = 0$
Correct Expansion	$0 = T_X(250mm) - T_Y(100mm) - 120kN(100mm)$
Correct Simplification	$T = \dfrac{12000Nmm}{(250mm - 100mm)}$
Incorrect Expansion	$0 = T_Y(100mm) - 120kN(100mm)$
Key Relation	$\sum F_X = 0$
Correct Expansion	$0 = F_{Cx} - T_X$
Correct Simplification	$F_{Cx} = T_X$
Correct Goal$_1$	$T = 80N$
Correct Goal$_2$	$F_{Cx} = 80N$
Correct Goal$_3$	$F_{Cy} = 40N$
Incorrect Goal$_1$	$T = 120N$

A consideration of the ways in which the research literature indicates that skilled problem solvers differ from less-skilled problem solvers led to the hypotheses in Table 2, which are visually exemplified in Figures 2A and 2B. Figure 2A, for a skilled problem solver, develops key equations through the application of simple algebra directly from well-known key relations, like *the sum of moments about a point equals zero*. The skilled problem solver follows an efficient path through the problem space, avoiding ineffective and incorrect expansions. In contrast, the less-skilled problem solver is susceptible to incorrect expansions and equation simplification, due to limited prior knowledge of the problem-solving domain and weak mathematical skills. The less-skilled problem solver is thus likely to execute more problem-solving steps, including some that are incorrect, and may not reach the goal (solution) for the problem.

Figure 2A. Depicts Flow of Problem-Solving Processes for an Expert

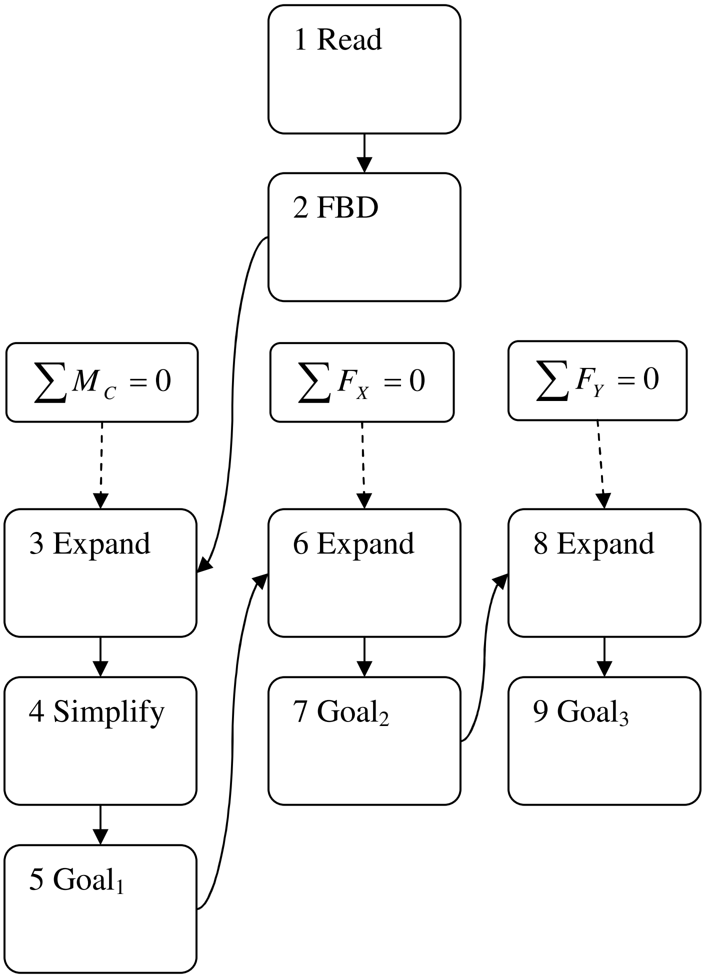


Figure 2B. Depicts Flow of Problem-Solving Processes for a Novice

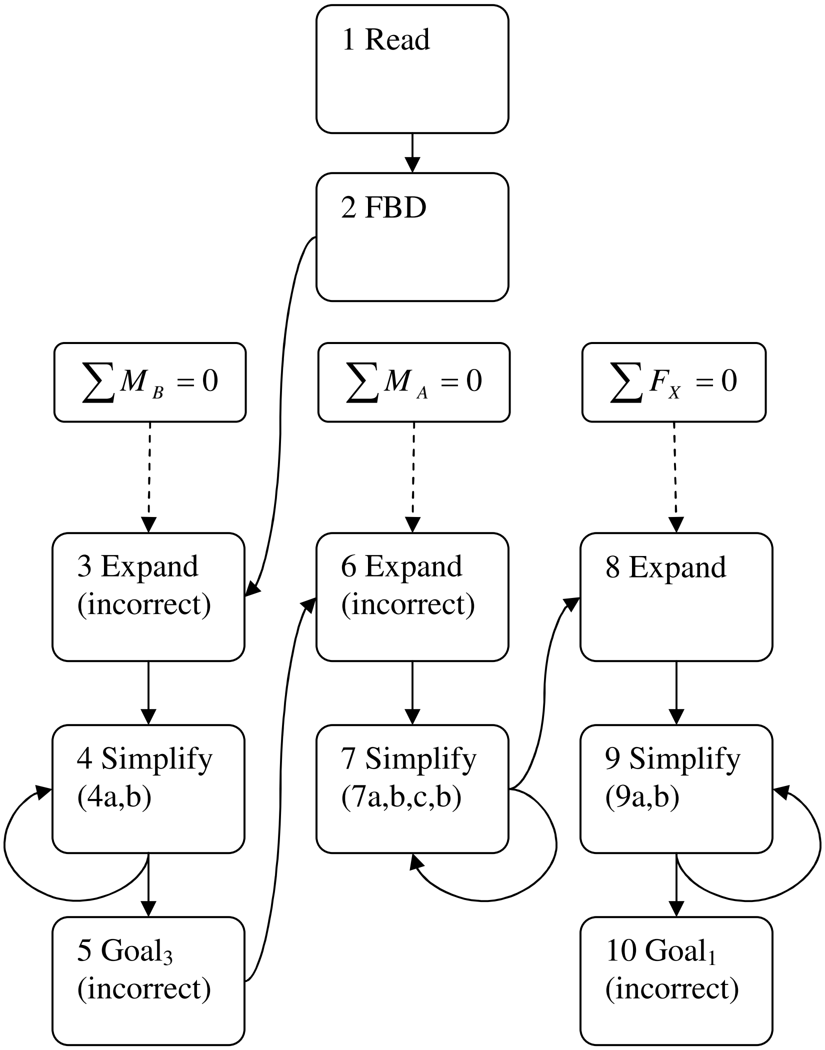


Table 1. Ordered Hypotheses for the Three Groups of Participants

Factor Tested	Hypotheses	Supported
Problem Solving Steps	Faculty < Higher-Level < Lower-Level	No
Lower-Level Processes	Faculty < Higher-Level < Lower-Level	No
Higher-Order Processes	Faculty > Higher-Level > Lower-Level	Yes
Incorrect Processes	Faculty < Higher-Level < Lower-Level	Yes

Case Study

Materials, Participants, and Procedure. Two problems were chosen as the primary materials in this study. The problems appeared in the exercises for Chapter 4 in *Vector*

Mechanics for Engineers: Statics and Dynamics, 6th Edition[17]. At the time of data collection, student participants had recently completed and had been tested in class on the material from that chapter. They had not, however, solved the two problems used in this study. The first problem statement read: *The 10-m beam* AB *rests upon, but is not attached to, supports at* C *and* D. *Neglecting the weight of the beam, determine the range of values of* P *for which the beam will remain in equilibrium.* The second problem read: *Neglecting friction, determine the tension in* cable ABD *and the reaction at support* C (see Figure 2). Both problems were presented with the corresponding diagram from the textbook, and were assigned to participants in the same order as presented here.

Eighteen undergraduate students who were currently enrolled in Mechanics I and five faculty members were recruited through the engineering college at a large public university by one of the experimenters. The undergraduates were paid $25 each for approximately one hour of participation. The five faculty members were from the Mechanical Engineering department. Additionally, two Mechanical Engineering faculty who did not provide verbal protocols graded participants' problem solutions, using a rubric provided by one of the experimenters, as follows:

> *Grade each problem as if it were an open-book homework problem that the students had completed just before the test over this material.*
> *1. Indicate whether the solution for each problem is correct or incorrect.*
> *2. Score the solution as you normally would, indicating whatever partial credit you would give. Present your score on a 100-point (or percent) scale.*
> *3. Mark up the solutions as you normally would, writing in any comments that you would normally give.*

The graders were unaware that faculty solutions were included, and there was no identifying information on the solution sheets. Student participants were assigned to either a high-performing or low-performing group, based on an average of the scores assigned to them on the two problems. Six participants were excluded from the analyses because their scores for both problems were not consistently high or low. Six high-performing and six low-performing students formed two experimental groups. The five faculty members formed the third group. On a 100-point scale, where 100 is a perfect score, the average of the lower-performing students, higher-performing students, and faculty, were 56.75, 88.17, and 95.4, respectively.

Participants met individually with an experimenter in a quiet room. They were instructed to think out loud as they solved the problems. Solving the two problems took approximately 50 minutes. Problem solving was video-recorded, with the permission of participants. The video recordings were used after the conclusion of the study to disambiguate interpretation of the written solutions. During data collection, the primary role of the experimenter was to prompt participants to continue to verbalize their thoughts if they fell silent for an extended period. The verbatim instructions to students were as follows, with the textbook opened to the problems at the end of the chapter:

> *In this study, you will be asked to solve two statics problems from this chapter in your textbook. Take a moment to page through the chapter to confirm that you have covered this material. Each problem will be presented on a sheet of paper. Extra paper is available if you need it. Solve the problem as you normally would. But try to*

> *neatly show your work. As you are solving these problems, say out loud what you are thinking. The more thoughts you verbalize, the better. Whatever you say should simply reflect what is going through your mind while solving the problem. If you fall silent for more than a minute or so, I will remind you to keep talking. The textbook is available for you to use, you can use your personal notes, and you should feel free to ask questions at any time.*

The textbook was also available to faculty participants, and they were likewise informed that they could use it as a resource.

Results and Discussion. An analysis of the worksheets began by identifying the key relations that applied to the problems, generating correct equations to solve the problems, and considering possible errors in expansion and simplification. The equations in the worksheet were the primary source of data. The experimenter consulted the video and verbal transcript to clarify elements of the solution that could not be directly derived from the worksheets. Relying on the video, the experimenter numerically labeled (1, 2, 3…) each equation in the order the participant generated them. These numbers allowed the experimenter to see what steps the participants were taking in their problem space. Then each equation was coded as a particular type of problem solving step. A simple count of the frequency of specific problem-solving processes was conducted for each participant. Mean frequencies, by group, are presented in Table 3. Based on existing problem solving research[16], and Bloom's taxonomy[13], simple processes were composed into lower-order processes, higher-order processes, and Errors and Guessing. Because of the small sample sizes, exact statistical tests were applied[18]. Because we had prior hypotheses about the order of effects (Table 1), one-sided probabilities at an alpha level less than 0.05 was adopted for tests of significance using the Jonckheere-Terpstra test[19]. As indicated in Table 2, two of the hypotheses were supported and two were not supported.

The analysis of participants' worksheets revealed that the three skill groups in this study did not differ in the overall number of problem states that they produced. This was contrary to the ordered hypothesis that faculty would be most efficient and lower-performing students least efficient. The data revealed that for ordinary problems even the most skilled problem solvers still need to engage in a fair amount of search to transform the problem from the initial to goal state.

Although the skilled groups did not differ in the overall number of problem states, there were significant differences in how the problem states were distributed. Lower-performing students, relying on prior knowledge and mathematical skills, were more likely to make errors in constructing a free-body diagram and in expanding on the key relations involving sum of moments and sum of forces. They incorrectly simplified equations, and guessed the value of variables. Somewhat surprisingly, the skilled groups did not differ significantly in terms of cognitive control processes – so-called metacognitive processes, like checking their work. In terms of Bloom's[13] taxonomy of mental operations, metacognitive processes can be regarded as higher-order cognitions, relative to processes that are more direct applications of factual knowledge and learned procedures. There was also no difference between the groups in terms of stating relevant assumptions.

A problem space conception of problem solving in statics leads to several important insights. Using chess as an example, it is clear that even a beginning player has sufficient operators to play the game. However, the quality and outcome of the game will differ. Similarly, the transformation of problem states as part of searching for a path to the solution in a problem space informs our understanding of less-skilled and more-skilled problem-solvers in statics.

Table 3. Average Participant Frequencies of Problem Solving Processes for the Three Groups of Participants

	Engineering Faculty	Higher-Performing Students	Lower-Performing Students	p-value*
Total Number of Steps	42.4	39.5	35.2	.31
Higher Order Processes				
FBD Construction (Correct)	3.00	2.30	1.80	**.03**
FBD Construction (Incorrect)	0	0	1.00	**.006**
Equation Expansions (Correct)	6.60	6.30	5.50	.24
Equation Expansions (Incorrect)	0.20	0.70	3.00	**.002**
Written Assumptions (Correct)	1.20	1.20	0.50	.31
Written Assumptions (Incorrect)	0	0	0.20	.35
Checks Work	2.00	0.20	0	.18
Corrects Work	0.60	0	0	.29
Lower Order Processes				
Equation Simplifications (Correct)	17.20	15.50	12.80	.12
Equation Simplifications (Incorrect)	0.60	0.20	1.50	.23
Equation Substitutions (Correct)	3.80	4.30	2.70	.46
Equation Substitutions (Incorrect)	0	0	0.20	.35
Equation Rewrites (Correct)	1.80	3.00	2.20	.23
Equation Rewrites (Incorrect)	0.40	0.20	0	.10
Guess Value and Trial & Error	0	1.00	0.30	.25
Solution				
Correct Goal Solution	4.20	3.30	0.30	**.0002**
Incorrect Goal Solution	0.80	1.30	3.20	**.006**

* p-values based on Jonckheere-Terpstra Exact Test (one-sided). p-values < .05 are statistically significant.

Conclusions

This paper introduces a new research approach for thinking about problem solving in statics. Compared to experts' conventional grading of students' work, it adds both theoretical depth and practical insights for answering why and where highly-skilled problem solvers are better than less-skilled problem solvers. Conventional grading focuses on identifying and penalizing errors. Our approach conceives of statics problem solving as a constructive process that is active and that may include false steps depending on prior knowledge and level of effective problem-solving monitoring.

Two limitations of this study are a relatively low number of participants in each of the skill groups and a focus on only two problems from the statics curriculum. We would expect that in future studies, higher-skilled participants would distinguish themselves more in the application of metacognitive processes, like those in the upper portions of Bloom's[13] hierarchy. In the present study, construction of the free-body diagram and correct expansion of key relations were critical aspects of reaching correct problem solutions. The higher-skilled participants distinguished themselves in making fewer errors in these operations, and thereby had significantly higher achievement of correct goal solutions.

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