Pursuing the Perfect Statically Indeterminate Bar: Model Versus Experiment

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Abstract

Engineers and technologists often now function in a professional environment where analytical modeling and simulation must serve the role previously filled by prototyping and experimentation. An inherent difficulty in the development of both conceptual and analytical models is recognizing the limitations on the applicability of the model to a given component or system. These limitations come from numerous sources, including simplification for ease of calculation, idealized dimensions, and neglecting the effects of manufacturing processes. A primary goal of basic mechanics courses is to teach students how to develop appropriate engineering models which will allow them to reasonably approximate “real-life” results. In a typical first laboratory-based strength of materials course, students conduct established experiments to verify simple engineering models, but may not consider the constraints imposed on the test specimen in order to obtain experimental results which match theoretically predicted values. In particular, manufacturing factors are frequently overlooked.

A laboratory exercise based on the statically indeterminate bar of multiple, equal length, adjacent materials subject to tensile axial loading was designed to help clarify the importance of recognizing the constraints of a theoretical model. This application was selected due to the difficulty many students have in correctly using the theory, as well as its sensitivity to deviations from the conditions assumed when developing the model. According to the conceptual model included in various textbooks, each material will experience equal strain, even though internal forces generally differ. A collection of bar specimens was produced to show some of the factors that can affect the validity of the application of the theory, including force balance; method of joining; tolerances; and strain gage mounting. The following paper describes the exercise and the significance of overlooking various engineering constraints as demonstrated through simple strain measurements. The experiment was originally intended as a review laboratory session for an elective upper division course in experimental strength of materials, but is also appropriate in an introductory mechanics/strength of materials course, with some simplification.

Introduction

The statically indeterminate bar which is subjected to axial loading with one degree of indeterminacy, such as that shown in Figure 1, is commonly an early topic in a first course in mechanics/strength of materials. Study of this topic affords students the opportunity to integrate the fundamental concepts of stress and strain, and to recognize more fully the effect of material stiffness on load-carrying capability. The necessary inclusion of material effects for determination of internal loads differs significantly from their previous mechanics coursework, so many students struggle with this initial encounter with interdependent variables. In order to aid the student as s/he strives to master this new idea, the typical elementary mechanics or

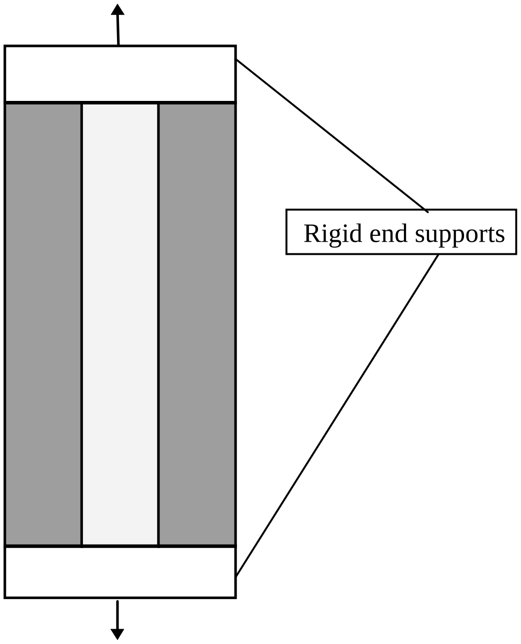


Figure 1: Statically indeterminate bar subjected to axial loading

strength of materials textbook author includes detailed directions on how to solve problems which correspond to this model, and may include numerous examples demonstrating how to define the governing equilibrium and compatibility equations.[1-11] In particular, authors of all of the textbooks checked include a combination of examples and homework problems based on the most simple theoretical model; that of a bar constructed of multiple linear, elastic, isotropic, homogeneous materials of equal length. Such an idealized bar will experience equal strain in each material, allowing for relatively easy determination of the axial stresses and loads induced in each material.

This model appears to be very straightforward, and students generally succeed in learning to solve well-defined statically indeterminate bar problems for equal length materials. However, using an engineering model to gain insight into the behavior of a real component or system demands a reasonable grasp of the explicit and implicit assumptions made in developing the model.[12] At the sophomore level, most students overlook all but the most obvious assumptions in any given problem. The statically indeterminate axial bar model is no exception. Because the typical elementary textbook author thoroughly explains the process of setting up and solving the ideal problem only, the course instructor must take full responsibility for teaching the student the identification and implications of the limitations on the model's assumptions. Even upper division students exhibit a high level of uncertainty regarding the reasons for making each assumption.

For the bar made of multiple linear elastic, isotropic, homogeneous materials of constant cross-section, a number of assumptions are made about the bar itself, as listed in Table 1. The validity and limitations of each of these assumptions is questioned as part of the experiment described.

TABLE 1: Model Assumptions and Limitations

1)	*All material lengths are equal*.	Machining tolerances make this unlikely.
2)	*Cross-sectional dimensions are constant*.	Machining tolerances again make this unlikely.
3)	*Published values for modulus of elasticity are applicable.*	Suppliers track yield strength and percent elongation, so the modulus of elasticity can vary significantly between lots.
4)	*Supports are rigid.*	The model version of “rigid” means that the support has no deformation or deflection, while in practice, “rigid” implies that the deflection is minimal compared to the bar sections.
5)	*The method used to join the supports to the bar (and the material sections together) is selected so that it has no effect on transmission of load to the component material sections.*	The type of joint and the tolerances allowed in the joint can greatly affect the load in each component section.
6)	*The forces in the model are balanced so that no bending is induced from the differences in material stiffness.*	Because this appears to occur in all textbook problems, and the topic is discussed before combined normal stresses are considered, students frequently fail to recognize this potential problem.
7)	*Poisson effects can be neglected*.	For cases in which one material component surrounds another, such as reinforced concrete columns, the induced transverse stresses may be significant.

Description of the Experiment

This experiment was developed to facilitate acceptance of the idea of indeterminacy, as well as recognition of the limitations of this particular engineering model when compared to actual components. The experiment is based on the case of the bar made of two adjacent materials of equal length, where axial loading should produce equal strain in each section of the bar. This case was selected due to the simplicity by which experimental deviations from theory can be detected, (i.e., visual inspection of data), as well as its widespread use in elementary mechanics/strength of materials textbooks. The experiment is designed to raise the students’ awareness of the constraints on the applicability of a model which result from experimental conditions and manufacture of a component, as well as improve their skill in applying engineering models.

Several versions of the experiment have been conducted. The most comprehensive version is presented here. The students measure the axial strain in each material in a series of axial bar specimens which increasingly deviate from the theoretical engineering model. The resulting load in each specimen is predicted from the theoretical model, then calculated from the measured strains. These loads will generally differ, so the students must perform a qualitative (and somewhat quantitative) “error analysis” to evaluate the source of the variation. “Error analysis” tools which can be used include visual inspection; measurement of all relevant specimen and fixture dimensions; strain gage alignment checks; non-standard tensile tests to verify moduli of elasticity; and calculation of approximate support deflections.

Test Procedure

A series of two material bar specimens and "dogbone" specimens equipped with clevis pin connections are pulled in a standard loadframe. Each material section of each specimen has a single strain gage mounted along the direction of axial loading. Axial load was monitored via a Measurements Group P-3500 Strain Indicator. The strain at each gage was measured at 50 lb_f or 100 lb_f load increments for all specimens, using a similar strain indicator in conjunction with a switch and balance unit to read the strains. The specimens tested are listed in Table 2.

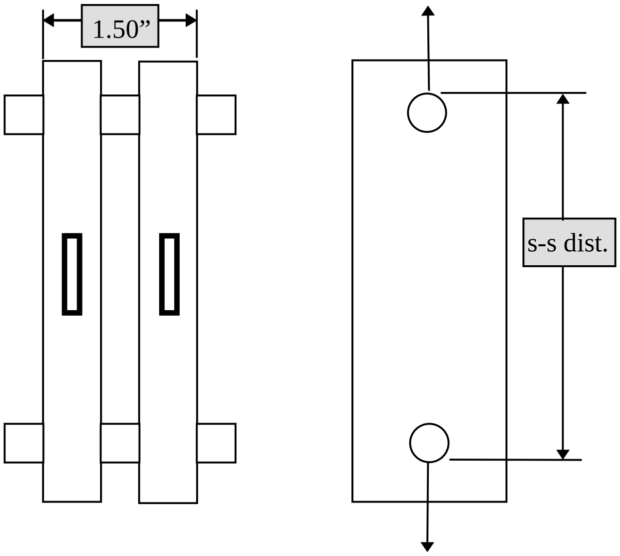
Table 2: Axial Bar Specimens

1)	*Statically determinate reference case*	Two aluminum bars supported by 0.375 in (9.5 mm) diameter stainless steel pins at top and bottom.
2)	*"Ideal" statically indeterminate case*	Three-component bar (aluminum-brass-aluminum) which are supported by 0.375 in (9.5 mm) steel endplates attached by machine screws, with fixturing for pins bolted to the endplates.
3)	*Machining tolerances case*	Three bars (aluminum-brass-aluminum) equipped with 0.375 in (9.5 mm) diameter stainless steel pins at top and bottom, where the pin contact surface to contact surface distance differs slightly in the bars.
4)	*Combined machining tolerances and force imbalance case*	Two bars (aluminum-brass) supported by the 0.375 in (9.5 mm) diameter stainless steel pins.
5)	*Miscellaneous deviations case (Transverse joining, force imbalance, etc.)*	One of several aluminum-brass and aluminum-steel flat dogbone specimens joined by rubber cement and supported by 0.375 in (9.5 mm) stainless steel pins.

Figures 2-6 show the specimens and representative test data. The standard test procedure is completed for each specimen during the first of two laboratory sessions. Between sessions, the students begin their calculations and determine what additional information will be needed as they attempt to account for the likely source of any differences in the strains measured and loads calculated. The second laboratory session is devoted to collecting any dimensions needed, conducting modified tensile tests, et cetera. The students have access to an optical comparator, a coordinate measuring machine, various micrometers and calipers, and the equipment required for the original tests. Some details of the manufacture of the bars are not available, so the students develop and try to confirm their own hypotheses regarding the processing and its effect on the strains present.

Findings

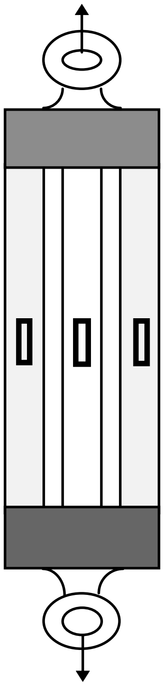
The students discover that carefully designed experimental specimens can provide data which closely correspond to the engineering model, but models rarely account for the variations which are typical of real parts. They gain an appreciation for the effects of manufacturing processes on the relevance of a given model. By conducting the experiment and analyzing the data collected, sufficient practice with statically indeterminate axial bars is obtained so that most students can solve basic problems of this type. In addition, the slight misalignment of some strain gages helps students gain awareness of "experimental error" where the error is not simply the difference between the theory and the reality. Hopefully, the lessons learned through this experiment carry over to the use of other engineering models throughout their careers as the students strive to understand the relationship between the components/systems they design and the products which are ultimately produced.



Two Al 6061-T6 bars, 0.25”x1.0”; contact surface to surface distance of 7.390” on left and 7.384” on right

Load, lb_f	StrainL,μ	StrainR,μ
0	-1	-1
51	9	10
100	18	20
153	27	31
201	37	41
253	47	52
299	56	63
350	67	74
401	77	84

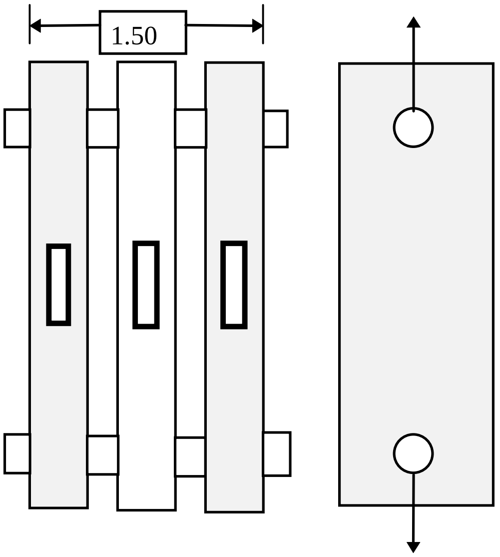
Figure 2: Statically Determinate Aluminum Bars (reference case)



Two Al 6061-T6 bars around C260 Brass bar; each 0.25”x1.00” by 7.0” screwed into 0.375”x1.0”x1.0” steel endplates

Load, lb_f	StrainAL,μ	StrainB,μ	StrainAR,μ
1	-1	0	2
51	8	8	8
102	15	15	14
152	23	22	20
203	30	30	27
254	37	38	33
303	44	44	40
348	50	52	46
404	58	60	53

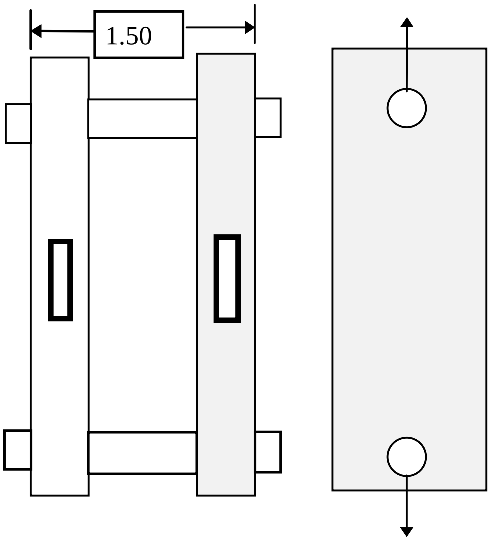
Figure 3: “Ideal” Statically Indeterminate Specimen



Two Al 6061-T6 bars around a C260 Brass bar, each 0.25”x1.00” with a nominal 7.0” center to center pinhole distance supported by 0.375” dia stainless steel pins

Load,lb_f	StrainAL,μ	StrainB,μ	StrainAR,μ
0	-1	-1	1
55	10	0	11
104	18	0	20
149	27	0	29
299	53	-1	60
330	59	-1	65
350	61	2	68
398	69	5	73
428	73	7	77
450	76	8	80

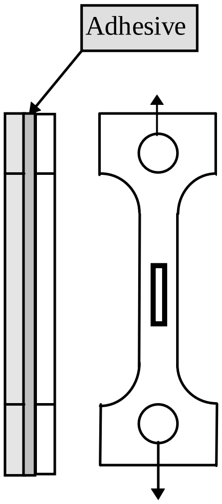
Figure 4: Machining Tolerances Specimen



One Al 6061-T6 bar and One C260 Brass Bar Each 0.25”x1.00” with a nominal 7.0” center to center pinhole distance supported by 0.375” dia stainless steel pins

Load,lb_f	StrainB,μ	StrainAR,μ
0	3	3
52	12	12
103	21	23
151	29	31
204	38	41
250	47	50
297	55	58
351	66	69
401	75	78
453	85	88

Figure 5: Combined Machining Tolerances and Force Imbalance Specimen



Al 6061-T6 0.518”x0.062” and C260 Brass 0.518”x0.067” joined by rubber cement

Load, lb_f	Al strain,μ	Br strain,μ
94	163	117
192	341	262
299	532	431
395	699	586
498	877	757
615	1076	950

Figure 6: Miscellaneous Deviations Specimen

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