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RANDOM BEAM PATTERNS FROM LINEAR ARRAYS

J. Au, O. Mudasiru, K. Chandra and C. Thompson
Center for Advanced Computation and Telecommunications
University of Massachusets Lowell
Lowell, MA 01854, USA

***Abstract*—The design of linear microphone arrays with randomly spaced elements is investigated. The probability distribution function for the element positions is derived in the continuum limit by matching the response of the array to an objective beam function. An analysis of the expected value and variance of the beam is presented as a function of the angular direction. Comparison of these metrics to those generated from uniformly distributed array positions is conducted. The number of elements and random ensembles required to meet a desired sidelobe level is obtained. It is found that using a probability density function of a particular beam pattern can reduce the side lobe levels of the random samples of the transducer position.**

Keywords—transducer arrays, transducers

I. INTRODUCTION

A microphone array is a device consisting of several microphones, referred to as array elements that can be jointly activated to receive sound from preferred directions. The configuration of microphone arrays so as to automatically tune to the direction of a sound source is an important problem in many classical applications such as teleconferencing and hearing devices as well as in emerging applications such as wireless acoustic sensor networks, spatial audio and computer mediated voice communications. The development of dense microphone arrays made up of low power elements offers the potential to form beams collaboratively using multiple arrays with randomly spaced elements.

Pei *et al.* [1] observed that directional antennas give a higher throughput gain because of reduced side lobes. The main-lobes extends towards the user giving maximum radiation or reception, while other lobes such as the side and back lobes, which represent lost energy are minimal. Lo [2] has shown that randomly spaced elements located along the aperture can lead to sidelobes with equal levels and with equal probability. This work examines the statistical characteristics of the beam pattern formed considering from randomly spaced elements. Section 2.0 presents the geometry of the problem and the analysis for developing the probability density function for element positions. Section 3.0 conducts the statistical analysis of the array response and Section 4.0 concludes the paper.

II. LINEAR ARRAYS AND BEAM FORMATION

A linear antenna array of length $2D$, comprised of $2N$ elements positioned at locations $x[n]$, $n = -N,-(N-1), \ldots, 1, 2, \ldots N$ is depicted in Fig. 1. The positions are normalized with respect to the half-length of the array yielding a normalized range of positions from -1 to 1.

The response of the array to a time harmonic plane wave at frequency ω radians per second and incident at an angle θ with respect to the normal to the array is

$$S(\theta) = \frac{1}{2N}\sum_{n=-N}^{N} a[n]e^{j\frac{\omega}{c}D\sin(\theta)x[n]} \tag{1}$$

with c meters/second being the speed of sound. The aperture function $a[n]$ will be assumed to be uniform for all elements, which are also taken to be omnidirectional. Defining

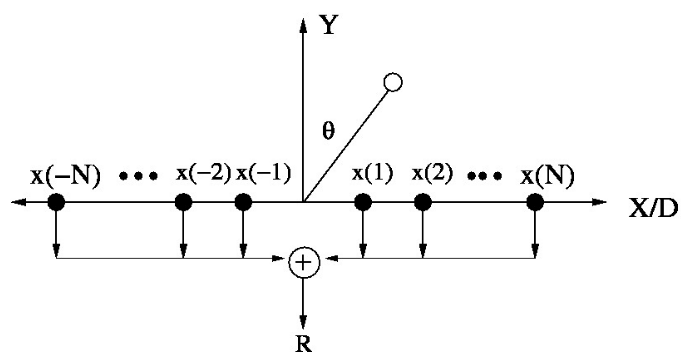

Fig. 1. Uniformly Spaced Linear Array

$u = kD\sin(\theta)$ where $k = \omega/c$ is the wavenumber, and by mapping the positions to continuum values, the response can be written as the integral

$$R(u) = \frac{1}{2N}\int_{-N}^{N} e^{jux(n)}dn \tag{2}$$

III. Random Positioning of Array Elements

The response in Eqn. 2 represents the case for a uniform distribution of length $2N$ with a cumulative probability $F(n) = n/N$, assuming the positions are symmetrical about $x = 0$. For element positions to be drawn from another probability distribution function $G(x)$, the transformation $G(x) = n/N$ yields, $G'(x)dx = 1/N\, dn$. Substitution in Eqn. 2,

$$R(u) = \frac{1}{2}\int_{-1}^{1} g(x)\, e^{jux}\, dx \tag{3}$$

where $g(x) = G'(x)$ is the probability density function (pdf) for the continuous valued array element positions x.

The objective is to determine the $g(x)$ that best matches a specified beam pattern. The method of solution for deriving $g(x)$ is given by the authors in [3]. The desired beam pattern $R(u)$ given by van der Maas [4] modified by Ishimaru [5] is applied for computing $g(x)$. The target response is

$$R(u) = sinc(u)\prod_{k=1}^{K}\left[\frac{1-\left(\frac{u}{u_k}\right)^2}{1-\left(\frac{u}{k\pi}\right)^2}\right] \tag{4}$$

where $u_k = \pm(K+1)\pi\sqrt{\frac{A^2+\left(k-\frac{1}{2}\right)^2}{A^2+\left(K-\frac{1}{2}\right)^2}}$, $K = 18$, and $A = 1.1$ is the sidelobe parameter.

Substituting the target response in Eqn. 3 and taking its inverse Fourier Transform, $g(x)$ can be evaluated using contour integration in the complex u plane with the condition that $g(x)$ is nonzero for $|x| < 1$. The result is

$$g(x) = x + \sum_{m=1}^{K} (-1)^m \frac{\sin(m\pi x)}{m\pi}\left(\left(\frac{m\pi}{u_m}\right)^2 - 1\right) \prod_{j\neq m}^{K}\frac{1-\left(\frac{m\pi}{u_j}\right)^2}{1-\left(\frac{m}{j}\right)^2} \tag{5}$$

Fig. 2 depicts the pdf $g(x)$ obtained from the method described above. This function serves to determine the random placement of array elements along its aperture. The transducer location $x(n)$ is a random variable that is drawn from the probability density function $g(x)$.

The next section compares the expected beam pattern and its variance as a function of u and determines the number of random ensembles and elements required to meet a set performance metric. The results for $g(x)$ are compared with the case for uniformly distributed element positions in the region where x : (0, 1).

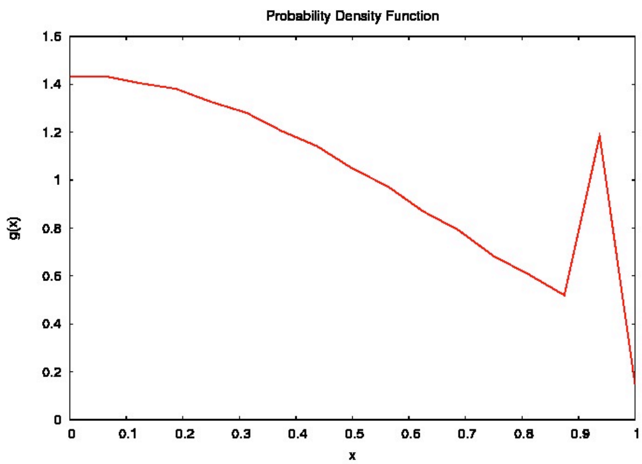

Fig. 2. Probability Density Function of Element Position x

IV. Beam Pattern Analysis

The beam response given by Eqn. 3 is a random function of the angle $u = kDsin(\theta)$. The expectation and variance with respect to u will vary with the number of array elements N. The convergence of these metrics will depend on the number of random ensembles of position vectors considered in the beam formation. Fig. 3 and Fig. 4 shows the rate of convergence of the maximum variance in u. Analysis was conducted for N ranging from 16 to 256. For each value of N, the number of ensembles denoted *NEN* was increased until the mean and variance converged. It can be seen that approximately 150 ensembles are required for convergences in both cases. However the magnitude of the variance is lower for the uniform distribution case.

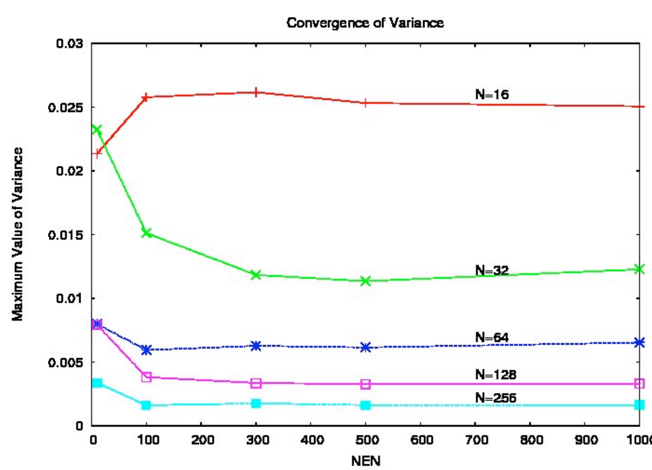

Fig. 3. Maximum Variance vs NEN for Uniformly Distributed x

The expected value $\mu_R(u)$ of the response and the one standard deviation $\mu_R(u) + \sigma(u)$ are shown in Fig. 5 for the uniformly distribution case. It is seen that there is a small probability that the first side-lobe level can fluctuate to levels above −30 dB although the successive sidelobe levels decrease with u.

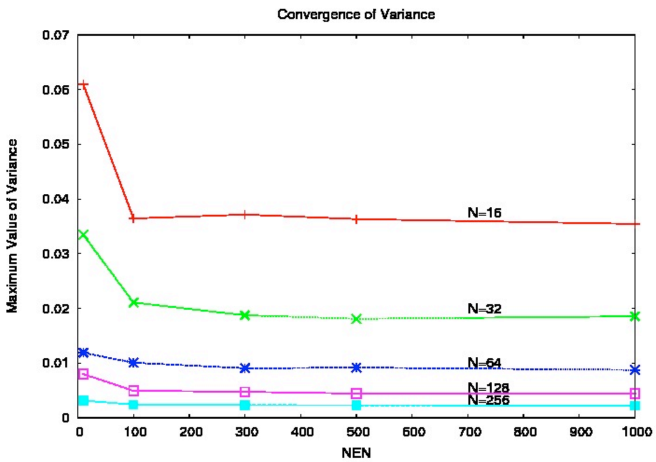

Fig. 4. Max Variance vs NEN for x Sampled from $g(x)$

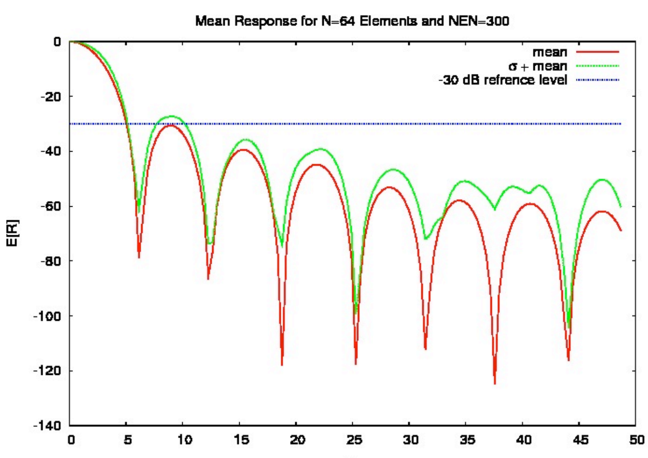

Fig. 5. Expected Beam Pattern and one σ deviation for Uniformly Distributed x

Fig. 6 shows $\mu_R(u)$ and $\mu_R(u) + \sigma(u)$ for the nonuniform case. Also shown in Fig. 6 is the target function given in Eqn. 4. The expected value is seen to match the target function well. The sidelobe amplitudes are all below the -30 dB level and are also of a uniform level in comparison with the results for uniform distribution.

Finally, Fig. 7 shows the manner in which the maximum variance decreases with the number of elements of the array. Although for small N the variance is lower for the uniformly distributed positions, the values are comparable with increase in N. The design considerations should take into account this tradeoff between the number of array elements and bound on the variance of the sidelobes levels.

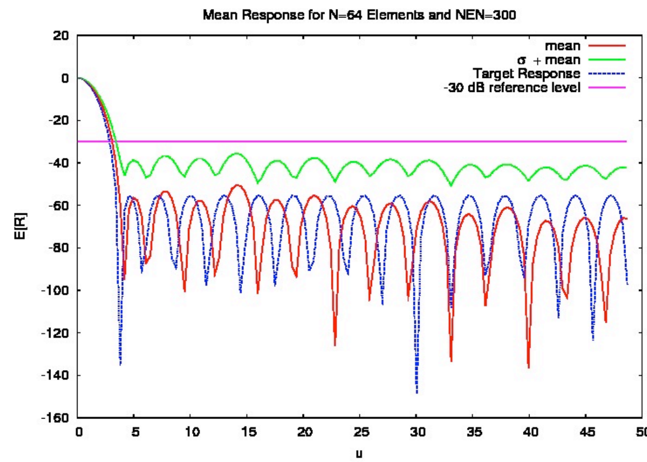

Fig. 6. Expected Beam Pattern and one σ deviation for Nonuniformly Distributed x

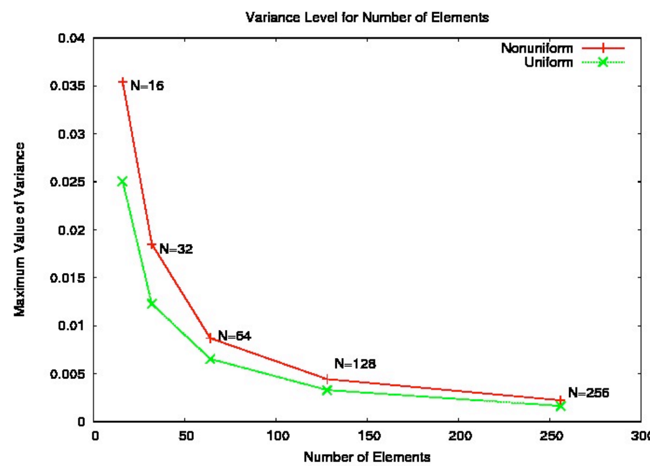

Fig. 7. Trade Off: Variance and Number of Elements

V. CONCLUSION

A statistical analysis of random beam patterns was conducted considering random positions of array elements. Two probability distributions, one uniformly distributed along the aperture length and another designed to patch a desired beam pattern were considered. The uniform case was found to deliver a smaller variance but the mean sidelobe level being non-uniform in amplitude with angle, could result in exceeding the threshold level of −30 db. The pdf designed to match a target beam pattern however results in a uniform sidelobe level and although the maximum variance was higher than the uniform case, with sufficient number of elements, the one standard deviation level $\mu_R(u) + \sigma(u)$ was below the reference level of -30 dB. To obtain lower variances for the nonuniform case, the number of transducers has to increase.

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