AC 2009-780: REAL OPTIONS AND THE USE OF DISCRETE AND CONTINUOUS INTEREST RATES

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Real Options and the Use of Discrete and Continuous Interest Rates

Abstract

Traditional engineering economics computes the net present value with a single interest rate. This is not the case in real options analysis. In options analysis, the present value of the benefits and the costs are needed in order to calculate the option value. In the last few years, the literature has had several examples where authors use multiple interest rates and different compounding assumptions for calculating present values. For example, "first" costs are almost always discounted using a continuous risk-free interest rate while later cash flows are often discounted using discrete market interest rates.

This paper focuses on the compounding assumptions. Two approaches are used: (1) Real option articles in *Harvard Business Review*, *Journal of Finance*, and *The Engineering Economist* are surveyed over matching periods to determine typical practices; and (2) A realistic delay option example is analyzed. The goal is to determine whether compounding assumptions are practically important or not. We conclude with a discussion of what should be taught in undergraduate and graduate engineering economy courses.

Introduction

Real options analysis differs from traditional engineering economics in that it attempts to provide a value for managerial flexibility. This value is called the option value, which is based on the mathematics used to determine the value of a financial option. The Black-Scholes equation is widely used to determine the value of financial options,[2] and has been adapted for use in real options. Binomial lattices can also be used to determine the value of either a financial or a real option, and some authors strongly advocate their use. Binomial lattices use discrete time steps to substitute for the Black-Scholes equation which uses continuous compounding. It has been demonstrated [12] that as the number of time steps increases, the binomial lattice method approaches the same answer as provided by Black-Scholes.

The Black-Scholes equation determines the value of a European call option, and is defined as:

$$C = S_0\phi(d_1) - Xe^{-rT}\phi(d_2) \tag{1}$$

where

$$d_1 = \frac{ln\left(S_0/X\right)+\left(r+\sigma^2/2\right)T}{\sigma\sqrt{T}} \qquad d_2 = d_1 - \sigma\sqrt{T} \tag{2}$$

The variables are defined as follows:

- C value of a call option
- S_0 value of the stock at the current time
- $\phi(d_x)$ cumulative standard normal distribution of the variable d_x
- X strike price
- r risk-free interest rate
- T time to option expiration

σ volatility of the stock price

The strike price is discounted to the present time using the risk-free rate of return, compounded continuously. The volatility is determined by one set method. The net result is that the Black-Scholes equation provides one unambiguous technique to determine the value of a call option.

Real options analysis is based on the same mathematics, but a new set of definitions:

C value of a deferral (or delay) option
S_0 present value of the future cash flows
$\phi(d_x)$ cumulative standard normal distribution of the variable d_x
X project cost
r risk-free interest rate
T time to option expiration
σ volatility of the project's rate of return

Unfortunately, the translation from financial options to real options adds several layers of ambiguity. In determining the present value of the future cash flows (S_0), what interest rate and what compounding technique should be used? In determining volatility, what method should be used? The answers depend on where you look. The management literature, engineering economics literature, and practitioner guides all present somewhat different answers. Unfortunately, there is no single approach to determining the value of a real option to guide the would-be practitioner.

In this paper, we examine the question of compounding: using either discrete or continuous interest rates to discount future cash flows, both positive and negative. In a later paper we will address the question of risk-free interest rates for some cash flows and market rates for other cash flows. The management and the engineering economics literature is first reviewed, along with practitioner guidebooks. A realistic deferral option is analyzed using both discrete and continuous compounding to determine whether a difference in compounding can change the investment recommendation. We approach this as engineering economists, where one of the first lessons is the connection between nominal and effective interest rates based on the number of compounding periods. Also one of the first principles is that nominal interest rates cannot be compared until converted to equivalent effective interest rates—which can be compared. We also approach this as researchers who followed typical practice where the interest rates were nominal rates—until we noticed that an answer that should have matched for two calculations did not. We conclude with a discussion of what should be taught in undergraduate and graduate engineering economy courses.

Literature Review

Harvard Business Review (HBR) was surveyed from 1998 through 2008. While this journal does not focus on mathematics, it contains some of the most influential articles on real options over the past ten years. One article[16] written for the practioner was reprinted as the real options chapter in Canada et al.[5] Luehrman describes discounting the future cash flows using a market interest rate using discrete compounding. While other HBR articles[1,11,17] discuss net present value and underlying project values, the other *Harvard Business Review* articles do not describe the method of discounting future cash flows.

The *Journal of Finance* was also surveyed from 1998 through 2008. There were several articles regarding real options, [6,7,8,9,13,14] and all started from a theoretical basis in differential equations. The perspective of the *Journal of Finance* authors was consistent: mathematics are based on calculus and discounting was performed on a continuous basis.

The Engineering Economist contains more real options articles (30) between 1998 and 2008 than any other single journal we have seen. Unfortunately, the approach to discounting future cash flows and future costs were not consistent from author to author. Overall, the split between discrete and continuous discounting of future cash flows is fairly even. However, the trend seems to be towards more use of continuous discounting.

Discounting future costs was also not consistent. The Black-Scholes equation uses continuous discounting of future costs, but many authors used binomial lattices. There was disagreement among those using lattices: some used continuous discounting (which was more common), and a few used discrete discounting.

Several books were also surveyed. Four practitioner guides were surveyed, [1,10,18,20] and they were consistent regarding the discounting of future values. Future net benefit cash flows followed discrete discounting, while future "initial investment" costs were discounted continuously. One widely used basic finance book [4] contained a chapter on real options analysis,\ and continued this approach. One engineering economics text [19] has a complete chapter on options analysis and also discounts future net benefit cash flows in discrete time while discounting future investment costs continuously.

There is no single approach to discounting in real options analysis. Those who focus on the use of calculus tend to use continuous mathematics throughout. Those who do not focus on calculus are divided, and it does not matter whether the approach uses Black-Scholes (a continuous application) or binomial lattices (a discrete time method). There is no agreement on a single approach to discounting in the literature to guide the practitioner.

Why This Lack of Consistency in Approach?

Does continuous compounding of cash flows make sense? For financial options, stock prices vary continuously in markets that are open essentially 24 hours a day. So for financial options, continuous compounding is appropriate. For engineering (real) projects, increased cash flows may be obtained from new products or product cost savings may be obtained from implementing a new technology. The improved cash flows may occur with every product that is sold, which occurs nearly continuously. So the use of continuous compounding can be justified in real engineering projects.

However, we suggest that two real drivers behind the use of continuous compounding in real options do not include the close link between continuous compounding and the above reality of distributed cash flows. Rather the two drivers are:

- Transfer of methodology from financial options to real options
- The greater mathematical ease of manipulating, differentiating, and integrating formulas containing e^r rather than $(1 + i)$.

Does discrete compounding of cash flows make sense? Our accounting systems are based on discrete increments of time. Budgets are set annually, and are usually updated monthly. Future information can not be obtained in increments smaller than what is reported, and in the case of real projects, this is done monthly or annually, not continuously. So discrete compounding may also be appropriate.

However, we suggest that the use of discrete compounding is not driven by the above link between compounding periods and accounting and budgeting. Instead we suggest that the key driver is spreadsheets — where existing formulas and cash flow tables are based on discrete compounding.

Given the above, it appears that the differences in compounding is driven by whether the application is theoretical or practical.

Since both the risk-free and the market interest rates are and should be based on the real world, they will be stated in consistent terms. Thus, we suggest that in reality either both rates are nominal interest rates or both rates are effective interest rates. These are being developed without reference to the kind of compounding that is being done.

There are three levels of consistency with interest rates in real options that we want to examine:

- Consistency with typical practice in the literature (continuous compounding for initial investment cost and common practice of discrete compounding for later “net” benefit cash flows)
    - Typical practice seems to conflict with the next level of consistency
- Consistency of interest rates with real world data
    - Stated as nominal interest rates ➔ ok for *r* in continuous, adjust *i* for discrete to match effective continuous rate
    - Stated as effective interest rates ➔ adjust *r* for continuous to nominal rate that gives that effective rate, ok for *i* as effective rate for discrete
- Consistency of interest rates for different types of cash flows (next paper).

Is the difference in compounding significant? To answer this we analyze a numerical example. Note: this example was created from our earlier work to analyze this question. It was not designed to show a problem exists.

Case Study [15]

A consumer products company is getting ready to start a new sunscreen product which blocks the sun's ultraviolet rays. Launching the product requires a current investment of $11.5 million. The company's hurdle rate for this type of project is 20%.

The company has recently identified a new sunscreen active ingredient, which is not yet available. Including it would delay the product's launch by one year. However, it would improve product efficacy and increase cash flows if it were used. The investment would be 5% higher if the project is delayed one year because the project would need to adopt a crash schedule. The life of the equipment and the new formulation technology is ten years in either scenario. Table 1 shows the expected cash flows and costs, along with their anticipated ranges. The present value (using the hurdle rate and discrete compounding) of the "Launch Now" option is -$1.45 million and the NPV of the "Delay 1 year" option is -$0.05 million. The NPV analysis recommends that the project be abandoned. The volatility of the project is estimated to be 0.136, or 13.6%, using the logarithmic present value returns method (as described in Copeland [10]).

Table 1. Sunscreen Cash Flows

	Cash Flows, $million		Lower	Upper
Year	Launch now	Delay 1 year	Limit	Limit
1	1.0	0.0	-40%	+20%
2	2.0	1.0	-40%	+30%
3	2.5	2.5	-40%	+40%
4	3.0	3.5	-40%	+40%
5	3.0	3.5	-40%	+40%
6	3.0	3.5	-40%	+40%
7	3.0	3.5	-40%	+40%
8	3.0	3.5	-40%	+40%
9	3.0	3.5	-40%	+40%
10	3.0	3.5	-40%	+40%
11	0.0	3.5	-40%	+40%
Initial Investment	11.50 million	12.08 million	-5%	+15%
Salvage value	0.75 million	0.75 million	-100%	+100%
Hurdle rate	20%	20%	-20%	+20%
Risk-free rate	5%	5%	-40%	+40%
Delay cash flow premium		0.5 million	-40%	+20%

Because this is a fairly simple delay option, the option value may be calculated using the Black-Scholes method. A binomial lattice could also be used, obtaining essentially the same answer if enough time steps were incorporated.

Matching the most common assumption in the literature and the Black-Scholes equation, the 5% risk-free rate was compounded continuously for the initial investment costs. Then the question is, does it matter whether the 20% market interest rate is compounded discretely or continuously for the delay option cash flows. For a valid comparison we must use the same effective interest rate. (Note: the compounding question also applies to the risk-free rate, but its much lower value means that the difference between nominal and effective interest rates is *much* smaller.)

If we assume the 20% is a nominal rate, the easiest way to convert from discrete to continuous compounding is to replace the discrete interest rate with its equivalent effective continuous rate of 22.14% using equation (3). If we assume that the 20% is an effective rate, then the direction of calculation is reversed and the nominal rate, *r*, for continuous compounding is 18.23%.

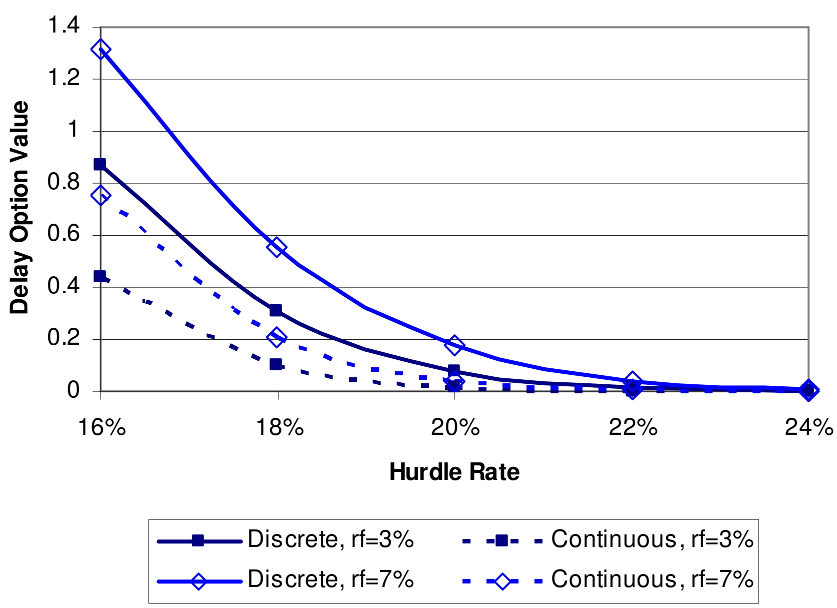
$$r_C = e^{r_D} - 1 \qquad (3)$$

Where r_D is the discrete interest rate

r_C is the equivalent effective rate compounded continuously

The option value was explored over the range of interest rates that would be expected within the case study. These were also investigated with three risk-free rates, again within the case's expected limits.

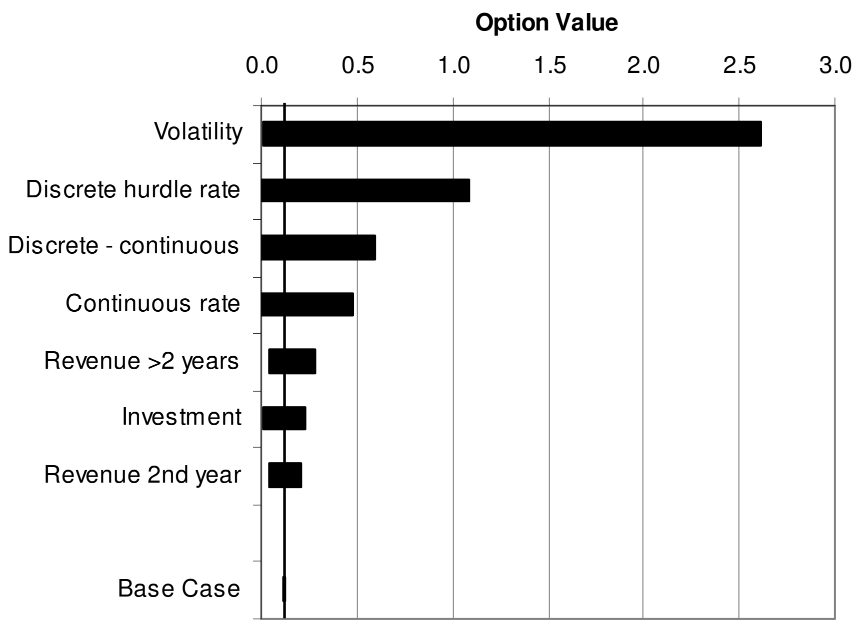
Figure 1. Effect of Discrete and Continuous Compounding, Sunscreen Project



The results are shown in Figure 1. As expected, continuous discounting of the positive cash flows decreases the option value. The impact is quite noticeable, especially at the lower hurdle rates. Real options have been criticized for overstating the value of a project, [3,21] and discrete compounding could be one source of this problem.

Is this a truly significant difference? Sensitivity analysis was conducted on the variables of the case, and a tornado diagram of the results is shown in Figure 2. Of the input variables, the volatility has the greatest impact on the option value and the hurdle rate (compounded annually) is the second most influential variable. Continuous discounting decreases the maximum option value, and the difference between discrete and continuous discounting is more significant than any remaining variable. *The compounding assumption has a significant effect on the value of the option.*

Figure 2. Tornado Diagram for the Sunscreen Case.



The expanded net present value (ENPV) is determined by adding the net present value with the option value. The ENPV is a primary tool for deciding whether to invest in the project. With discrete discounting of the future cash flows, an ENPV of $0.07 million results with a (weak) recommendation to invest. With continuous discounting of the cash flows, the ENPV drops to – $0.03 million, with a (weak) recommendation to abandon. Because real options analysis is used primarily when NPV is near zero, the method of discounting can change the decision outcome.

Recommendations for the classroom

We believe that a detailed study of real options analysis is not appropriate for an undergraduate course. Real options requires an understanding of sensitivity analysis and decision trees, and these should be taught at the undergraduate level; however, options analysis should probably be limited to making the student aware of its existence. We believe that real options does have a role to play in advanced graduate courses. While the use of options analysis remains controversial, it is a subject that the advanced engineering economics student should understand. If engineers are to take part in the debate, then we must first understand the methodology and test it on real world applications to projects. That is our domain as engineering economists and engineering managers.

Finance academics tend to view the world from the perspective of calculus and continuous equations. While this may be confining, it is at least consistent. As engineering economists, we deal with both discrete and continuous problems; this unfortunately can lead to inconsistency.

There is no single approach to solving real options problems. This lack of uniformity may be one of the many issues hindering more widespread adoption.

Most of the literature discounts future costs using continuous discounting. This is true when directly applying the Black-Scholes equation (and related models), and continuous discounting is the predominant technique when using binomial lattices. While not universal, continuous discounting of future costs is practiced by a strong majority.

For realistic numerical examples, spreadsheets are and will continue to be the modeling tool used. Thus compounding of future net benefit cash flows will be discrete. However, we can still be consistent with reality by adjusting all interest rates to be an effective rate first. This may require adjusting the "market" interest rate or the "risk-free" rate.

Conclusions

There is no uniform approach to using either discrete or continuous discounting when using real options. The business literature consistently uses continuous discounting, in keeping with their calculus-based approach to options analysis. Practitioner books are consistent in continuous discounting of future costs while using discrete discounting of future positive cash flows. The engineering economy literature is not consistent.

The use of discrete or continuous interest rates can have a significant effect on the value of the option, and the choice of compounding method can change the decision outcome. This is not a meaningless topic. We recommend ensuring that consistent assumptions about compounding and effective interest rate values be made.

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