Real-World vs. Ideal Op-Amps: Developing Student Insight into Finite Gain-Bandwidth Limitations and Compensation

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Abstract

In learning circuit design using operational amplifiers (op-amps), most Electrical Engineering students are taught about inverting and non-inverting amplifier design, summing amplifier design, and simple filter design (first and second order), all assuming the use of an ideal operational amplifier. While this is a great place to start a design discussion, students can be left with the lingering impression that a µA741 op-amp (which worked fine for anything they needed to accomplish in their carefully scripted labs from sophomore or junior year) is a generic op-amp that will do anything they will ever need to do!

This paper presents some of the successful design and compensation techniques from one laboratory in a junior-level Linear Circuits class at the U.S. Coast Guard Academy. In this lab, students are asked to design two Sallen-Key second order low pass sections, using the µA741 op-amp, in order to meet two specific desired resonant frequencies. Students usually can meet a design specification at low frequency on their first attempt; however they typically fall short of the resonant frequency for the higher resonant frequency by 15% or more. This creates a teachable moment, as students reflect on their success in one design, and their failure to meet specifications in another design. In this paper a graphical technique is introduced that helps students to predict the impact of Gain-Bandwidth Product limitation of a µA741 op-amp.

Recognizing that the real world op-amp itself can be modeled as a first order transfer function, this paper presents graphical techniques that can be used early in the design process to “pre-compensate” for gain-bandwidth product limitations. Using these techniques, students are then able to meet specifications with their µA741 op-amps. Finally, using a more expensive (LM318) op-amp, with 15MHz gain-bandwidth product illustrate that minimal (if any) compensation that is required to meet the design specification.

This paper illustrates measurement results, along with some of student feedback that suggests this exercise does reinforce their learning with respect to real-world characteristics of op-amps.

Introduction

Electrical Engineering students typically learn about operational amplifiers in their sophomore or junior years. First they learn the “ideal op-amp assumptions,” and begin to analyze circuits by assuming these devices have infinite input impedance, zero output impedance, and an infinite gain-bandwidth product. To reinforce student learning, faculty often develop laboratory exercises by carefully choosing signal amplitudes and frequencies so that slew rate or gain-bandwidth product limitations are never observed for the op-amp selected. Even “ideal op-amp
assumptions” are a reasonable place to start a discussion about circuit design; however, students should learn the conditions under which those assumptions are valid.

Some of the early ideas for considering the effects of design with “real-world” op-amps introduced by Peterson, Hartnett, and Gross at the U.S. Coast Guard Academy, however the authors primarily focused on the theoretical analysis and did not discuss measurement methodologies. Classic references such as Schaumann and Van Valkenburg and Van Valkenberg also discuss the idea of “one-pole roll-off” and “two-pole roll-off” real-world op-amp models, but present no measurements or methodologies. In this paper is the extended of our laboratory in Linear Circuit course at the U.S. Coast Guard Academy.

The current paper presents some of the successful design and compensation techniques from one laboratory in a junior-level Linear Circuits class that reinforces student learning about finite gain-bandwidth product limitations. Students learn graphical technique in the classroom that predicts the impact of Gain-Bandwidth Product limitation of a µA741 op-amp.

In this lab, students are asked to design two Sallen-Key second order low pass sections, using a µA741 op-amp, in order to meet two specific resonant frequencies. In their first design \((f_o = 72.3\text{kHz and } Q = 2)\), students typically fall short of the desired resonant frequency by 15% or more, while coming reasonably close to their desired \(Q\). In their second design \((f_o = 7.23\text{kHz and } Q = 2)\), students typically come very close to meeting the desired resonant frequency and \(Q\) specifications on the first try. This creates a teachable moment as students reflect on their failure in design #1, and their success in meeting specifications in design #2. First our students calculate a parameter we call “\(G\)”, which is the gain-bandwidth product of the op-amp relative to the desired resonant frequency. Using this calculated value \(G\), they employ graphical techniques early in the design process to predict how much the closed-loop transfer function pole locations will shift from the desired locations because of a finite value for \(G\). This information can then be used to “pre-compensate” for gain-bandwidth product limitations. Using these techniques, students are then able to meet specifications for design #1 with their µA741 op-amp. Finally, students use a more expensive (LM318) op-amp, with 15MHz gain-bandwidth product, to illustrate that minimal (if any) compensation is required to meet design specification #1.

This paper is organized as following. First the Theoretical Background is presented. Then the laboratory procedures and its results are presented this procedures are discussed in five subsections. Finally, the conclusion of these results is summarized.

**Theoretical Background**

A 2\text{nd} order Sallen-Key low pass section is shown in Figure 1. This circuit is used for this laboratory.
Figure 1: Sallen Key low pass filter

The system transfer function $H(s) = \frac{V_{\text{out}}}{V_{\text{in}}}$ for this filter (assuming an ideal op-amp) is:

$$H(s) = \frac{A_0 \omega_0^2}{s^2 + (3 - A_0) \omega_0 s + \omega_0^2}$$  \hspace{1cm} (1)

The relationship between the desired resonant frequency ($\omega_0$), the resistors (R), and capacitors $C_1 = C_2 = C$, is

$$\omega_0 = \frac{1}{RC}$$  \hspace{1cm} (2)

The DC gain of the filter, $A_0$, is calculated as

$$A_0 = 1 + \frac{R_b}{R_a}$$  \hspace{1cm} (3)

where $R_a$ and $R_b$ are shown in Figure 1. The quality factor, $Q$, is calculated as

$$Q = \frac{1}{3 - A_0}.$$  \hspace{1cm} (4)

Assume a real-world op-amp ($\mu$A741) with a 1MHz gain-bandwidth product and a DC gain of 200,000. The Sallen-key circuit in Figure 1 can be modeled as a “non-inverting amplifier” portion shown in Figure 2. The transfer function, $A(s) = \frac{V_{\text{out}}}{V_{\text{in}}}$ for this circuit is given by
\[ A(s) = \frac{\text{GBW}}{s + \text{GBW}} \left( \frac{\text{GBW}}{\omega_o} \right) = \frac{s}{\omega_o} + \left( \frac{\text{GBW}}{\omega_o} \right), \]  

where GBW is the gain-bandwidth product of the op-amp (expressed in rad/sec), and the non-inverting amplifier gain becomes \( A(s) \) instead of the constant \( A_o \).

Figure 2. Non-inverting amplifier portion of Sallen-Key circuit from Figure 1.

The variable \( G \) is defined as the gain-bandwidth product (in radians/sec) relative to the designed resonant frequency (\( \omega_o \))

\[ G = \frac{\text{GBW}}{\omega_o} \]  

and

\[ A(s) = \frac{G}{s/\omega_o + G/A_o}. \]  

Assume \( \omega_o = 1 \) and substitute \( A(s) \) from (6) into (1),
The 2nd order Sallen-Key section now exhibits a 3rd order transfer function characteristic because of the “real-world” op-amp first order open-loop transfer function. Of particular interest is to plot the complex poles of the transfer function from (9) as a function of the gain-bandwidth product relative to resonant frequency ($G_o$). This plot is shown in Figure 3. This graph shows the second quadrant of the s-plane that can be used to predict how much the pole location ($\omega_o$, $Q$) shift from their desired pole location, due to using a real-world op-amp with a finite value for $G$. Curved “vertical” lines in Figure 3 represent lines of constant $Q$, while curved “horizontal” lines represent lines of constant $G$.

Example 1:

Design a 2nd order Sallen-Key section from Figure 1 for a $Q=2$ and $f_o=72300$Hz, using a µA741 operational amplifier with a 1MHz gain-bandwidth product. Calculate the expected values for $Q$ and $f_o$.

Solution:

The desired pole location corresponds to $Q=2$ and (normalized) $\omega_o=1$ (shown as the “X” at radius of 1 at $Q=2$ in Figure 2). From (6) $G = \frac{1 \times 10^6}{72300} = 13.8$, so the DESIRED pole location (upper “X” in Figure 3) shifts to the intersection of the $Q=2$ curve and a $G=13.8$ curve, resulting in an ACTUAL pole location (lower “X” in Figure 3). This predicted actual pole location at a radius of $\omega_o = 0.82$ ($f_o = 0.82 * 72.3kHz = 59.3kHz$) and a $Q = 2.2$, as seen by the lower pole location (X) in Figure 3.

\[
H(s) = \frac{V_{out}}{V_{in}} = \frac{G}{s + \frac{G}{A_o}} \left( s^2 + \frac{3 - \frac{G}{A_o}}{s} \right) + \frac{1}{s + \frac{G}{QA_o}}
\]

(8)

\[
H(s) = \frac{G}{s^3 + \left( 3 + \frac{G}{A_o} \right) s^2 + \left( 1 + \frac{G}{QA_o} \right) s + \frac{G}{A_o}}.
\]

(9)
Note that as $G \to \infty$ (ideal op-amp assumption), error becomes negligible between desired and actual pole location for this Sallen-Key low pass filter section.

**Laboratory Procedures**

The lab procedure in this lab consists of five steps:

1. Design a Sallen-Key low-pass filter section with a µA741 op-amp for a resonant frequency of 72.3 kHz and $Q = 2$, given capacitors of 220pF (design #1).
2. Design a Sallen-Key low-pass filter with a µA741 op-amp for a resonant frequency of 7.23 kHz and $Q = 2$, given capacitors of 220pF (design #2). (Note design #2 is a factor of 10 lower in frequency than design #1.)
3. Construct these circuits, and measure the magnitude and phase responses of your designs in steps (1) and (2), and compare the measured results with MultiSim™ and Matlab™.
4. If it is observed that either design does not meet resonant frequency or Q specifications, re-design that Sallen-Key low-pass filter section to meet specifications.
5. Finally, return to the original designs from steps (1) and (2), and replace the µA741 op-amp with a LM-318 op-amp. Measure the magnitude and phase responses of your designs, and compare with earlier results.
After verifying proper circuit operation, students use the Agilent 35670 Dynamic Signal Analyzer (DSA) to measure magnitude and phase responses. This DSA is a two channel analyzer, which has a source output (in this case, a periodic chirp applied to the circuit input), and is able to perform simultaneous measurement of circuit output and input (and hence frequency response) from DC out to 51.2kHz. Unfortunately a wider frequency range is needed for step (1) in this lab, so the students configure the analyzer to single channel mode, allowing a single channel frequency domain measurement from DC to 102.4kHz. The initial DSA setting has shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>On/Periodic Chirp</td>
</tr>
<tr>
<td>Window</td>
<td>Uniform</td>
</tr>
<tr>
<td>Input</td>
<td>200 mVpk</td>
</tr>
<tr>
<td>Measure</td>
<td>Magnitude or Phase Response (Ch1)</td>
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<tr>
<td>Freq</td>
<td>Start/0&amp;Span/102.4k Hz</td>
</tr>
<tr>
<td>Trigger</td>
<td>Source trigger</td>
</tr>
</tbody>
</table>

Table 1
Initial DSA Setting for Resonant Frequency Response Measurement

Students configure the analyzer to trigger on the periodic chirp, and they store the complex frequency domain data from the circuit input into one of the DSA data registers. They then move the probe to measure the circuit output, and configure the analyzer to display a “math function.” This math function enables the analyzer to perform the (complex) mathematical function of measuring the output complex frequency domain data, and dividing by the (stored) input complex frequency domain data, thereby computing real-time complex frequency response. Students then choose to display the magnitude as one trace, and the phase as the other trace, and then transfer data to Matlab via an IEEE-488 to USB connection between instrument and computer.

A. Measure the frequency response (both magnitude and phase) for Sallen-Key design#1 (with desired resonant frequency of 72.3 kHz and $Q = 2$). Students design and build the Sallen-Key low-pass filter shown in Figure 1 using a µA741 op-amp, and assuming that $C_1 = C_2 = 220 \ pF$, and using equations (2)-(4), they found $R = 10k\Omega$, $R_o = 2k\Omega$, and $R_b = 3k\Omega$.

The measurement magnitude and phase results are shown in Figure 4. The frequency response of this circuit shows that the measured resonant frequency is 60.16 kHz and the measured $Q = 1.93$. Note that the students obtain an approximate value for the resonant frequency by measuring the frequency at which the phase response transitions through -90 degrees. Students measured the exact gain-bandwidth products for the op-amps they used in a previous lab, and for results shown here, the measured gain-bandwidth product was 985.6 kHz.
As mentioned in Example 1, the predicted values for resonant frequency and $Q$ are calculated as follows:
\[ G = \frac{985.6 \times 10^3}{72.3 \times 10^3} = 13.63 \]  \hspace{1cm} (5)

The desired pole location corresponds to \( Q = 2 \) and (normalized) \( \omega = 1 \), however our pole shifts to the intersection of the \( Q = 2 \) curve and the \( G = 13.63 \) curve, resulting in an actual predicted pole location corresponding to \( f_\omega = 0.822 \times 72.3 \text{kHz} = 59.4 \text{kHz} \) and a \( Q = 2.17 \). At this point, students realize that the theory is predicting the results they are measuring, and they begin to brainstorm how one might modify the design in order to meet the resonant frequency specification.

**B. Measure the frequency response (both magnitude and phase) for Sallen-Key design #2 (with desired resonant frequency of 7.23 kHz and \( Q = 2 \)).** Students design and build the Sallen-Key low-pass filter shown in Figure 1 using a \( \mu \)A741 op-amp by realizing that the only change needed to lower the resonant frequency from the previous design was to set \( R = 100k\Omega \). Measured magnitude and phase response measurements for Sallen-Key design #2 are shown in Figure 5.

From Figure 5 they find the measured resonant frequency is 7.36 kHz and \( Q = 1.78 \). Note that the resonant frequency is within 2% of the desired frequency for this design. This result also could be predicted from Figure 2. The gain relative to the resonant frequency \( f_\omega = 7.23 \text{kHz} \) is \( G = 138 \). The intersection of \( Q = 2 \) and \( G = 138 \) in Figure 2 gives a normalized resonant frequency of \( f_\omega = 0.98 \). Substituting this value into (6) gives the predicted resonant frequency as \( f_\omega = 0.98 \times 7.23 = 7.09 \text{kHz} \), also within 2% of the desired resonant frequency.

The students’ scores ranged from 65% to 98.75% for parts A and B. The average grade was 88.75%, and 20 out of 22 of students scored greater than 70%. Overall, 90.9% of our students scored above a 70% comprehension level for this lab.
Figure 5. Measured results of Sallen-Key circuit frequency response for design #1
\((f_0 = 7.23 \text{ kHz}, \text{ and } Q = 2.)\)

C. **Compare the measurement results with MultiSim\textsuperscript{TM} and Matlab\textsuperscript{TM}:** This part of the lab consists of using MultiSim\textsuperscript{TM} circuit analysis and Matlab\textsuperscript{TM} to model the real-world op amp frequency responses for both designs, and compare those responses to the measurement results. As you might imagine, measured results compare very favorably with MultiSim\textsuperscript{TM} results if students remember to use a \(\mu A741\) op-amp (instead of an ideal op-amp) for the simulation. The results of theoretical frequency response in Matlab\textsuperscript{TM} (using both an ideal and real op-amp) compared with the measurement data for the resonant frequencies of 72.3 and 7.23 kHz are shown in Figures 6 and 7, respectively.
Students’ scores ranged from 75% to 100%. The average grade was 93.2%, and 22 out of 22 of students scored greater than 70%. In average 100% of students’ grade was above 70%.

Figure 6. Measured frequency response (red), predicted response using ideal op-amp (green), and predicted response using real op-amp (blue) for Sallen-Key section ($f_o = 72.3$ kHz).

Figure 7. Measured frequency response (red), predicted response using ideal op-amp (green), and predicted response using real op-amp (blue) for Sallen-Key section ($f_o = 7.23$ kHz).

D. Redesign the Sallen-Key low-pass filter from step (1) to meet the resonant frequency specification of 72.3 kHz: At this point, students are looking for “the magic answer” regarding
how to proceed, and instructors would provide little guidance, other than to encourage them to brainstorm. Soon students start sharing ideas, and usually arrive at the idea of designing for a higher resonant frequency within a few minutes. Some use an intelligent “guess and check” method, and some use an arithmetic strategy, where if their first design was B Hz below the desired frequency, they would add B Hz to the desired frequency in iteration #2. Some students choose to use a logarithmic strategy, such that if the resonant frequency for design #1 ended up a factor of $K$ too low, the next design frequency would be $(1/K)$ times the design frequency. Some of the most insightful students then begin to realize that the larger they try to make $G$, the smaller $G$ becomes, and the more the poles shift from the original desired locations. These students often begin to think about writing some code to perform the iterative design, and some begin to ponder closed form solutions to the problem. No matter how students tackled the problem, most likely, they were able to compensate, arriving at a final $f_o = 72.3 \text{kHz}$ by using a value of $R$ of roughly $R = 7.2k\Omega$.

The data from students’ performance in this part of lab showed 17 out of 22 students understood how to modify their designs in order to meet the specifications for this part of lab. For this part of the lab, 77.3% of students scored higher than our 70% of achievement threshold.

E. Finally, measure the Sallen-Key low-pass filter original design with LM-318 op-amp for the given specification in step one: Here, instead of using iterative methods to design for a higher frequency, students solve the gain-bandwidth product challenges by designing the circuit with a LM318 op-amp. The LM318 has a gain-bandwidth product of 15MHz.

In this part of lab students are not required to predict their results. Assuming a typical 15MHz gain-bandwidth product, $G = 207.5$, and the intersection of $Q = 2$ and in Figure 2 gives a normalized resonant frequency of $f_o = 0.985$. Substituting this value into (6) gives the predicted resonant frequency as $f_o = 0.985 \cdot 72.3 = 71.2 kHz$. The measured resonant frequency for the response using the LM318 is 70.7 kHz and $Q = 2$ (Figure 8), which compares reasonably well with the predicted result, and is a significant improvement over the response obtained from the $\mu$A741. Unlike the previous iterative solution, this method is simpler and does not require additional calculations and circuit redesign. The cost, of course, is that the design requires a more capable (expensive) op-amp. Naturally our students thought the LM318 design experience was better, however they appreciated the learning experience where they all witnessed that (1) not all operational amplifiers perform the same, and (2) theory can be used to predict results and/or choose an appropriate op-amp. The measurement of frequency response for the LM318 design is shown in Figure 8.

Students’ scores ranged from 60% to 98.6% for this part of lab. The average grade here was 85.45%. , and 20 out of 22 of students scored above our 70% threshold.
Conclusion

This paper presents some of the successful design and compensation techniques from one laboratory in a junior-level Linear Circuits class at the U.S. Coast Guard Academy. Students
learned first-hand how finite gain-bandwidth limitations can limit bandwidth in a Sallen-Key second order section, and they learned two ways to deal with these limitations. Students also learned that one can use graphical technique either to perform compensation, or to predict results when using real-world op-amps with finite gain-bandwidth products. Finally, students’ feedback coupled with hourly examination results suggest to us that this laboratory does reinforce student learning with respect to real-world op-amp applications.

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References: