

Redesigned Application-oriented Integral Calculus Curriculum

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Selected Recent Publications

Theresa Atkinson, Leszek Gawarecki, Massoud Tavakoli (2016) "Paired Vehicle Occupant Analysis Indicates Age and Crash Severity Moderate Likelihood of Higher Severity Injury in Second Row Seated Adults in Frontal Crashes" *Accident Analysis and Prevention*, 89, 88–94,

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Redesigned Application Oriented Integral Calculus Curriculum

Abstract

This paper presents the development of an application based curriculum for an integral calculus course, a project funded by the KEEN Foundation. Textbook examples are frequently disconnected from students' immediate environment or use past data of little interest. In addition, information given on the subject is at most sketchy and the practical purpose of solving these examples is not clear. This lack of vivid applications in calculus courses motivated us to develop content that can be used by instructors to enhance students' learning experience by engaging them directly in solving problems and applying attained skills to real life situations relevant to the students' environment. The main focus of this work is to make the integral calculus course current, engaging, and relevant for students. The objective is to create a learning environment where calculus comes to life through real world examples, real and relevant data, and through the use of physical objects. We do not adopt the Project Based Learning approach however, but rather supplement the traditional lecture with motivating examples and multiple projects. Our pedagogical approach includes curriculum enhancement, deep learning, student engagement and entrepreneurship. We provide examples and projects which have the potential to capture students' attention and will be useful in other courses in mathematics, science, and engineering. This approach is intended to spark curiosity in students, demonstrate usefulness of the theory they study, and to answer the question "Where am I going to use this?" We divided the curriculum into three fundamental modules: (1) Integration of basic functions resulting from mathematical models and from observed data; (2) Advanced applications of integration; (3) Applications of infinite sequences and series. The proposed course learning outcomes (CLOs) are linked to KEEN Student Outcomes, particularly those that are measurable. Assessment will include student surveys, retention, grades, achievement of CLO's, and quality of students' projects. In order to keep a balance between mathematical rigor and engineering and science relevance, the team was designed to consist of engineering and mathematics faculty. Similar lesson developments are conducted in single variable differential calculus and multivariate calculus to create a consistent approach in the entire calculus sequence. All motivating examples and mini projects for the entire calculus sequence are available on line at <https://newb.kettering.edu/wp/experientialcalculus/>.

Introduction

The major goal we attempt to achieve is enhanced student learning experience through creating a learning environment where students are engaged. Student engagement is addressed in the literature with increased emphasis and frequency. The NSSE (National Survey of Student Engagement ⁶) specifies four themes: Academic Challenge, Learning with Peers, Experiences with Faculty, and Campus Environment. In our work we targeted the theme of Academic Challenge, which includes four engagement indicators: Higher-Order Learning, Reflective and Integrative Learning, Learning Strategies, and Quantitative Reasoning. We attempted to improve in our students taking calculus courses the Higher-Order Learning component: Applying facts, theories, or methods to practical problems or new situations, the Reflective and Integrating

Learning component: Combining ideas from different courses when completing assignments, and the Quantitative Reasoning component: Reaching conclusions based on own analysis of numerical information.

Following the revised Bloom's taxonomy of educational objectives, we targeted levels three – Applying, and four – Analyzing, Anderson *et al.*¹, Bloom *et al.*², or level three – Relativism, as proposed in Gainsburg⁵, specifically its components: Attempts to relate mathematical procedures to physical phenomena, to guide solving, and Appreciates instructor's derivations.

College success, student learning and student engagement are generally viewed as correlates, Chen *et al.*⁴, Kuh *et al.*⁹. In fact, authors agree that “student engagement is a precursor for knowledge and understanding”, Shulman, L.S. as cited in Kuh *et al.*¹⁰. Student engagement can be seen as predominantly influenced by the institutional context (i.e. facilities, from libraries and laboratories to residence halls, and administrative support) and by the faculty, through the quality of instruction and student-faculty interactions. The reader will find interesting historical perspective on student engagement in Chen *et al.*⁴, where the authors discuss the role of student-faculty interactions, curricula, and classroom learning.

Similar need for connecting students to community and real world problems is actively researched for other levels of education. For example, the K-12 level in Bouillion *et al.*³, Taylor *et al.*¹⁵, and graduate education in Newswander *et al.*¹¹ (see also references therein).

We have developed a new approach to teaching Calculus II by providing academically challenging motivating examples and projects demonstrating the use of calculus methods. We have created supporting instructional materials. The course content has been divided into three modules:

- Module 1. Integration of basic functions resulting from mathematical models and from observed data.
- Module 2. Advanced applications of integration.
- Module 3. Applications of infinite sequences and series.

Every module contains leading motivating examples and mini projects to be selected by instructors and students. The motivating examples are meant to be introduced at the beginning of a module and developed throughout, while the mini project should be assigned to demonstrate specific methods.

The justification for our work comes from the fact that one of the biggest challenges in teaching calculus along with the conceptual understanding of the subject is showing its relevance to student experience in real life situations. The textbook by Smith and Moore¹² attempts to provide engaging examples for single variable calculus. Smith¹³ emphasizes using projects as an important pedagogical approach. Clutier *et al.*⁵ provide an example of an outreach program where short instruction was followed by a variety of practical activities. A similar attempt to our design is available at <https://services.math.duke.edu/education/ccp/>.

Each mathematics instructor has faced a student who has posed this or a similar question “Where am I going to use this stuff?”, Krantz³. The answers range from a dispassionate “I don't know.

This is not my problem.” to an informed, for example, “Riemann sums will be used to derive formulas for volume and surface area of a solid. They are useful for solving many problems in physics and engineering, examples will be provided in this course.”

The use of real examples, data, and physical objects, helps the instructor to provide an informed answer to students’ questions about applications of the theory and decrease the gap between the theory and practical applications.

- We have used real world problems to motivate students, to spark their curiosity, and show the course relevance in real life situations.
- We have used examples from science, engineering, economics, and finance to demonstrate the relationship of integral calculus to various broad areas of interest to students.
- We have used examples from future courses, especially in the area of engineering, to justify calculus methods being studied and to instill engineering mindset.
- We have created only three modules with practical applications to help students focus on major techniques without dealing with minute details of separate sections.

The learning outcomes include, but are not limited to, the proposal’s connections to selected KEEN Student Outcomes, particularly those that are measurable.

The following student and KEEN outcomes are expected:

1. Students will understand how calculus is used to support solutions to real life problems that they find relevant.
2. Students will be exposed to highly motivating examples which will increase engagement and facilitate deep learning.
3. Through creating working groups, sharing responsibilities, and solving a complex problem together, students will develop collaboration and communication skills.
4. Students will apply creative thinking to solving ambiguous and complex problems with unknown solutions.
5. Students will learn teamwork skills through productive collaboration in the framework of a working group setting.
6. The KEEN network will have access to these materials to incorporate into their coursework.

Module 1. Integration of basic functions resulting from mathematical models and from observed data.

This module is designed to motivate students to learn how to use Riemann sums. Many students who study integral calculus already know some theory of integration, those who do not quickly understand the major dilemma. Why study Riemann sums when in just a few days they will learn much easier technique of finding antiderivatives using the Fundamental Theorem of Calculus (FTC)? A typical answer to this question is that Riemann sums will be used when certain measures need to be computed, like “lengths of curves, volumes of solids, center of mass, force due to water pressure, and work, as well as other quantities.”¹⁴.

There are three motivating examples and five mini projects in Module 1.

Motivating Examples,

1. The electric field of a line of charge.
2. Work done by a force in moving an object.
3. Volume of a solid.

Mini Projects,

1. Automobile velocity.
2. Crash safety test.
3. Design of a rollercoaster.
4. Gini index.
5. Total cost and profit.

In the electric field of a line of charge example, a thin, uniformly charged rod of length L with total charge Q generates an electric field. Students are guided to find the strength of the field at distance d in the plane that bisects the rod.

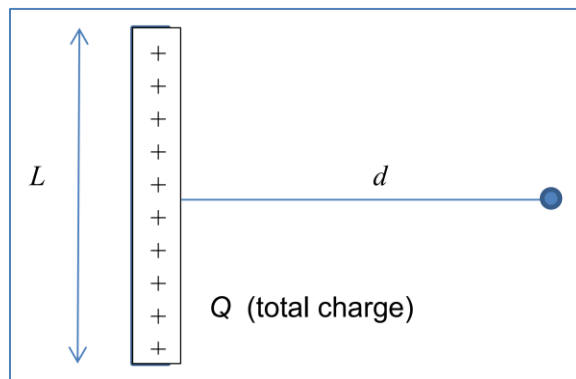


Figure 1 Line of Charge

The total charge Q is divided into many small point-like charges ΔQ located at points y_i and the electric field from each of them is determined. The net field can be found by summing the fields of all the point-like charges ΔQ and forming a Riemann sum. By taking the limit as the number of point-like charges ΔQ increases to infinity, the Riemann sum will converge to a definite integral. The integral can be evaluated using trigonometric substitution and trigonometric integration.

$$E^x = \frac{(Q/L)(d)}{4\pi\epsilon_0} \sum_{i=1}^n \frac{\Delta y}{(y_i^2 + d^2)^{3/2}} \rightarrow \frac{(Q/L)(d)}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{dy}{(y^2 + d^2)^{3/2}} \quad (1)$$

This example can be presented to students very early in the course. Students can be asked to work in small groups and draft their solutions to this problem. Most likely, they will not be able to make any substantial progress. During the course, students can be reminded about this

example. As they learn about Riemann sums and techniques of integration, their appreciation for the course may grow since they will observe that they are learning useful material. This motivating technique is described in Section 3.6.3 in Felder⁷.

During the course students will choose one mini project from every module. The mini projects cover a wide variety of topics. For this article, the authors describe the crash safety test. This project demonstrates the following concepts in integral calculus;

1. The concept of *numerical integration as a discrete and finite Riemann sum* (that is, the evaluation of a definite integral using a finite set of discrete values rather than the anti-derivative of a function).
2. The concept of the *anti-derivative of a function* and *specific integration techniques*.
3. The calculation of the *definite integral using the anti-derivative of a function*.

Other key concepts students will be exposed to are;

1. The practical concept of *functions defined only via data table*, rather than via an equation or formula for the function.
2. The concept of *a family of functions with a parameter* and their graphs.
3. The concept of *interpolating a set of experimentally obtained data values* to improve on the precision of the numerical calculations above.
4. The concept of *data modeling (curve fitting)* in an attempt to come up with an equation (formula) that models the discrete set of data points and generates a continuous curve that best matches the observations.
5. The concept of the *average value of a function*.

Students are asked to calculate the Head Injury Criterion (HIC). The HIC number is based on the average value of time-dependent acceleration $a(t)$ experienced by the head of a person during an impact. The HIC associates different likelihoods of head injury to different ranges of values of the HIC number.

In a car safety crash test, *anthropomorphic test devices*, or simply *dummies* are placed in the driver's and/or passenger's seat(s). A demonstration video is available for students at <https://www.youtube.com/watch?v=kj9xqrRskrY>.

The HIC number is defined in equation (2) as follows,

$$\text{HIC} = \text{Max}[(t_2 - t_1) \times (\bar{a})^{2.5}] = \text{Max} \left\{ (t_2 - t_1) \times \left[\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) dt \right]^{2.5} \right\} \quad (2)$$

where the maximum is taken over all time intervals.

The project requires students to download the LoggerPro software by Vernier (available for free to our students) and to download real data from NHTSA SaferCar.gov data base. The authors emphasize the use of real data as much as possible. The students need to calculate the HIC number using Riemann sums over selected intervals and confirm the result using LoggerPro in two ways; using the built in integration feature and by fitting a curve of the from

$$a(t) = \frac{b}{(t-c)^2+d} \quad (3)$$

and integrating the fitted function using learned techniques of integration. This project, just like the motivating example, can be assigned at the beginning of the term and revisited as students acquire more skills.

Module 2. Advanced applications of integration.

We designed this module to show the vast applicability of integrals to solving real world problems, emphasizing those applications that are relevant to our students. Some of the practical examples we considered in this module are applications to;

1. Business and Economics – Capital Value.
2. Volume of a solid.
3. Statistics – Computing Probabilities.
4. Modeling.
5. Medicine – Cancer Treatment.

Motivating Example – Volume of a Solid

Our science and engineering students take an engineering graphics course in their freshman year using Computer Aided Design (CAD) software. To make calculus more interesting, a motivating example, calculation of the volume, is designed to demonstrate two different approaches. One approach is to calculate the volume using pure mathematics student learn in integral calculus. The other approach is to employ CAD, a tool that students use in many courses as well as in their co-op jobs in engineering design and analysis. Results of these approaches can be compared to validate their solutions, and many interesting competitions and games can be integrated in these activities to make the calculus class fun. This case study describes the following steps and concepts in integral calculus;

1. Definition of volume.
2. Calculation of volume by integral.
3. Computer Aided Design (CAD).
4. Parametric design of a solid.
5. Determination of volume in CAD.
6. Comparison of the results in #2 and #5.

Definition of Volume

The volume of a solid obtained by rotating the area bounded by $y = \sqrt[3]{x}$, $x = 8$, and $y = 0$ about x-axis can be computed using calculus. The area of the cross section of the solid through x is

$$A(x) = \pi y^2 = \pi(\sqrt[3]{x})^2 = \pi x^{2/3} \quad (4)$$

The volume of the solid is

$$V = \int_0^8 A(x) dx = \int_0^8 \pi x^{2/3} dx = \frac{96\pi}{5} \quad (5)$$

Determination of Volume by CAD

In the parametric design of CAD, mathematic equations can be used to define and generate curves. Figure 1 shows the region bounded by the same curve and lines described above.

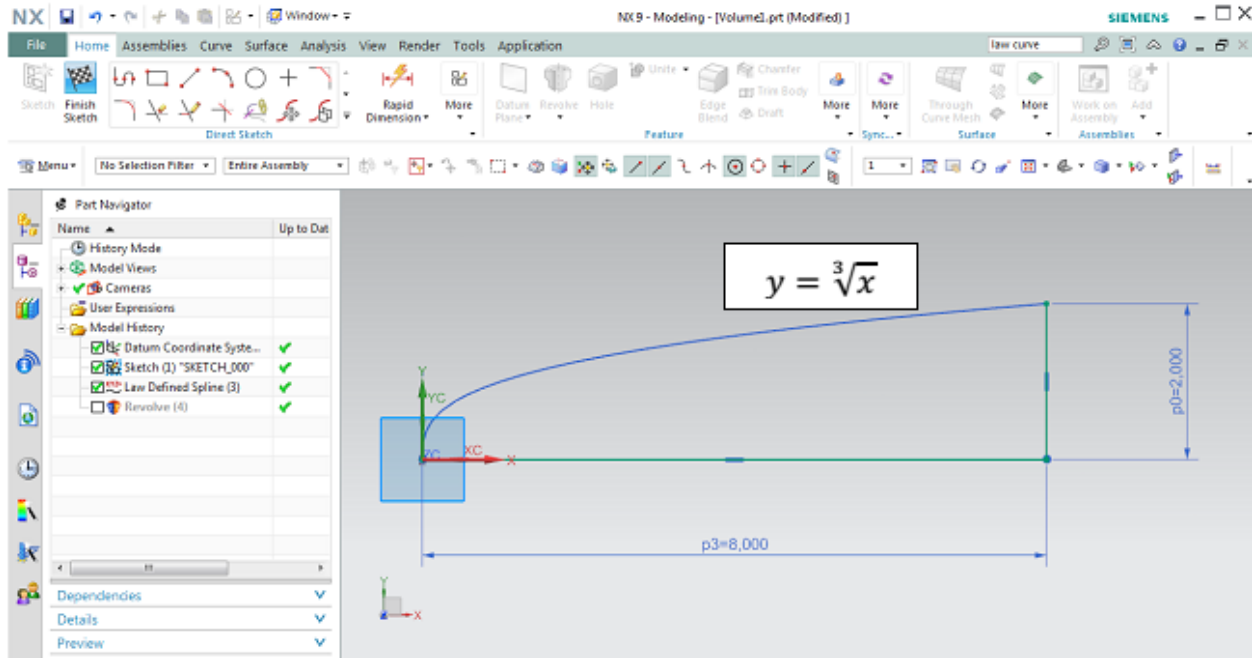


Figure 2 Parametric Design in CAD

Using the solid modeling technique in CAD, the region is revolved about the x-axis to create the axisymmetric solid, as shown in Figure 2. Using the built-in “Measure Bodies” feature in CAD, the volume (as well as many other geometrical and physical characteristics) can be directly measured. Note that the volume in equation (5) does not have to have unit. In CAD, a unit must be used when creating any geometry. In this example, inches is used for the unit for length, therefore the volume has a unit of cubic inches in Figure 3.

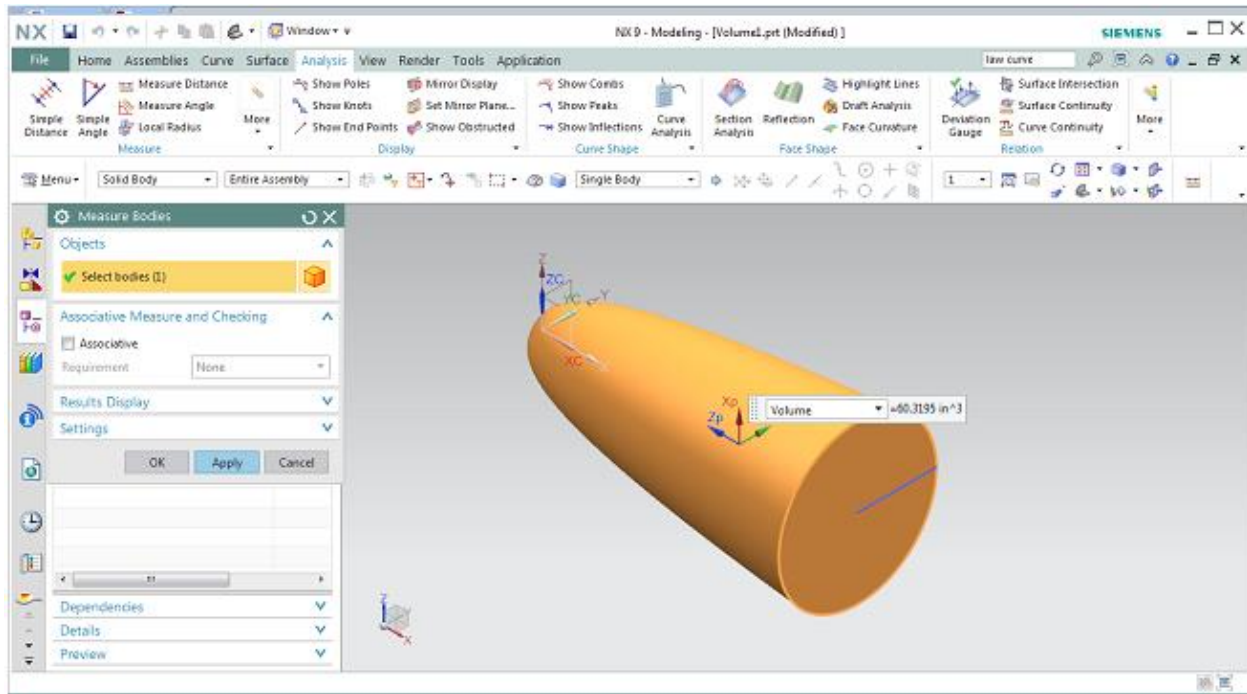


Figure 3 Volume of a solid calculated in CAD

More Fun Activities

Comparison of results from integration and CAD can be used to validate the solutions. In addition, more fun activities can be included. For example, students are encouraged to compare which approach is faster - “pure mathematics” versus “CAD”;

- When the law curve equation changes
- When the limits of the law curve changes
- When hollowing the solid
- When drilling a hole in the solid
- When adding a boss to the solid
- When creating a pocket in the solid

This activity can be used as an individual assignment, or a team project. Instructors can determine the scope of work based on the amount of time students are expected to spend.

Following is an application of improper integrals to Business and Economics - Capital Value. This project demonstrates the following concepts in integral calculus;

1. Definite Integrals
2. Improper Integrals

The definite integral can be used in a number of applications in business and economics. For example, the definite integral can be used to find the total income over a fixed number of years from a continuous income stream. The definite integral can also be used to find the present value,

the value today, of a continuous income stream that will be providing income in the future. The present value is useful in deciding when to replace machinery or what new equipment to select. When the notion of the present value is extended to an infinite time interval, the result is called the Capital Value of the income stream and is given by

$$\text{Capital Value} = \int_0^{\infty} f(t)e^{-rt} dt \quad (6)$$

where $f(t)$ is the annual rate of flow at time t , and r is the annual interest rate, compounded continuously .

For these projects, the student must find business data on the annual rate of flow and assume the value of the annual interest rate.

Module 3. Applications of infinite sequences and series.

We included in this module applications to finance, physics, and other areas of mathematics. Our financial applications concern computation of present and future values of annuities, and the present value of a perpetuity, which involves infinite series. Another financial application uses regression to predict future United States debt based on the historical government data.

Physical applications include motion of a bouncing ball and the period of a pendulum where the following integral is obtained

$$T = 4 \sqrt{\frac{L}{g}} \int_0^{\pi/2} \frac{dx}{\sqrt{1-k^2 \sin^2 x}} \quad (7)$$

for a pendulum with length L and the maximum angle θ_0 , where $k = \sin(\theta_0/2)$ and g is the acceleration due to gravity. The period is then approximated by using an infinite series

$$T = 2\pi \sqrt{\frac{L}{g}} \left(1 + \frac{1^2}{2^2} k^2 + \frac{1^2 3^2}{2^2 4^2} k^4 + \frac{1^2 3^2 5^2}{2^2 4^2 6^2} k^6 + \dots \right) \quad (8)$$

In the bouncing ball project, students are asked to bring a ball and make measurements to estimate the total distance traveled by the ball when dropped on the floor.

Assessment.

In order to assess the level of student engagement and interest, a set of Reflective Survey questions have been designed, as shown below.

1. Participating in a calculus project enhanced my learning (1 -not at all, to 5 - to a great extent)
2. Which project did you do?
3. Please reflect on how this project did or did not enhance your learning. (For example,

what aspects of this project did you find most/least interesting or exciting?).

4. Participating in a calculus project made me more engaged in the course (1 -not at all, to 5 - to a great extent)
5. Please reflect on how this project did or did not make you more engaged. (For example, what aspects of this project did you find most/least related to the course content?).
6. Are there any changes you recommend in the way the project was assigned/supervised by the instructor and how it was graded?
7. Is there any project you would like to recommend?

The survey was conducted in June of 2017 in a multivariate calculus class of 36 students, with 27 students participating in the project. The answers to the survey questions are either a numerical value based on a scale of 5 (questions 1 and 4), or descriptive sentences. Due to the length limit of the paper, only questions 1-5 are presented in this paper. Figure 4 shows the ratings of Question 1. The average rating of 3.30 based on a scale of 5.

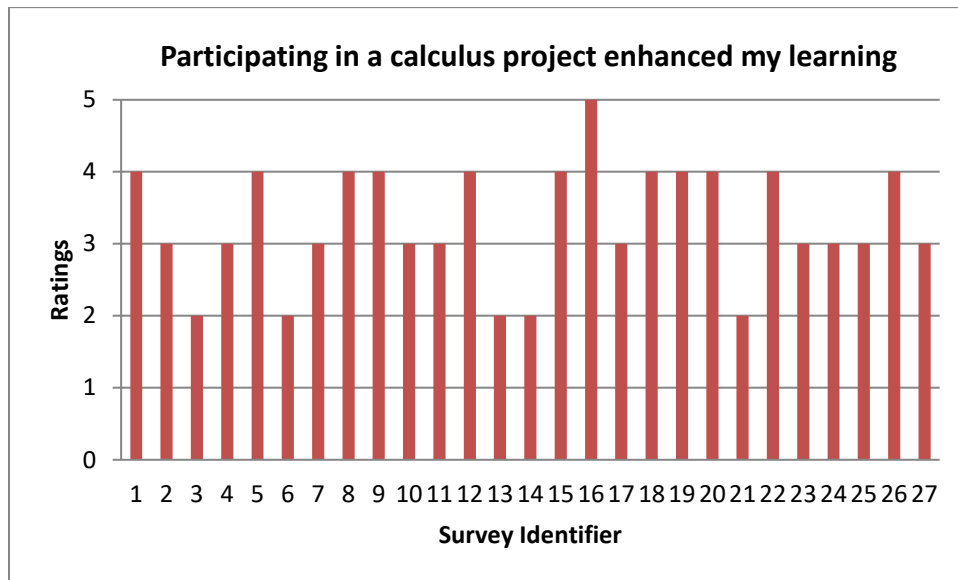


Figure 4 Ratings of Question 1

Feedbacks to questions 1-3 are combined and tabulated in Table 1. Very interesting responses and constructive suggestions to improve these projects were received from the students.

Table 1 Feedback from Questions 1 – 3.

Question 1 - 3: Please reflect on how this project did or did not enhance your learning?		
Score (1-5)	Project	Responses
4	Biot Sawart Law; Lagrange Multipliers & Sufficient Conditions (the most referred to)	It made me excited about math. I began to look at other formulas and question how they were derived.
3	Biot Sawart Law; Lagrange Multipliers and Sufficient Conditions	The projects worked well, however, a large portion was actually figuring out what these projects meant. More clear instructions are needed.
2	Cobb-Douglass Function	The project showed me that this function existed, but it did a poor job explaining what to do.
3	Hot Air Balloon – Module 3	Being a chemistry major, using ideal gas laws and $PV = NRT$ made this a little bit more relevant than the other projects to my major; not sure the Calc III application.
4	Biot Sawart Law and Cobb Douglass	It was interesting to see how calculus is applied in ways that are broad and very significant.
2	Designing a Parabolic Antenna	This project failed to enhance my learning in calculus, but did increase my learning in physics.
3	Biot Sawart Law; Economics	Helped solidify understanding.
4	The Economics Project and Hyperbolic Paraboloid	Going over Lagrange Multipliers helped me retain the information.
4	Seotic 2, Project 3	It required concepts that we are learning in class (derivatives) but what does it still mean?
3	Design a Container	This did not enhance my learning as the project was very simple and I had done something similar in previous classes.
3	Module 1	While I found it interesting/relevant, I didn't really enjoy the length of it; it could take way too long.
4	Submarine Project	It forced me to do something more difficult and to explore solutions outside the domain.
2	Project 7: Economics	I found the project extremely exciting, but I did not have the time and energy to keep enjoying the work on it.
2	Biot Sawart Law	It had very little to do with what was being taught in class. The application portion was interesting but I couldn't find much about it on-line.
4	Submarine Project – still finishing	Helped my MatLab skills; Never learned MatLab in any class.

5	Module 2: Project 7: Economics	It helped me to understand the concept better because it gave me a reason to look more into it to solve the project.
3	Cobb-Douglass Project	It gave me an understanding that calculus can relate to very simple real world things. (The end product was most exciting.)
4	Parabolic Antenna	I thought it was interesting seeing what goes into designing a parabolic antenna.
4	Antenna	It reinforced concepts taught in class and made me relearn them in a sense.
4	Design a Container	This project made me think about and find the different ways to find the area, volume, and centroid of an object. It was interesting to find so many different ways to do my project.
2	Hot Air Balloon	The ideas that were most useful for this project were taught at the end of the semester.
4	The Hall Effect	The problems I solved made me feel like I was doing it right.
3	Biot Sawart Law	There was no clear due date and modules were announced before the material was covered.
3	Designing an Antenna	It was a good review of some concepts, but it turned me off from math because there are other ways to do this.
3	Involving the uncertainty of circuit components in a bandwidth filter.	I found it very interesting that each component in a circuit has a significant difference in the effect which it has on a circuit.
4	Cobb-Douglass – Profit Maximizing/Elasticity	Presenting was not useful.
3	Parabolic Antenna	The wording on question 3, especially, confused me. It was a struggle to find out what to do to answer some of the questions.

Feedbacks to questions 4-5 about engagement are combined and tabulated in Table 2, where the scores are reported in the first column and the comments in the third column.

Table 2 Feedback from Questions 4 – 5.

Question 4 - 5: Please reflect on how this project did or did not make you more engaged?		
Score (1-5)	Project	Responses
3	Biot Sawart Law; Lagrange Multipliers & Sufficient Conditions (the most referred to)	The project is extremely daunting, more so when you don't know where to start. These are a few pointers.

3	Biot Sawart Law; Lagrange Multipliers and Sufficient Conditions	It forced me to go outside class for information and look at what other people have done for these types of projects.
1	Cobb-Douglass Function	Due to the lack of instruction on how to work through the project, it was very difficult to get engaged in the process of completing the project.
1	Hot Air Balloon – Module 3	The topics are not very relevant to any life scenarios such as biochemistry or medicine so it was really difficult to stay engaged. However, with some of the projects I could see the relevance.
4	Biot Sawart Law and Cobb Douglass	I thought everything related to the courses in an interesting way that displayed the vast applications.
2	Designing a Parabolic Antenna	The project felt very disconnected from the course even though the course material was vital to the project.
3	Biot Sawart Law; Economics	It gave me a better understanding of why I learned all of this.
4	The Economics Project and Hyperbolic Paraboloid	The Cobb-Douglass Function was more for economics or business classes.
3	Seotic 2, Project 3	Doing it was a priority. I was more engaged with the WebAssign because it was sort of hard to relate to these projects. Yes, we for the most part, are all ME's in an advanced math class, but that doesn't suggest that we will understand where all of this is
3	Design a Container	This project made me more engaged in class as I was able to relate what we learned to real life.
1	Module 1	It was relevant to what we were learning but it didn't engage me as the project's subject wasn't what class was doing at the time.
3	Submarine Project	It was nice to see a practical application of the calculus.
1	Project 7: Economics	The project got in the way of the calc homework that was already assigned.
1	Biot Sawart Law	Lack of connection with the class.
3	Submarine Project – still finishing	Made me work outside of class.
5	Module 2: Project 7: Economics	I used the concepts learned in class to solve it.
3	Cobb-Douglass Project	It helped me understand applying partial derivatives.
3	Parabolic Antenna	It helped me understand double integrals more when I had to find the surface area.
3	Antenna	Figuring out the formulas both helped and distracted from the calculus.

3	Design a Container	When we started finding volume by integration in class I was better able to understand how it could be applied.
2	Hot Air Balloon	Helped me to research more ideas on how to apply math.
1	The Hall Effect	It made me engaged in research but not the class.
1	Biot Sawart Law	I had to put off doing a second project until the material was covered.
1	Designing an Antenna	It added more stress, taking my attention off from other issues.
3	Involving the uncertainty of circuit components in a bandwidth filter.	The relationship of frequency and bandwidth doesn't really relate to Calc III. This equation should be given, not something that should be found by the student.
4	Cobb-Douglass – Profit Maximizing/Elasticity	Used exactly what we learned in a more applicable fashion.
2	Parabolic Antenna	The calculations were similar to what we've done, especially the gradient and tangent plane.

The distribution of grades in this section was as follows;

Grade	Frequency
A	9
B	7
C	6
D	5
F/W	9

The results are not indicative of any change in grades distribution in this course.

The Likert-type scale results for question on engagement and enhanced learning are statistically significantly positively correlated with the Spearman correlation coefficient $\rho = 0.688$ (p – value < 0.005).

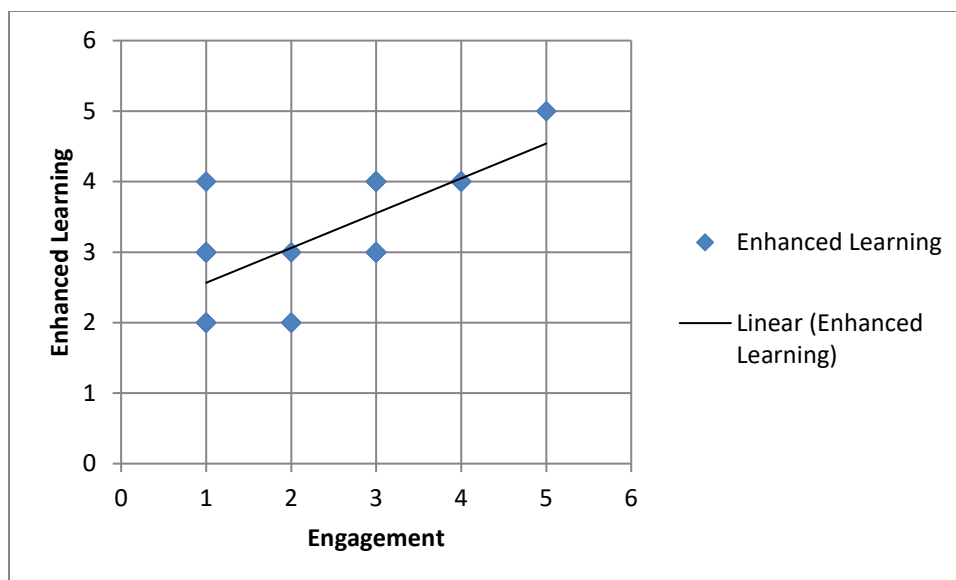


Figure 5 Enhanced Learning vs Engagement

It is interesting to compare student answers for students who reported high engagement and learning enhancement against those who reported low values in both categories. It is also interesting to analyze responses of students who reported high learning enhancement but low engagement.

Students reporting high learning enhancement and high engagement exhibit behaviors of *deep learners* who try “not just to learn facts but to understand what they mean, how they are connected, and how they might relate to ... experience”, Felder ⁷, pg. 152. This is reflected in the following student comments: “It was interesting to see how calculus is applied in ways that are broad and very significant”, “It helped me to understand the concept better”, “The projects are difficult but they should be and I think that is the point”.

In contrast, students who’s rating were low were taking a *surface* approach to learning, that is “memorizing facts but not trying to fit them into coherent body of knowledge”, Felder ⁷, or a strategic approach; “finding out what the instructor wants and delivering it”. Felder ⁷. This is demonstrated in the following student comments: “The project showed me that this function existed, but it did a poor job explaining what to do”, “It had very little to do with what was being taught in class. The application portion was interesting but I couldn’t find much about it on-line”, “Due to the lack of instruction on how to work through the project, it was very difficult to get engaged”, “The project got in the way of the calc homework that was already assigned”, “Lack of connection with the class”.

The group of students reporting high learning enhancement but low engagement exhibit some features of *deep learners* mixed with behaviors of *surface* and *strategic learners*, commenting for example as follows: “Being a chemistry major, using ideal gas laws and $PV = NRT$ made this a little bit more relevant than the other projects to my major”, “While I found it interesting/relevant, I didn’t really enjoy the length of it”, “The problems I solved made me feel like I was doing it right”, “It was a good review of some concepts, but it turned me off from math because there are other ways to do this”, “The topics are not very relevant to any life scenarios

such as biochemistry or medicine so it was really difficult to stay engaged. However, with some of the projects I could see the relevance”.

From student comments it becomes clear that the relevance of projects to both, student interests and course learning objectives, is critical to turning *surface* and *strategic learners* into *deep learners*.

Future research.

The authors, in cooperation with other instructors in the Mathematics Department, will continue to assess student interest in motivating examples and mini projects. Some material may be replaced with better examples. The level of student engagement will be measured by conducting new surveys.

Conclusions.

The redesigned courses in the calculus sequence have been developed, including one-dimensional differential calculus, one-dimensional integral calculus, and multivariate calculus. The content, motivating examples and mini projects, have been uploaded to an experiential calculus web site and a presentation to all mathematics instructors and selected engineering faculty members has been organized. The work on this project has just concluded and so far only one instructor has provided initial feedback. Student reactions to the assignment have been mixed, from very enthusiastic to the point that some students decided not to do it, even though the assignment has been graded for credit. However, student feedback was generally positive. The authors will use results of future surveys to assess the success of this project and to improve student engagement and hence, to enhance learning.

We have also learned that the relevance of projects to both, student interests and course learning objectives, is the key to promoting student engagement.

The repository of motivating examples and mini projects for the entire calculus sequence is available on line at <https://newb.kettering.edu/wp/experientialcalculus/> and will be complemented by additional materials in the future.

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