



Representations, between engineering design and engineering analysis

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Abstract

This is a survey paper submitted to the Design in Engineering Education Division (DEED). As a survey paper, it provides a comprehensive review of the literature on the topic of representations between engineering design and engineering analysis. This research aims to characterize the overlap as well as distinctness between *engineering design thinking*, on the one hand, and *engineering model-based reasoning*, on the other hand. The 1990s witnessed the rise of a transformative wave to the engineering curricula, where the “engineering science” model became dominant in engineering curricula. In this model, the focus in the first two years of the curriculum is placed on the “engineering sciences,” or, alternatively, “analysis,” with the expectation that students would apply the learned scientific principles to solve technical problems. However, a segregation problem between “design” and “analysis” started to emerge. The problem was caused not only by lack of appreciation for the complexities associated with design teaching and learning, but also by lack of students’ fluency to apply their learned mathematical modeling skills in complex, open-ended design problems. In this paper, we develop a “representations framework” to study the relationship between engineering design thinking and engineering model-based reasoning. It is the focus of this study to understand the role of multiple representations in problem solving, in order to characterize the overlaps and the distinctiveness in the use of the term “representation” in the contexts of mathematical modeling and design processes. Engineering design is a systematic, intelligent process that aims to solve ambiguous problems. In the majority of current engineering education curricula, a major emphasis is placed on the traditional view where prerequisite ideas are taught in decontextualized situations. While students in their courses interact with models in varying contexts, teaching focuses on algorithmic steps to find a solution. In this paper, we develop a framework to understand how representation is described, taught and learned in analysis-focused classes and in design-focused classes.

1. Introduction—Nature of the problem

“Engineers create the world that never was,” famously stated Theodore von Karman, comparing engineers with “scientists [who] discover the world that exists” (“Foundation”, n.d.). Arriving at Caltech in 1929 coming from Aachen, Germany, he restructured aerodynamics education placing an emphasis on the scientific and mathematical foundation (“JPL”, n.d.). Overall, the American engineering education experienced a first transformative wave in the beginning of the 1900s, and especially later after World War II, when American engineering colleges embraced the analytical mode of engineering science (Seely, 1999). The role of European-educated engineers, such as von Karman, Stephen Timoshenko and Harald Westergaard was impactful in transforming the engineering curricula. Westergaard, for example, realized that his mastery of mathematical theories to study concrete structures was a more powerful problem solving tool compared to rules of thumb and design experiences that American engineers relied upon in their engineering work (Seely, 1999).

The 1990s, however, witnessed the rise of another transformative wave to the engineering curricula. By that time, the “engineering science” model became dominant in the engineering curricula, where focus in the first two years of the curriculum is placed on the “engineering sciences,” or, alternatively, “analysis,” with the expectation that students would apply the learned scientific principles to solve technical problems (Dym, Agogino, Eris, Frey, & Leifer, 2005). The shift in the early 1990s came about by

observing the imbalance in the engineering curriculum where the focus is placed on the engineering sciences and students were required to master the scientific theories underlying natural phenomena. The scale of the problem reached, according to Evans and colleagues (1990), a degree of “segregation” between ‘design’ faculty and ‘analysis’ faculty in engineering departments (Evans, McNeill, & Beakley, 1990). Initially, the problem was recognized as lack of appreciation for the complexities associated with design teaching and learning (Dym et al., 2005), but the problem was also recognized as the lack of students’ fluency to apply their learned mathematical modeling skills in complex, open-ended design problems (Carberry & McKenna, 2014). Overall, the problem is characterized as lack of clear understanding of the overlap as well as distinctness between engineering design thinking and model-based reasoning.

2. Purpose of the paper

This is a survey paper submitted to the Design in Engineering Education Division (DEED). As a survey paper, it provides a comprehensive review of the literature on the topic of representations between engineering design and engineering analysis. The study of how engineering students use multiple representations in their design is lacking. As a survey paper, this work is intended to motivate future discussions and efforts on the topic. Based on the literature surveyed, we identified gaps that are promising for future research.

3. Characterizing the problem

In this section, we provide an overview of what we know and what we do not know about the problem. This review is intended to (1) providing a background that is grounded in the literature for developing the theoretical framework; and (2) providing a focused direction for future steps. The section is organized under two major headings: features of engineering design and features of mathematical modeling.

3.1 Features of engineering design

One of the definitions for engineering design is the “systematic, intelligent process in which designers generate, evaluate, and specify concepts for devices, systems, or processes whose form and function achieve clients’ objectives or users’ needs while satisfying a specified set of constraints” (Dym et al., 2005, p. 104). Design pedagogy is enhanced through project-based learning (PBL), where students engage in real-life projects that motivate learning by doing (Kolb, 1984). Providing design experiences that promote the learning of the different aspects of the design process; e.g., creative thinking and teamwork, have existed in the engineering curricula in the US since 1997 (“NSF”, 1997). The original (PBL) model, founded in Aalborg University in Denmark in 1974, featured two unique themes: (1) *design-oriented projects* which focused on the know-how of synthesizing knowledge from different disciplines, and (2) *problem-oriented projects* which focused on the know why of solving theoretical problems through relevant knowledge (Kjersdam & Enemark, 1994; Luxhol & Hansen, 1996). These two themes point to early attention in the design pedagogy to the relationship between design and analysis. Dym et al. (2005) described this relationship to “seemingly parallel the idea of integrating divergent and convergent thinking” (p. 109).

While introducing a comprehensive design project experience was exclusive to the senior year, recent trends emerged to weave PBL in the first year and throughout the curriculum in semester-long projects. Students are being introduced to design methods, team dynamics and ethics in parallel with the analysis-focused courses (Dym et al., 2005). However, the systematic integration of design and analysis in courses is still lacking (Carberry & McKenna, 2014). Some of the outstanding research questions about PBL as

proposed by Dym et al. (2005) include: “What are the best proportions of problems, projects, teamwork, technology, and reality for a given state of student development,” and “how do the proportions change with regard to the context of different engineering disciplines and institutional missions?” (p. 112).

There are some unique features to the design process; an overview is provided in Figure 1. Some of the features include tolerance to ambiguity; big-picture, systems thinking; handling uncertainty; making decisions; thinking as part of a team in a social process; and the ability to communicate in different languages; that is, representations. Dym et al. (2005) provided a thorough literature review on each of these features. For the purposes of this study, I shall highlight two observations in the overview provided in Figure 1. First, while mathematical modeling is cited as one form of representations, there are common features of the design process that overlap with mathematical modeling which are cited elsewhere in the map. For example, and as will be described in the following section, both the design process and mathematical modeling share the notions of *reasoning about uncertainty*, *making estimates*, *handling uncertainty* and *making decisions*. Second, mathematical modeling and dealing with numbers are seen as part of communicating languages of design, which can be an externalized way of thinking similar to *verbal* and *graphical* representations. However, and as explained in the following section, mathematical modeling is not just a way to communicate design, as Dym et al. (2005) suggested, but it also is a way for model-based reasoning.

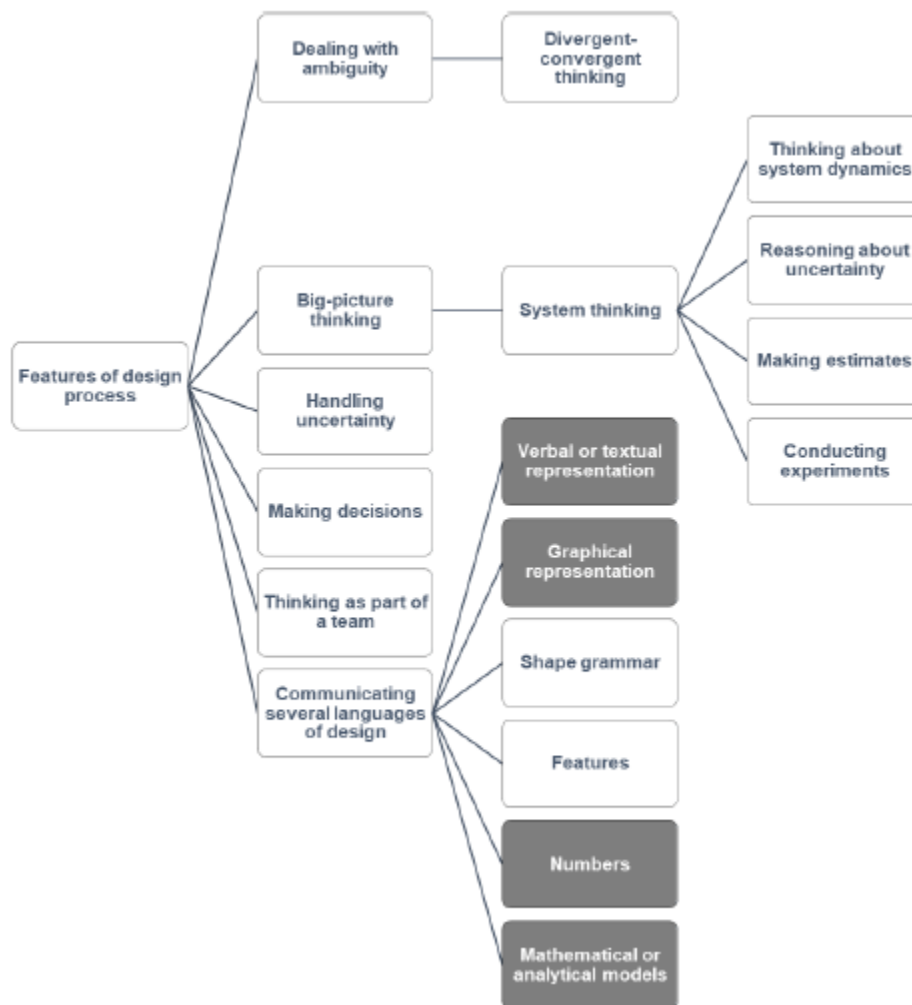


Figure 1. Features of the design process. Highlighted boxes are discussed in more depth.

The idea that reasoning exists in mathematical modeling while designing is not exclusive; reasoning in designing can also be seen in visual synthesis and in designers' use of textual language. In visual synthesis, Juhl & Lindegaard (2013) studied how engineering students incorporated analysis in the creation of design synthesis. During synthesis in design, visual representations served as a way to "present and organize recognitions so that they are recognizable across other disciplines and can be integrated into new recognitions" (p. 20). Juhl & Lindegaard proposed an epistemology of design synthesis that is composed of the following cognitive processes: exploring—inscribing—collecting—combining and refining. They suggested this framework as a way to "integrate content from more than one course in synthesis-oriented design courses" (p. 46). Juhl & Lindegaard (2013) identified the need to study the role of representations in both learning and communicating results.

In addition to visual synthesis, Atman, Kilgore, & McKenna (2008) studied designers' use of language. They found that students' engagement in design learning result in acquiring the language of design, which in turn shapes the knowledge that students have about design. Therefore, the language of design is not just a superficial representation of knowing; rather, it reciprocates in the students' ability to developing solutions to engineering design problems because language is critical in activities in design. Atman et al., (2008) recommended the study of language as a structured way of inquiry about design expertise.

Throughout this overview of features of design process and the current state of research on the topic, it is recognized that there are commonalities between the design process and mathematical modeling, mainly through issues around handling uncertainty and reasoning in the process of finding a solution. The research also pointed to the fact that the multiple representations used in designing, including visual synthesis and language, are venues to study design expertise development and processes. However, a gap still exists on understanding how *different, multiple* representations are used simultaneously in problem solving, especially in the process of integrating analysis and synthesis.

3.2 Features of mathematical modeling

Modeling efforts attempt to translate natural phenomenon or real-world problems into representation systems, including mathematical and computational models (Gainsburg, 2006). In educational settings, the process of "constructing, describing, explaining, manipulating, predicting, or controlling mathematically significant systems" is called *model-eliciting activity*. The process aims to train students in the "process" of creating models while practicing model-based reasoning. This is different from the *traditional view* where students are expected to seek out one "final answer" (Lesh & Doerr, 2003). Figure 2 depicts the difference between the two approaches in terms of the relationship between the real world and the model world in both traditional and model-eliciting activities.

In the majority of current engineering education curricula, a major emphasis is placed on the traditional view where prerequisite ideas are taught in decontextualized situations (Moore, Miller, Lesh, Stohlmann, & Kim, 2013). While students in their courses interact with models in varying contexts, teaching focuses on algorithmic steps to find a solution (Carberry & McKenna, 2014). Figure 3 provides an overview of the progress in the state of research on the topic, moving from *the traditional view* (Circle 1), to research on understanding how multiple representations *provide deeper understanding of concepts* (Circle 2), to research on understanding how models and modeling *are used by students in design* (Circle 3).

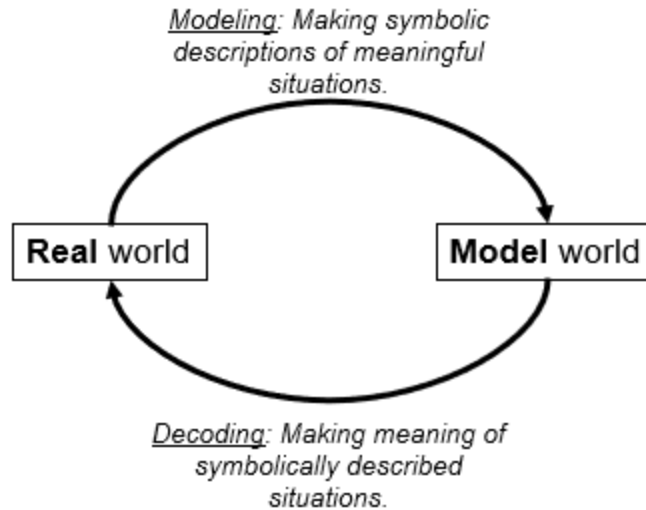


Figure 2. The relationship between the real world and the model world in model-based reasoning. The “decoding” path represents the traditional view.

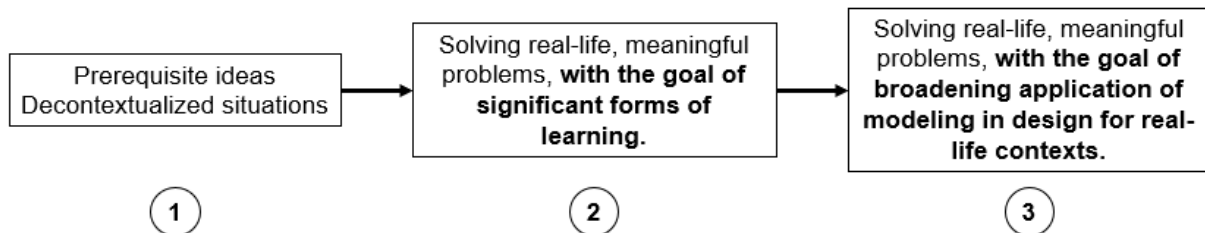


Figure 3. Progress of research on the topic—an overview.

The traditional, decontextualized view on mathematical learning (Circle 1) has been described by Lesh & Doerr (2003) as one that gives little attention to real-life situations. In the traditional view, learning to solve real life problems is viewed as a subset of a more general problem-solving skills that relies on heuristics. Solving real life problems, in this view, is only pursued in the classroom if time permits and viewed only as an application of the preceding learned ideas, skills and heuristics (Lesh & Doerr, 2003). Lesh & Doerr (2003) found model-eliciting activities to involve the development of ideas and understanding as intermediate steps throughout the activity of solving a real-life problem. Most importantly, they found that students invent “constructs that are more powerful than anybody has dared to try to teach them using traditional methods” (p. 5).

Building on these ideas, Moore et al. (2013) attempted to understand engineering students’ fluency in using models to deepen conceptual understanding (Circle 2). They tried to understand the process through which students develop models, initially as “a collection of fuzzy, intuitively functioning, undifferentiated, poorly integrated and relatively unstable partial interpretations, which are expressed using a variety of interacting representations” (p. 142). Interestingly, their work, in the context of engineering, extended the work by Lesh & Doerr, which was mainly in the context of mathematical education, in that students’ use of modeling in problem solving involves developing, testing and revising multiple immature interpretations, not just a single model (Moore et al., 2013). Their work recommended

the development of problems that engage students with careful modeling activities to allow for conceptual development to occur.

In the context of design education and learning Carberry & McKenna (2014) studied the utilization of modeling as an engineering tool in students' work in design (Circle 3). Carberry & McKenna's (2014) work extends on the previous work of embedding modeling interventions as model-eliciting activities into engineering instruction. Their work builds on previous research on aspects that students think should be modeled when designing, how they create and use models, and how students critique models. Carberry & McKenna (2014) found that students do not realize the full power that models and modeling can bring to design, and they recommended that explicit instruction of modeling in formal engineering education. They recommended a curriculum-approach of being explicit in analysis-focused courses on how modeling can be used in solving open-ended problems as well as incorporating modeling as a component in design-focused course.

While the overview of the previous studies has focused on features of mathematical modeling in educational settings, Gainsburg (2006) studied engineers' use of modeling in their work setting. She found that engineers use and adapt models of various levels of abstraction, with the major challenge of understanding inaccessible phenomena where "forms and behaviors of the proposed elements and structures [which make up the model are] essentially inaccessible" (p. 14). She also shed light on how engineers in their real-life problem-solving work with abstract and theoretical concepts: "some were fragmented, their representation and storage distributed in parts among multiple artifacts, with some parts only existing conceptually" (p. 14). Gainsburg (2006) synthesized work on steps for mathematical modeling, including that of Lesh & Doerr (2003), and identified the following cyclical steps of the process:

1. Identify the real-world phenomenon
2. Simplify/idealize the phenomenon
3. Express the idealized phenomenon mathematically (i.e., "mathematize")
4. Perform the mathematical manipulations (i.e., "solve" the model)
5. Interpret the mathematical solution in real-world terms
6. Test the interpretation against reality

4. Framework for representations, between engineering design and engineering analysis

It is the focus of this study to understand the role of multiple representations in problem solving. In order to characterize the overlaps and the distinctiveness in the use of the term "representation" in the contexts of mathematical modeling and design processes, it is useful to understand the perspectives of the various stakeholders that come at the intersection of education, psychology, mathematics and engineering. For example, Lesh & Doerr (2003) observed how "parents, policy makers, community leaders, teachers, administrators, teacher educators, curriculum designers, and others" associated different meanings to the language used in communicating ideas around education (p. 8). Consequently, a term such as "representation" may evoke certain meanings to a group of stakeholders and may fail to elicit other meanings for others. Here, we only focus on two major perspective to compare and contrast: mathematical modeling and design process. One of the aims of this research is to characterize the similarities and differences in the use of the term in order to accurately and effectively develop the students' skills to translate between and within different representational forms.

The term "modeling" has been described by Lesh & Doerr (2003) to have been chosen to characterize this line of research based on "assumptions that sound sensible and useful to ordinary people—but that also lead to implications that are powerful and nonobvious for decision-makers whose ways of thinking influence schools" (p. 8). The problem with the use of language to describe the phenomenon in this study

is not only present in mathematics education. Juhl & Lindegaard (2013) observed a similar problem in engineering education: when searching for the term “representations” in the *Journal of Engineering Education*, they found that the majority of the results describe “means for communicating research results such as experimental data” (p. 23). They elaborate on this narrow view which attempts to only capture the final results of the work with no interest in or attention to the intermediate steps that lead to the achievement of the outcome. The problem of the lack of sufficient studies on representations as pathways to bringing together analysis and synthesis is further complicated by the nature of the various disciplines that tend to emphasize different aspects of knowing (Juhl & Lindegaard, 2013).

Table 1. Framework integrateing the use of “representation” between design and mathematical modeling along five dimensions.

Dimension	Description
Purpose	The reason that a representation exists in the two modes of problem solving: design process and mathematical modeling
Representing what?	The content of a representation
Types	The common representation categories that each mode of problem solving uses
Transitions	The process of changing from one representation to another
Challenges	The challenges associated with creating representation in design and in mathematical modeling

The framework we propose integrates the use of “representation” between design and mathematical modeling along five dimensions: *purpose*, *representing what?*, *types*, *transitions* and *challenges*, Table 1.

- Purpose:** This dimension addresses the reason that a representation exists in the two modes of problem solving: design process and mathematical modeling. In *design*, a representation can serve as a way to articulate design aspects, describe objects, constraints or limitations, and communicate different ideas between stakeholders (Dym et al., 2005). As a language, specifically, representation can aid in ascribing to a community of practice by sharing same syntax (Atman et al., 2008). A representation can encourage aggregation of ideas, moving from one recognition to another, as well as the social synthesis of ideas (Juhl & Lindegaard, 2013). Representations exist throughout the design process and they enable convergent-divergent modes of thinking (Dym et al., 2005). In *mathematical modeling*, Gainsburg (2006) observed that engineers “use, adapt and create models of various representation forms and degrees of abstraction” (p. 2) with the purpose of understanding inaccessible phenomena. In mathematics education, Lesh & Doerr (2003) described modeling as a way to develop ideas, constructs, processes and systems that enable deeper understanding. In engineering education, modeling is conceived of as an essential part in the design process (Carberry & McKenna, 2014), making a clear link between the purposes of the two modes of solving problem (Moore et al., 2013).
- Representing what?:** This dimension addresses the content of a representation. In *design*, some of the aspects that a representation represents, as in Figure 1, are system dynamics, uncertainty, estimates, and experiments (Dym et al., 2005). Representations can be used to characterize, share and develop forms of recognition of the design problem (Juhl & Lindegaard, 2013). Representation can also, indirectly, indicate to students’ level of expertise and internalization of the design knowing and practice, through their use of the design language (Atman et al., 2008). In building *mathematical models*, the process of mathematizing usually involves “quantifying, dimensionalizing, coordinatizing, categorizing, algebraizing, and systematizing relevant objects, relationships, actions, patterns, and regularities” (Lesh & Doerr, 2003, p. 5). Representations in mathematical models show early thinking about a situation, with interacting representation aimed at developing integrated understanding (Moore et al., 2013). Gainsburg’s (2006) steps for mathematical modeling involve identifying which aspects of reality needs to identified, simplified and modeled.

- Types:** This dimension addresses the common representation categories that each mode of problem solving uses. In *design*, as discussed before, representations can be verbal or textual statements, graphical representation, and mathematical or analytical models (Dym et al., 2005). Other representations include: shape grammars, features and numbers (Dym et al., 2005). Representation in design can be used to explore, inscribe, collect or combine and refine ideas (Juhl & Lindegaard, 2013). In *mathematical modeling*, representations can be of varying degrees of abstraction (Gainsburg, 2006). Furthermore, they can be in the form of equations, graphs, diagrams, models, metaphors, spoken language, tables and written symbols (Lesh & Doerr, 2003).
- Transitions:** This dimension addresses the process of changing from one representation to another. In *design*, the process is usually referred to as iteration between convergent-divergent modes of thinking and take place throughout the design process (Dym et al., 2005). Moore et al. (2013) described the process as an indication of fluency in representing. However, research is lacking on how transition between different forms of representations takes place, especially, between design and analysis modes of thinking. In *mathematical modeling*, Gainsburg (2006) recognized the cyclic nature of the steps of creating models. Similarly, Moore et al. (2013) attempted to capture the multiple modeling cycles, Figure 4. Lesh & Doerr (2003) described the transitions as ones that take place across “meanings of conceptual systems” and ones that are “distributed across a variety of representational media” (p. 12), Figure 5.

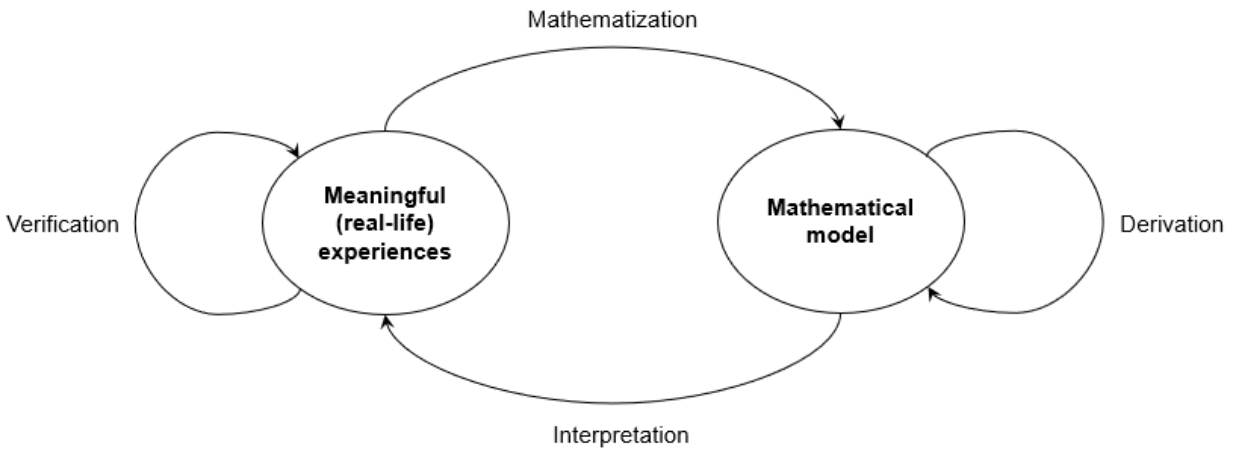


Figure 4. Multiple modeling cycles according to Moore et al., (2013, p. 143).

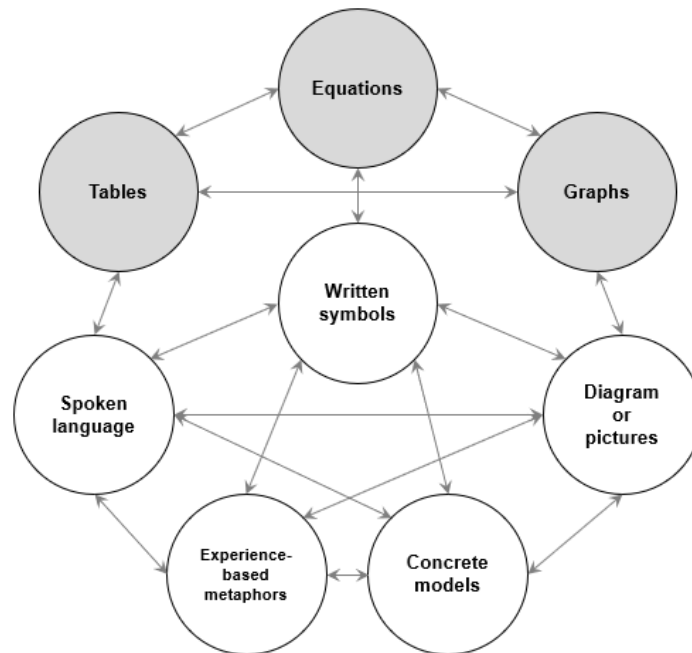


Figure 5. Mathematical modeling involves shifting back and forth among a variety of relevant representation, Lesh & Doerr

- Challenges:** This dimension addresses the challenges associated with creating representation in design and in mathematical modeling. In *design*, a major challenge exists because of the lack of training, promotion and emphasis on representations as both an analytic and collaborative tool (Juhl & Lindegaard, 2013). Students' conceptions and skills of linking different modes of representation; e.g., mathematical with design, is lacking (Carberry & McKenna, 2014; Moore et al., 2013). Furthermore, developing the balanced pedagogy that integrates project-based and problem-based learning is still not explored (Dym et al., 2005). In *mathematical modeling*, a challenge exists in realizing that understanding of concepts occur in solving real-life problems more effectively when compared to the traditional approach of teaching decontextualized ideas and situations (Lesh & Doerr, 2003). In addition, one of the challenges in modeling is the ability to understand inaccessible phenomenon (Gainsburg, 2006). Another challenge is keeping track of different kinds of models as the solution to the problem progresses (Gainsburg, 2006). Furthermore, just like design, students' ability to link modeling with real-life problem solving is lacking (Carberry & McKenna, 2014; Moore et al., 2013).

5. Future work

The study of how engineering students use multiple representations in their design is lacking. Although there are studies that indicated the importance of mathematical modeling as one representation in design; others focused on visual synthesis in representation; and others studied language as representation of knowing, studies are lacking on integrating the different levels of representation. More specifically, understanding how representation is described, taught and learned in analysis-focused classes and in design-focused classes is lacking. Some research has pointed to the disciplinary nature of this understanding. Furthermore, a larger view at the curriculum level of the links of representation between analysis and design needs more study.

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