

**AC 2008-1289: RETENTION AND TRANSFER OF LEARNING FROM MATH TO PHYSICS TO ENGINEERING**

**Sanjay Rebello, Kansas State University**

**Lili Cui, University of Maryland Baltimore County**

# Retention and Transfer of Learning from Mathematics to Physics to Engineering

## Abstract

Retention and transfer of learning is particularly important in the education of future engineers. Engineering majors are required to apply what they learn in their mathematics and physics courses to their engineering courses. We report on two studies. The first focuses on transfer from calculus to calculus-based physics. The second focuses on transfer from physics to engineering. We use a theoretical framework that distinguishes between two kinds of transfer processes. Based on this theoretical framework and using both qualitative and quantitative methods we examine the difficulties that learners have when they attempt to transfer their learning from mathematics to physics to engineering. Our results indicate that although students have the requisite calculus knowledge they have difficulties in applying this knowledge to physics and engineering courses. Our results also indicate that although learners see similarities between problems encountered in physics and engineering differences in notation and assumptions appear to impede transfer.

## Overview

All engineering majors are required to take multiple calculus courses and two semesters of calculus-based physics as a pre- or co-requisite for their engineering courses. In our research we have investigated the extent to which these students retain and transfer their learning from calculus to physics and also from physics to engineering courses. While retention is the ability to recall your knowledge at a later point in time, transfer of learning is defined as the ability to apply what one has learned in one situation to a different situation.

Many introductory calculus-based physics students have difficulties when solving physics problems involving calculus. The participants in this study were students enrolled in a second-semester physics course taken by future engineers and physicists, calculus instructors and physics instructors. A total of 416 students' exam sheets were collected and reviewed. A total of 28 students and nine instructors were interviewed.

Most of the students enrolled in the calculus-based introductory physics sequence often go on to major in engineering. To investigate retention and transfer from physics to engineering, we created an inventory of questions based on concepts that instructors in engineering courses believe that students entering their courses should know. We then surveyed engineering students as they began their *Statics & Dynamics* and *Electromagnetics* courses to assess the extent to which they retained the relevant physics knowledge. In all over 149 students were surveyed and six students and faculty were interviewed.

Since this paper is an overview of two large studies, we do not provide detailed data or examples. Rather we describe general trends in our data and discuss how these trends shed light on what educators in mathematics, physics and engineering can do to facilitate transfer from one discipline to another more effectively. In this paper we first present our research questions.

Then we present the theoretical underpinnings of the research on transfer. Next we describe the methodologies and results from studies on transfer from mathematics to physics and physics to engineering. Finally we conclude with a description of the implications of our results for instruction.

## Research Questions

We seek to answer the following overarching questions:

- To what extent are learners able to retain and transfer what they have learned in mathematics to physics? Physics to engineering?
- What difficulties do students have in transferring what they have learned in mathematics to physics? Physics to engineering?

## Theory

Transfer of learning is often defined as applying knowledge and skills learned in one situation to another situation.<sup>1,2</sup> Typically, researchers have pre-defined what they hope students will transfer. They have viewed this as a static, passive process and focused mainly on the cognitive aspects of transfer. Lack of evidence of transfer in many studies based on traditional models has led researchers to consider other perspectives of transfer. Contemporary perspectives have gone beyond the cognitive aspects of transfer. The socio-cultural perspective asserts that the social and cultural environment affects transfer through language, cultural tools and interaction with people. Transfer is the extent to which participating in an activity while being attuned to the affordances and constraints in one situation influences the learner's participation in a different situation.<sup>3</sup> The actor-oriented perspective conceives transfer as the personal construction of similarities between activities where the 'actors,' i.e. learners, see situations as being similar.<sup>4</sup> Preparation for future learning focuses on whether students can *learn* to solve problems in transfer situations.<sup>5</sup> In all contemporary perspectives transfer is a dynamic process of reconstruction of knowledge in a new situation rather than merely applying previously learned knowledge intact to a new situation. To measure transfer we must investigate when, how and why learners activate certain small grain resources, what contextual characteristics cue activation, how they coordinate these resources with new information and what factors control the coordination. These variables are similar to ones in a model proposed by Redish.<sup>6</sup> Further, we must investigate the extent to which this activation and coordination results in compilation of larger grain size knowledge structures such as coordination classes<sup>7,8</sup> or mental models<sup>9</sup> that are not overly contextualized and can be used appropriately in future contexts. Transfer is a complex, dynamic process that must be probed through *in vivo* techniques focusing on process over product.<sup>10</sup>

Our model of transfer<sup>11</sup> incorporates two qualitatively different processes. Vertical transfer is the activation of small grain resources from long term memory and their association or coordination to compile larger grain size knowledge structures that are more stable. Horizontal transfer is the application of well developed compiled knowledge structures to new contexts. Other researchers have made similar distinctions, e.g. Salomon and Perkins<sup>12</sup> contrasted 'low road' transfer and 'high road' transfer. Bransford and Schwartz<sup>5</sup> discuss transfer in terms of 'sequestered problem solving' which promotes horizontal transfer while 'preparation for future learning' promotes

vertical transfer. diSessa and Wagner<sup>8</sup> have applied coordination class theory<sup>7</sup> to distinguish Class A transfer which is applying well prepared knowledge and Class C transfer, which is constructing new knowledge. Similarly, Schwartz, Bransford and Sears<sup>13</sup> discuss transfer in terms of efficiency and innovation.

To help students become both efficient and innovative problem solvers, or in other words develop adaptive expertise, Schwartz, Bransford and Sears<sup>13</sup> posit that we must guide students through an optimal adaptability corridor (See Figure 1) that balances both innovation and efficiency and thereby facilitates the development of adaptive expertise. Thus, this model of transfer<sup>11</sup> consolidates both traditional and contemporary views of transfer and serves as a building block for the theoretical framework in our studies.

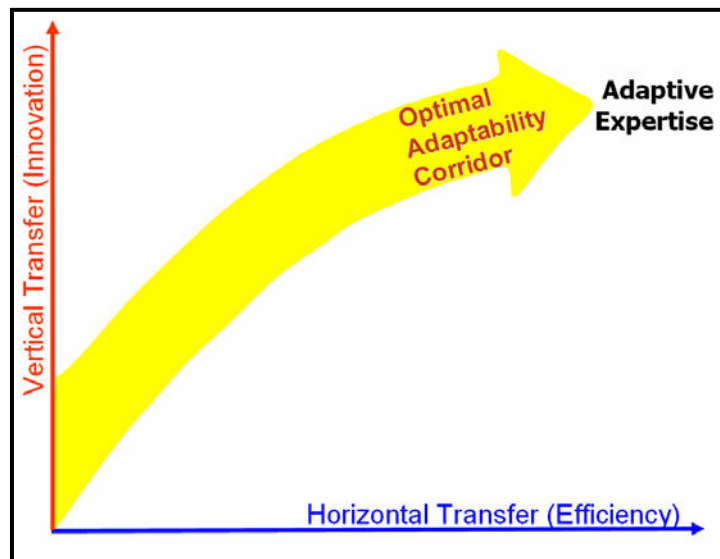


Figure 1: Horizontal & vertical transfer and the optimal adaptability corridor

### Transfer Studies from Calculus to Physics

Most science and engineering majors are required to take calculus-based physics. Students are usually required to concurrently take both their first calculus and physics courses, or take at least one calculus course prior to taking physics. While a few integrated curricula<sup>14,15</sup> have been developed and have been found useful in teaching calculus and physics, in most universities, calculus and physics are taught as two separate subjects in their respective departments.

The connection between calculus and calculus-based physics is obvious both from the historical view and practical perspectives. Anecdotally we have often found that some physics teachers claim that their students do not have the pre-requisite calculus knowledge to help them master physics. Is this the case? There has been no significant research on transfer of learning from calculus to physics. Therefore, assessing transfer of learning from calculus to physics is the central focus of this study.

We use a combination of qualitative and quantitative methods to examine transfer in the context of problem solving. The participants in this study were students enrolled in a second-semester

physics course taken by future engineers and physicists, calculus instructors and physics instructors. A total of 416 students' exam sheets were collected and reviewed. Statistical methods were used to analyze the quantitative data. A total of 28 students and nine instructors were interviewed. The video and audio recordings were transcribed and analyzed in light of the aforementioned theoretical framework.

A three-phase research plan was used in this study. Phase I was designed to assess horizontal transfer of knowledge using traditional physics problems. Phase II was designed to assess vertical transfer of knowledge using non-traditional physics problems—"Compare and Contrast" problems, Jeopardy problems and Graphical Representation problems. In Phase III, we interviewed both physics and calculus instructors with regard to transfer of learning from calculus to physics.

Individual clinical interviews with student volunteers were conducted in each phase of this research. The interviews were analyzed using a phenomenographic approach. According to this approach the researcher categorizes different statements in the transcript. The categories arise from the data and are not created in advance by the researcher. The categories are then collapsed into emergent themes. The emergent themes are discussed below. This being a qualitative study with only 28 students, the relative percentage of students in each theme is not necessarily representative of the percentage of students in the population.

We found students appeared to retain their calculus schemas to solve calculus problems in Phase I. When interviewees were given pure calculus problems, they were able to solve the problems quickly and correctly. Furthermore, students self-reported that they were confident in their calculus knowledge retention because they remembered what they had learned in their calculus class and were able to do the calculus operations such as integrations and differentiations. On an average they ranked their calculus knowledge retention as 7 on a scale from 0 (most dissatisfied) to 10 (most satisfied). A majority of students believed that the calculus knowledge they retained was enough for physics courses since they had not come across any situations in physics that required a level of calculus with which they were not comfortable. This result is consistent with previous research on transfer of learning from algebra to physics.<sup>16</sup>

We found that students had difficulty associating their physics problem variables with their calculus schemas. Students were not confident in setting up calculus-based physics problems; even though they may have seen similar problems previously. Students typically appeared to be misled by the various numbers or constants in the physics problems and they could not decide what variable they were looking for. They tended to resort to novice problem-solving strategies such as means-ends analysis. Students had difficulty reading out of information from the given physics problems and aligning it with their calculus schemas. More specifically, students could not decide the variable of integration and limits of the integral. These results are also consistent with Tuminaro's<sup>16</sup> research for algebra-based physics courses, in which he found that students often failed to interpret their mathematics knowledge in a physical context.

Students did retain their calculus schema for performing integration and differentiation. But students had difficulties in transferring their calculus knowledge when solving a physics problem. We also found that assessing transfer of learning from calculus to physics must be

examined from multiple perspectives of transfer and use multiple research methods. Our results showed a statistically strong correlation between students' calculus and physics performance on physics exam problems. This appeared to indicate the possibility of transfer when viewed from the contemporary Actor-Oriented Transfer (AOT) perspective<sup>17</sup> which focuses on students dynamic constructions of similarities between two aspects of their knowledge. However, the correlation between student performance on physics exams problems and their calculus course performance was not as significant. We found relatively weaker evidence for the possibility of transfer when viewed from a traditional perspective and Preparation for Future Learning (PFL) perspective.<sup>5</sup> This weaker evidence for the possibility of transfer when viewed from a more traditional perspective compared with the evidence from a contemporary perspective is consistent with previous research on transfer.<sup>18</sup> Student difficulties in Phase I appeared to indicate that from students' point of view end-of-chapter physics problems involved vertical and not horizontal transfer as we had previously assumed. This observation is consistent with our flexible framework of vertical and horizontal transfer which allows for divergent interpretations of the same task as involving either horizontal or vertical transfer when examined from the perspective of varying levels of expertise.

We found that students had difficulty deciding when to activate appropriate calculus schemas. More than half of the interviewees admitted that they did not know the reason why they used integration in a given physics problem, other than they mimicked the strategy used in a similar sample physics problem from the lecture or a textbook. Thus, students often resorted to pattern matching while approaching their problems.

Our interviewees generally had difficulties solving the non-traditional 'Compare and Contrast' physics problems. They commented that they had not received any specific formal instruction on why to use integration instead of summation, and so they once again resorted to pattern matching by trying to recall similar problems that they had seen and use them as a guide to decide whether integration was important. Most interviewees stated that they had not addressed these issues in their physics course.

We also found most of our interviewees tended to use pre-derived algebraic relationship rather than calculus to solve the problem. They were unable to explain the conditions under which the closed form expressions were applicable. So it appears that students did retain their calculus schemas, but they did not have a clear understanding of when to activate their calculus schemas.

Based on our results that indicate students' failure to successfully complete non-traditional Jeopardy problems,<sup>19</sup> we conclude that students had difficulty in deconstructing and reconstructing their schemas. Again, students tended to rely on ends-means analysis without invoking deeper conceptual understanding. When trying to construct an appropriate physical situation corresponding to a given Jeopardy expression, we found students tended to focus on a limited numbers of constants rather than the variable of the integration or differentiation to help them construct the physical scenario. They often used dimensional analysis and unit matching to find out the physical quantity that was being calculated in the expression. Thus, students had difficulty in deconstructing their calculus schemas in Jeopardy problems of navigating multiple representations in the graphical representation problems.

Based on our findings we can conclude that students had difficulty in engaging in both horizontal as well as vertical transfer of learning from calculus to physics. We observed that our interviewees were relying on the equation sheets and doing pattern matching. They were confused about the meaning of different symbols and lacking a big picture of the problem. Most of the interviewees were able to perform calculations when solving traditional physics problems. In case of horizontal transfer we found that students had difficulties in associating physics variables with their calculus schema, although they appeared to have no difficulty in recalling the required calculus schema for integration or differentiation. In case of vertical transfer we found that students were unable to articulate a set of criteria that would enable them to decide when to activate the appropriate calculus schema. They also faced difficulties in deconstructing and reconstructing their schemas. Finally, students also appeared to have difficulties constructing connections between the meaning conveyed by the graphs and the corresponding symbolic representations.

Overall, students' problem solving behaviors appeared to suggest that they often resort to naïve strategies such as pattern-matching or ends-means analysis to solve problems. These problem solving behaviors appear to suggest that students searched for an appropriate schema to help them solve their problem. When they were unable to find the appropriate schema to solve a problem, they were often unable to construct or deconstruct an existing schema to address the problem at hand.

### **Transfer Studies from Physics to Engineering**

Almost all engineering students are required to have two semesters of calculus-based physics courses. These two semesters cover many of the concepts that are revisited in later engineering courses. For instance, the first semester of calculus-based physics focuses on topics that are revisited in the *Statics and Dynamics* course taken by engineering majors one semester after they have completed the physics course. Similarly, second semester calculus-based physics focuses on topics that are revisited in the *Electromagnetics* course taken by engineering majors two semesters after they have completed the physics course. Our intent was to measure the extent to which students were able to retain what they had learned in physics to the engineering courses.

In Phase I of this study we began with a survey of five engineering faculty members who were either currently teaching or had previously taught the above classes to find out what topics they would like their students to have already learned as pre-requisites in the physics class before taking the engineering course. Our surveys revealed that there was broad overlap between the topics that are typically covered in a physics course and those that the engineering faculty members would want the physics courses to cover. However, the engineering faculty members also expected that the students leaving the physics class would have already learned many of the topics covered in the earlier part of the course. For instance, in the *Electromagnetics* course, instructors expected that students should already possess an understanding of topics such as electric charges, force-at-a-distance, light as a wave and vector fields. They would then build on this understanding to cover topics such as Coulomb's Law, Gauss's Law, Ampere's Law and Maxwell's equations.

Based on the topics selected by the engineering faculty we designed two conceptual surveys. The first in *Electromagnetics* and the second in *Statics and Dynamics* focused on the topics that engineering faculty expected students to have learned from the physics class. The surveys included mainly multiple-choice but also some open-ended questions. We borrowed conceptual questions from Mazur's book on Peer Instruction.<sup>20</sup> We also utilized questions from the Force Concept Inventory.<sup>21</sup> The survey administered in the *Electromagnetics* course had 17 open-ended questions and was administered to 41 students. The survey administered in the *Statics and Dynamics* course had 14 multiple-choice questions and was administered to 108 students. We also administered the same surveys to students in a physics class after they had completed the relevant topics to gauge the extent to which the performance of the physics students who had recently learned the material would differ from those in the engineering course who had completed the material at least six months to one year prior to taking the survey.

Our results indicated that there were some differences. In six out of 17 questions in the *Electromagnetics* survey and seven out of 14 questions in the *Statics and Dynamics* survey a statistically significant percentage of engineering students answering the questions correctly was smaller than the percentage of physics students answering the questions correctly. In two of the 14 questions in the *Statics and Dynamics* survey the proportion of incorrect choices for the engineering students was skewed toward stating that not sufficient information was available. For instance, in a question asking students to find the center of mass of a composite object a statistically significantly higher percentage of engineering students versus physics students (43% versus 30%,  $z = 2.01$ ) responded that there was not sufficient information to answer the question. We speculate that this was because their training in engineering courses has taught students to question simplifying assumptions such as the uniform density of the material of a composite object. This is a type of assumption that physics classes typically tend to promote, e.g. considering an object to be a point mass to solve the problem. However, we speculate that engineering courses which are tied more to the real world prepare students to question these simplifying assumptions. Therefore a question that required students to implicitly assume that the density was uniform across a composite object was answered with a response of "not enough information" much more frequently by the engineering students as compared to the physics students.

In Phase II of this study we used clinical interviews to understand how engineering students related their experiences in their engineering courses to experiences in their physics courses. We recruited three students in *Electromagnetics* who had taken physics previously and interviewed them three times during the semester to talk about their experiences. Each interview was scheduled after students had taken an hour exam in their *Electromagnetics* class. Students were asked which exam questions were easiest and which were hardest. They were also asked which questions were least similar and which were most similar to those they might have encountered in a previous physics class. In addition to interviewing the students, we also interviewed three physics faculty to obtain their perspective. In each interview the physics faculty members were shown questions on the *Electromagnetics* exam. They were then asked which of these questions were most related and which questions were least related to questions in their physics class. Finally, they were also asked to describe the general similarities and differences between the questions in *Electromagnetics* and those in physics.



Results from the student interviews indicated that students often found the qualitative questions to be the easiest. The most difficult questions were the questions pertaining to vector calculus. Students were also able to describe the general conceptual area in physics that each of the questions in *Electromagnetics* was drawing from. However, they were unable to describe the connections in detail. When asked about the differences between the questions in *Electromagnetics* and physics, all students pointed to three major differences. The first was differences in mathematical notation. The same physical quantity was often described using different notations in physics and *Electromagnetics* and students found that this difference in notation posed a barrier to their ability to make connections between the two fields. The second difference was the way in which equations were expressed in physics versus *Electromagnetics*. We believe that this too is related to the difference in notation and therefore students were unable to realize that the equations in both the courses were in fact conveying the same meaning. Finally, the last difference that students pointed out was the difference in the kinds of examples in the two courses. They were referring to the differences in the context of the questions. Physics questions were more rooted in abstract contexts that were unrelated to everyday life. Questions in the *Electromagnetics* course were related to everyday examples, although they covered the same concepts. Again, students pointed to the lack of concrete examples in physics as making it more difficult to apply what they had learned in physics to engineering. In addition to the differences between the types of problems that students were asked to point out, at least one student strongly alluded to what he believed to be another important difference, i.e. the difference in the culture of the two disciplines. He went on to point out that in their engineering courses students were often required to work on projects in small groups. Thus they developed a sense of camaraderie which helped them get together to solve problems and prepare for the class in general. He contrasted this with his experiences in physics where group work was encouraged in the laboratory but beyond that students were not provided with context to work in groups. In fact, to a large extent students were asked to work on their homework separately. Thus the social dynamic in physics and engineering was quite different.

The results of the interviews with physics faculty members indicated that the faculty members were more likely than the students to see similarities and connections between the problems on the *Electromagnetics* exam and the physics problems that they cover in their courses. The physics faculty members pointed to three major differences between the problems in *Electromagnetics* and physics. First, they observed that the mathematical formalism was more sophisticated in the *Electromagnetics* course than in the physics course. This can be attributed to the fact that the *Electromagnetics* course is at a higher level than the physics course. Second, the physics faculty recognized that physics questions tended to be accompanied with pictures and diagrams – visualization that helped describe the situation. Finally, the physics faculty agreed with the students that the examples in *Electromagnetics* were much more realistic than those in physics. Overall, the physics faculty members felt that these distinctions between the problems were trivial and should not pose a major barrier to the students.

Overall, we can interpret the results above in light of our theoretical framework on horizontal and vertical transfer. We find that students have difficulties in transferring what they have learned from one course to another. Because students do not have well developed knowledge structures in physics that they can apply intact to problems in engineering, they often have to reconstruct their knowledge in engineering courses. In other words, transfer from physics to

engineering from students' perspective is primarily vertical transfer. Seemingly minor differences in notation and kinds of examples in physics and engineering pose an additional challenge to students as they try to reconstruct their physics knowledge in the context of their engineering course. From the physics faculty members' perspective, however, the knowledge transfer is virtually obvious, i.e. it is horizontal transfer. Because faculty members already possess well developed knowledge structures in physics they have no difficulties in applying them to problems in engineering courses. The differences in notation or the contextualization of the examples does not pose a barrier to them. The main challenge for educators then is to enable students to eventually develop the same kinds of expertise that faculty members possess.

### **Implications for Education**

Based on the results discussed above there are several concrete implications that could be adopted in mathematics and physics courses that would help facilitate transfer from physics to engineering. Faculty in mathematics courses should spend more time discussing the applications as well as the conceptual underpinnings rather than focus solely on strategies and techniques to solve problems. They should also introduce more 'word' problems as these problems are commonly encountered in physics and engineering courses. Learning the strategies of solving word problems in mathematics would better prepare students to solve these problems in later courses. Faculty in physics courses should focus on helping students learn how to interpret information in a word problem and to set up the solution. They should also be more mindful of notational and representational differences between physics and engineering courses. Faculty in both physics and mathematics courses should provide more opportunities for group work both in and out of class thereby preparing students for the collaborative learning experiences that they would encounter in their engineering courses as well as beyond. We believe that all of these issues would be better addressed if mathematics, physics and engineering faculty got together to plan and design the learning experiences for the students who take their classes. We have not made any changes in our courses yet based on this study, but these issues would well be considered in course or curriculum modifications that we might plan in the future.

In addition to the suggestions above, we must also view our results in the context of our theoretical framework as such an analysis might elucidate the big picture of the issues involved in educating our future engineers. When viewed through the lens of our theoretical framework of horizontal and vertical transfer, this study seems to suggest that educators should balance both horizontal and vertical transfer when helping students transfer their learning from calculus to physics to engineering, or more broadly from any structured domain to a relatively semi-structured or ill-structured domain. From a physics educator's perspective, our current mathematics education and physics education to some extent is mostly focused on horizontal transfer. However, when students come to our engineering courses, we expect them to engage in vertical transfer, i.e. solve problems that are more unstructured and contextualized than they did in their previous courses. We speculate that the rather abrupt change in focus from horizontal to vertical transfer in going from one course to another causes students to have difficulties because they have not gained enough training to engage in vertical transfer in their previous course.

Schwartz, Bransford and Sears<sup>13</sup> suggest that educators should follow what they call an Optimal Adaptability Corridor (OAC), as shown in Figure 1. The OAC provides a learning trajectory to

develop from a novice into an adaptive expert through the balance of efficiency and innovation, or say, a balance of horizontal and vertical transfer as per our framework. The mathematics, physics and engineering departments should work together to develop students' ability of horizontal and vertical transfer of learning.

How does one facilitate students' navigation through the OAC? Educators can adapt proven successful pedagogical strategies such as the Hestenes' Modeling Cycle<sup>22</sup> to foster both horizontal and vertical transfer in the OAC through incremental steps of Model Development and Model Deployment in the OAC, which correspond to iterative modeling cycles. In the Model Development step students develop a model on the new problem situation, while in the Model Deployment step they apply the developed model in a different situation.

The point of transition between the deployment of an old model and development of a new model is not arbitrary. Here too, proven theories and pedagogical strategies of conceptual development provide some clues. Piaget<sup>23</sup> suggests that an internal conflict or cognitive dissonance due to a discrepant event -- a contradiction between observations and expectations -- provides the necessary motivation for students to abandon or modify their existing model or schema (which provided the basis for their expectations). Piaget's ideas of cognitive dissonance can be adapted to this model of instruction in that by demonstrating the limitations of a particular model, we can provide students with the necessary impetus to modify their model. This realization of the inadequacy of a given model provides the necessary discrepant event that generates a point of inflection in students' learning trajectory and motivates them to develop a new model to address the new problem scenario.

This entire process entails students to reflect on their own thinking. They are now able to engage in metacognitive self reflection of the models that they construct and discuss the underlying assumptions and recognize the extent of applicability of these models. The students would thus navigate an OAC as shown in Figure 2.

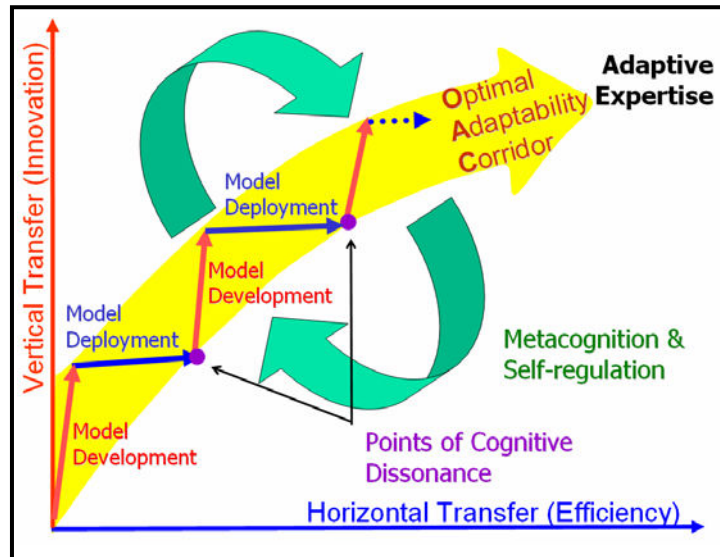


Figure 2: Navigating the Optimal Adaptability Corridor through successive iterations of Model Development and Model Deployment

Mathematics, physics and engineering courses should use small steps of Model Deployment following Model Development as shown in Figure 2 to promote horizontal and vertical transfer coupled with cognitive dissonance and metacognition to help guide students along an OAC toward developing expertise. For example, after students learn how to find the electric field by a point charge using Coulomb's law, educators should give students several point charges to deploy their understanding of Coulomb's law. Similar exercise and homework problems do exist in most physics textbooks, usually with the limit of three charges. After students have applied Coulomb's law to find electric field at a certain distance from several point charges, one should ask what they would do if they were given many closed spaced point charges. Students would realize it is unrealistic to add up a large amount of point charges and would be more amenable to develop an alternative model to calculate the electric field. This would provide a reason to use integration over summation. These kinds of experiences would help students learn to both construct new models as well as recognize their underlying assumptions and limitations. Thus these kinds of learning experiences will facilitate both horizontal and vertical transfer of learning as they move across multiple courses from mathematics to physics to engineering.

## Summary

We have investigated student transfer and retention of learning from mathematics to physics and physics to engineering courses. Our studies were informed by a theoretical framework based on contemporary ideas of transfer of learning. Based on these ideas we delineate two kinds of transfer of learning. Horizontal transfer involves applying previously learned knowledge to a new situation without modifying or restructuring the knowledge in any way. Vertical transfer involves restructuring or modifying our knowledge schemas to address the new problem at hand. We utilized both qualitative and quantitative methods in our studies and analyzed our data in the context of our theoretical framework. Our results indicate that students clearly have greater difficulty in accomplishing vertical transfer compared to horizontal transfer. While students are generally able to successfully recall what they have previously learned they have

difficulty adapting this knowledge to new situations. Differences in types of problems as well as notations between mathematics, physics and engineering contribute to these difficulties. The theoretical framework offers some insights into the kinds of strategies that might be successful in accomplishing both horizontal and vertical transfer by guiding students through successive iterations of small steps in both horizontal and vertical directions that would lead them to acquire adaptive problem solving expertise. While we have not implemented any changes in our current courses based upon this study, we believe that it provides some useful insights into the kinds of interventions that might be successful in the future.

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