Rethinking Precalculus: A Thematic Approach

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Abstract: Over the last decade, Precalculus at Louisiana Tech University has seen many revisions as needs and demands shift for the many STEM majors it services. Originally a 4 SCH course, it was changed to 3 SCH due to hour restrictions in some curricula but did not receive a proportional reduction in content. This dense course contained the majority of college algebra, trigonometry, logic, and a few topics from linear algebra, resulting in an overload in content for most students. Students with strong fundamentals were already experienced with the majority of the content, making the course uninteresting, while those who could benefit most from these topics were often overwhelmed by the rapid pace of the course. These issues, combined with faculty frustrations that Precalculus had become primarily review and remediation, led to a curriculum redesign to be piloted in fall 2022. The goal of the redesign was to create a course which was themed around a three-part core of logic, area under a curve, and limits while integrating algebra and trigonometry review. Emphasis is placed on exploration, rigorous derivations, and proofs to develop mathematical thinking.

In fall 2022 the pilot was administered to six sections of Precalculus. The progress of the students from each section was tracked through the 2022-2023 academic year. Data from exams in their subsequent calculus courses was collected and compared to their peers from non-pilot sections of Precalculus to determine if there were statistically significant differences in performance. This paper will outline and detail the curriculum. Statistical results from a preliminary study of effectiveness will be presented and discussed.

Introduction/Motivation

Louisiana Tech University operates on a quarter calendar but awards semester credit hours (SCH). This is accomplished by extending the meeting time for classes. For example, a 3 SCH class will typically meet for 75 minutes three times a week, or for 110 minutes twice a week for 10 weeks. One advantage this affords is students majoring in engineering and other STEM programs can begin in Precalculus without being behind in their curriculum, leading to benefits in retention [1]. This and other contributing factors have led to Precalculus serving as a gateway for most incoming freshmen into engineering and other STEM programs.

In the 2022-2023 academic year, well over 50% of incoming students in the College of Engineering and Science started in Precalculus. The grade these students receive in their first mathematics class can be an important indicator of success and retention [2] [3] [4]. This effect is more pronounced for incoming freshmen in engineering programs. Calculus I is a corequisite for the second course in the freshmen engineering sequence, making Precalculus a potential roadblock that prevents many students from progressing in their degree path. This contributes to an already high attrition rate among first year engineering students [5] [6]. These and other

factors have made Precalculus at this institution the target of frequent incremental reforms over the past few decades.

In the spring of 2022, a committee was formed with the goal of redesigning Precalculus from the ground up. Several key issues with the previous version of the course were identified. Most prominent among these was the pace. To be fair on faculty workloads, in 2011 the class was reduced from 5 to 3 contact hours to match the credit hours of the course. While the material covered was somewhat reduced, reluctance to trim anything perceived as necessary by various faculty resulted in a course that covered most of a traditional college algebra class, a trigonometry class, and a week of linear algebra within the time constraints of a 3 semester credit hour course. Another side effect of this was many of the more advanced concepts in the course being cut in favor of keeping the more fundamental algebra and trigonometry skills viewed as essential to success in calculus. The resulting course was focused on the idea of remediation or review and heavily emphasized procedural skills.

The goal of the redesign was to reduce the pure focus on procedures while keeping the course accessible to students placed into Precalculus by integrating review of trigonometry and algebra with topics that have a direct connection to calculus, such as limits and area under a curve. An increased emphasis on exploration, derivations, and proofs were used to develop mathematical thinking. This has the additional benefit of exposing students to topics which are more engaging and less focused on relearning what they would have already seen in high school, which is seen as a possible shortcoming in standard precalculus courses [7]. The key topics to be covered were mathematical logic with functions, area under the curve, and conic sections with trigonometry.

Initial Course Design (Summer 2022)

The design committee met regularly in the summer of 2022 to create the initial version of the pilot. A study of mathematical logic was the central focus for the beginning of the course. Functions and related properties such as factoring, roots, and inequalities were discussed while analyzing the logical consequences of statements. Students are asked to explore statements such as, “Let x be any real number that satisfies a given function f(x), where $f(x) > 0$.” Consequently, students are required to discover methods for obtaining the roots of polynomials, inequalities, and properly handling universal and existence logic. By understanding the relationship of the x-axis to the function behavior students can establish intervals on which the function is above or below the x-axis. This led to a conversation considering the area under the curve of a function on a specified interval, which began with geometric understanding about area.

The middle of the course focused on building up the tools needed to calculate the exact area under a curve. It began by establishing the approximate area as a series of approximating rectangles that the students were asked to seek ways to improve upon. This led naturally into discussions of summation, sequences, mathematical induction, and limits of sequences at infinity. As a bridging topic, limits at infinity of function and the end behavior of functions were established to extend from the limits of sequences. Finally, conic sections and three-dimensional space were utilized to introduce a discussion of trigonometry.

The course was viewed as a “preparation for calculus” course as opposed to a traditional precalculus course. The initial goal was to prepare students for calculus and reinvigorate the

discovery in mathematics. In all subject matters and discussions, the content was contained to the class and there was not a published text or video series required as part of the course to further their studies. The aim of this pedagogy was to allow the instructors the flexibility to answer the deeper questions that were relevant to the discussion without discouraging students to think freely by narrowly focusing on the content that needed to be taught on a specific day.

Pilot Group (Fall 2022)

In the fall quarter of 2022, six sections of Precalculus consisting of 176 students were chosen to pilot the new curriculum. Four different instructors taught these six pilot sections. The key topics previously mentioned were established as the three core studies. Each of these cores spanned roughly one third of the course and concluded with a 150-point, 75-minute exam. At points halfway through each core topic there was a 50-point, 30-minute test to ensure that students were maintaining their studies. The course concluded with a comprehensive final worth 250 points.

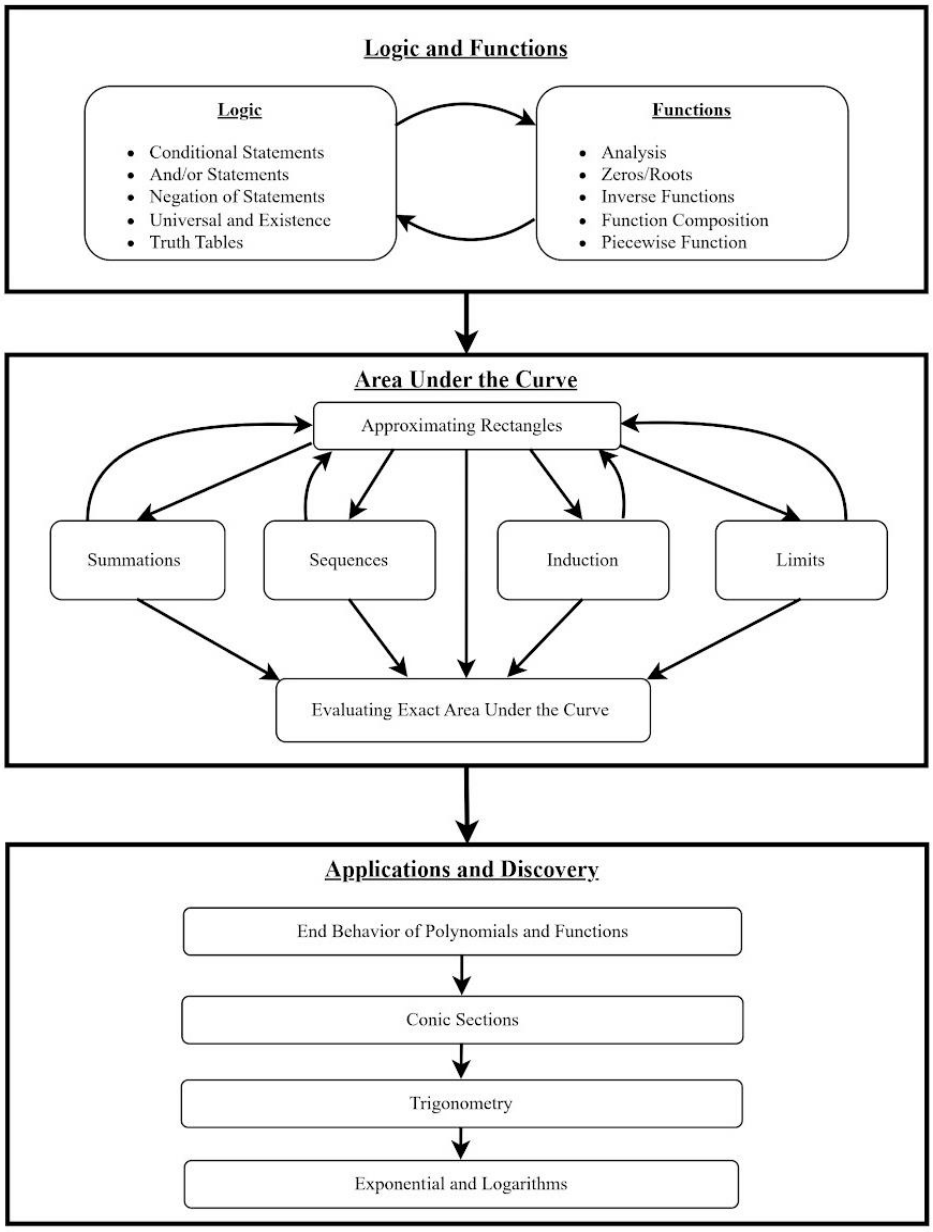
Homework was assigned in class using worksheets and students were required to write their work into a bound notebook (3-ring binder, science notebook, or spiral bound). This handwritten homework approach was used to develop student's ability to express their work clearly. During each test, the notebooks were collected and scored. The instructor gave feedback on errors that were noticed and gave a score that reflected the student's ability to communicate and execute the material. The scores did not impact the student's course grade; however, if a student earned a passing score on all the notebook checks, then the final's scaled percentage was able to replace the lowest exam grade.

Second Iteration (Winter 2022-2023)

In the second quarter, two sections consisting of 62 students were chosen for the pilot. Several incremental changes were made to refine the content and delivery. Content revisions included an increased emphasis on analyzing roots of higher order polynomials and examining end behavior of functions to connect the concept of limits with a review of various types of functions. Minor adjustments to conic sections were made to smooth the transition to trigonometry. The structure of the final course design can be seen in Diagram 1. Homework content and delivery were made more consistent with the content from the lectures. A formal notebook was established that was given to students with formatted problems and spacing for work. These notebooks were checked as in the previous term.

A common issue identified in the fall quarter was feedback on homework not being given until students had already taken the associated exam unless students requested it during office hours. Weekly 15-minute tests were implemented to give instructors the opportunity to deliver regular feedback to students and discourage procrastination. Each test was given on the same day of the week and was worth 50 points. The course grade now consisted of these weekly exams, a 75-minute common midterm worth 150 points, and a 75-minute cumulative common final exam worth 200 points. The homework was scored in the same manner the previous term.

Diagram 1: Final course curriculum for 2023-2024.



Third Iteration (Spring 2023)

For the spring quarter of 2023, all 4 sections of Precalculus with 110 students were moved to the pilot course. These sections were taught by 3 instructors, one of whom was teaching this curriculum for the first time. Few content revisions were made this quarter as the question of how scalable the course format would be to 14 sections in the fall was the focus. The exam structure from the winter quarter was carried over. Due to varied class times, and in preparation for fall, the midterm and final exams were not proctored during class times but during an evening

time slot for 120 minutes. This was more in line with the policy and procedures of later calculus courses in the sequence.

Although the weekly exams were continued into the spring quarter, the format made it impossible to maintain verbose feedback on homework notebooks as given in the fall. It was also seen as redundant to give feedback on both homework problems and weekly exam questions. Thus, the decision was to not implement any scored feedback to the homework. Furthermore, homework notebooks in the winter and spring term were produced by the department and/or the instructors. This was not considered scalable to the fall term where approximately 500 students were expected to enroll in precalculus. Talks with an independent publisher were established for students to be able to purchase homework notebooks through the on-campus bookstore.

Analysis

Different curriculum and testing structures between the pilot and non-pilot sections of Precalculus made a direct comparison of student performance in the two impossible. Instead, performance on exams in the subsequent Calculus I and Calculus II courses over the 2022-2023 academic year were used as a source of comparison. The Calculus I data only contains students that passed both versions of Precalculus in the fall, and the Calculus II data only contains students that both passed Precalculus in the fall and Calculus I in the winter. Two groups were created for comparison composed of non-honors students who took either the pilot class or the non-pilot Precalculus class. For all exams, the groups' performance on each exam is compared through a two-sample z-test.

Performance in Calculus I

A typical first exam in the Calculus I course would cover topics such as limits of functions and the definition of a derivative. The second exam covers differentiation rules, related rates, linear approximation, and differentials. Lastly, the third exam tests optimization, L'Hospital's rule, curve sketching, and definite integration via Riemann sums.

	Exam 1		Exam 2		Exam 3	
Group	**Exam Average**	**Sample Size**	**Exam Average**	**Sample Size**	**Exam Average**	**Sample Size**
Pilot	79.58	118	74.48	109	67.67	106
Non-Pilot	78.09	84	78.27	77	68.88	73
Pilot vs Non-Pilot	**p-value** = 0.261931		**p-value** = 0.077221		**p-value** = 0.350229	

Performance in Calculus II

The first exam in Calculus II covers antidifferentiation, the fundamental theorem of calculus, u-substitution, area and volumes. The second exam covers techniques of integration and improper

integrals. The third exam, during the period in which the data was collected, consisted of an introduction to probability and statistics where continuous distributions are integrated.

	Exam 1		Exam 2		Exam 3	
Group	**Exam Average**	**Sample Size**	**Exam Average**	**Sample Size**	**Exam Average**	**Sample Size**
Pilot	78.71	78	72.64	73	78.63	67
Non-Pilot	75.98	63	70.21	63	75.82	58
Pilot vs Non-Pilot	**p-value** = 0.111765		**p-value** = 0.179730		**p-value** = 0.209049	

Discussion

The students in the pilot version of Precalculus performed neither better or worse than non-pilot students at a significance level of 5%, though a significant increase in pass rate was observed in the pilot section over the non-pilot. Whether the curriculum of the course or other factors such as more frequent testing is primarily responsible for this increased pass rate is difficult to determine. Some studies have shown little to no correlation between taking a traditional precalculus course and success in the following calculus courses [8] [9]. The integration of algebra and trigonometry review with introductory calculus concepts makes this course closer to a calculus course with integrated review, which has shown more positive results [10] [11] [12]. Future studies comparing students who are recommended placement in Precalculus on entrance exams but are allowed to start in Calculus I by AP or transfer credit with those who start in Precalculus may be used to further evaluate the effectiveness of the curriculum and examine the necessity of a precalculus course. A deeper look at the relationship between performance in Precalculus and the freshman math sequence with graduation and retention rates for engineering students at Louisiana Tech University is also planned. Despite no change in performance in subsequent calculus courses, enthusiasm among faculty for the pilot led to full adoption in fall 2023 with the goal of further refining the curriculum and developing more supporting resources in the future.

Table 1: Pass rates for pilot and non-pilot sections of Precalculus for the 2022-2023 academic year.

2022-2023	Pass rate	Sample Size
Pilot	69.25%	348
Non-Pilot	60.21%	377
Pilot vs Non-Pilot	**p-value** = 0.005509	

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