

Rethinking the Curricular Complexity Framework for Transfer Students

Dr. David Reeping, University of Michigan

Dr. David Reeping is a Postdoctoral Fellow in the Engineering Education Research Program at the University of Michigan. He earned his Ph.D. in Engineering Education from Virginia Tech and was a National Science Foundation Graduate Research Fellow. He received his B.S. in Engineering Education with a Mathematics minor from Ohio Northern University. His main research interests include transfer student information asymmetries, threshold concepts in electrical and computer engineering, agent-based modeling of educational systems, and advancing quantitative and fully integrated mixed methods.

Dustin Grote, Weber State University

Dustin currently serves as an Assistant Professor in Teacher Education at Weber State University and leads the higher education leadership program. He holds a PhD from Virginia Tech in Higher Education. His interdisciplinary research agenda includes graduate funding in STEM, transdisciplinary, experiential and adaptive lifelong learning, undergraduate education policies, systems thinking, organizational change, broadening participation in engineering, improving community college transfer pathways in engineering, curricular complexity in engineering, and assessment and evaluation in higher education contexts. Prior to pursuing a Ph.D., Dustin served as a Postdoctoral Research Associate in Engineering Education at Virginia Tech, Director of Admissions at Community College of Denver, and in Outreach and Access Initiatives for the Colorado Department of Higher Education.

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Abstract

This theory paper explores how we can extend the *curricular complexity framework* to better capture vertical engineering transfer students' experiences – i.e., students transferring from a community college to a four-year university. Most prior research using the framework focuses on characterizing what makes university engineering programs “complex” for first-time-in-college (FTIC) students by quantifying the interconnectedness of their prerequisite structures. These measures have been correlated with other metrics like retention, time-to-degree, and program quality. Unlike FTIC students, transfer students can enter the curriculum at multiple points in time and often bring course credit that is applied to requirements, relegated to electives, or lost entirely. As currently conceptualized, exploring transfer student pathways appropriately with the current version of the curricular complexity framework is difficult without constraining assumptions - e.g., analyzing the effects of a curriculum revision and assuming no courses are transfer-friendly. To address this gap in the literature, we adapt the curriculum complexity framework to capture challenges vertical engineering transfer students may encounter in their pathway to a four-year degree.

Introduction

Suppose we wanted to quantify how much more accessible a curriculum becomes when removing a prerequisite to a particular course. How would we do it? We could wait a few years to calculate the typical metrics related to retention, observing the flow of students through that portion of the curriculum. On the other hand, a method drawn from graph theory does not require us to wait. In fact, all we need is the plan of study itself.

The method is formally called *Curricular Analytics*, which defines a *curricular complexity framework* of metrics that measure different facets of a plan of study. These calculations aid practitioners and researchers in examining how a curriculum's accessibility changes when prerequisite structures or sequencing patterns are altered. The curricular complexity framework is composed of two primary metrics used to describe a curriculum's interconnectedness [1,2], which have been used to predict program quality, retention rates, and completion rates [1,3,4]. An interactive web tool (<https://curricularanalytics.org/>) is available from the creators to conduct descriptive and predictive analyses.

The two measures in the curricular complexity framework are the *blocking factor* and the *delay factor*. The *blocking factor* of a course refers to the number of courses inaccessible to a student who fails the course in question. The *delay factor* of a course is the longest prerequisite chain flowing through it. When added together, the two metrics form the course's *cruciality*. A demonstration of the calculation for the *blocking factor* and *delay factor* is given in Figure 1. To characterize the overall curriculum's complexity, we can sum the crucialities together to form an aggregate measure called the *structural complexity*. Previous research has shown that structural complexity correlates well with FTIC student graduation rates [5] but does not for transfer

students [6], suggesting that the current metric might not account for structural nuances in transfer student pathways.

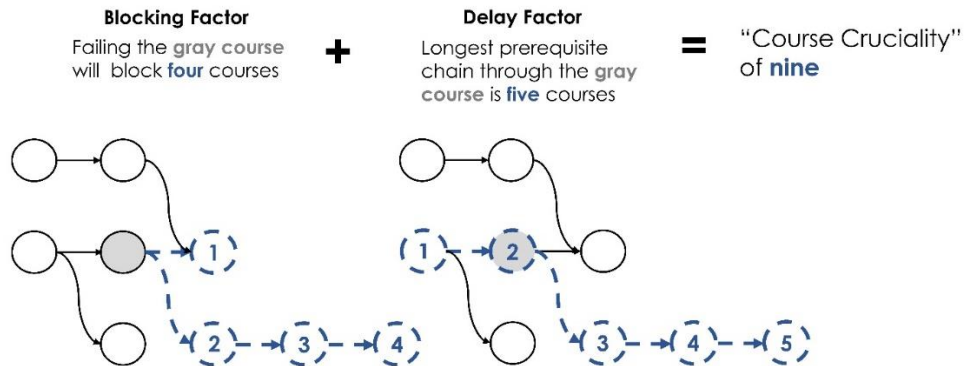


Figure 1. Example calculation of *course cruciality* using the *blocking factor* and *delay factor*

To provide grounding for what kind of values to expect from structural complexity, Table 1 presents a series of examples that increase in interconnectedness. Empirical values of curricular complexity for four-year programs from 63 schools ranged between ~50 and ~500 with an average of 273.6 in Heileman et al.'s program quality study [3]. Within institution variation is also notable; the range was 191-618 in a study by Grote et al. at Virginia Tech [6]. Note that the metric depends on the number of courses in the plan of study, so comparisons using the raw measure between plans of study with vastly different sizes are not appropriate.

Table 1. Structural complexity behavior under different prerequisite configurations

Configuration					
<i>Structural Complexity</i>	4	7	10	11	12
<i>Blocking Factors Sum</i>	0	1	2	3	4
<i>Delay Factors Sum</i>	4	6	8	8	8
<i>Explanation</i>	No prereqs, so all courses have delay factors of 1 by default because the longest prereq chain through them is just themselves.	Adding 1 prereq makes one course blockable, and the connected courses have a prereq chain of 2 running through them - increasing the delay factor sum by 2.	Adding another prereq in this fashion blocks another course, increasing the sum of blocking factors by 1, but the delay factors cannot increase because the longest possible chain is 2 courses here.	Having 2 prereqs coming from a single course means it can block 2 courses now, increasing the sum of blocking factors by 1.	Finally, saturating the set of four courses with prereq chains increases the blocking factor sum by one.

Research Aims

This theory paper aims to extend the structural complexity metric's theoretical underpinnings by conceptualizing a transfer student structural complexity model. Specifically, we considered three transfer-centric issues highlighted in [6]: 1) timing of courses being offered (e.g., fall vs. spring semesters); 2) strings of prerequisite courses not offered at the community college that may add additional semesters to a degree plan; and 3) credit loss. We leveraged these suggestions to form our target conceptual model, as shown in Figure 2.

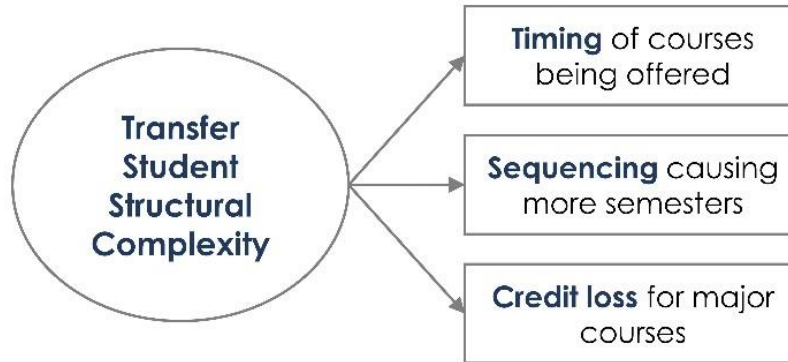


Figure 2. Our conceptual model to operationalize *transfer student structural complexity*

We explored ways to address each of these concerns mathematically, translating each transfer-specific curricular issue into tangible sub-metrics of structural complexity. For example, we introduce a metric that accounts for whether courses are being offered each semester (e.g., Fall or Spring). Next, we constructed a measure called the *transfer delay factor* that accounts for prerequisite strings of courses at the University that extend students' expected time to degree. We also incorporated a method for tabulating credit loss.

We present our model's coherence through test cases and highlight where these metrics can better capture transfer student structural complexity. Building on these results, we offer suggestions for the future development of the curricular complexity framework. Moreover, we highlight avenues for researchers and practitioners to apply these metrics in transfer student receptivity and retention studies.

Operationalizing *Transfer Student Structural Complexity*

This section will describe our rationale for operationalizing the three components specified for *transfer student structural complexity*. Table A1 in Appendix A summarizes the notational convention we will use, staying consistent with [1], throughout our mathematical descriptions in roughly the order of appearance.

Timing of courses being offered

Unlike FTIC students, vertical transfer students may enter the 4-year institution at varying points in the engineering curriculum. For example, students may complete only the first year of introductory coursework in engineering at a community college before applying for transfer to the 4-year institution. Other students may seek to transfer via articulated degree pathways that prescribe completion of an Associate Degree at the community college, which are typically designed for completion in two years. Other 4-year institutions may allow for Spring admission for transfers or even rolling admissions for students to enter at any point in their degree progress. Simultaneously, prescribed plans of study of 4-year engineering degrees usually consider the timing of courses being offered by departments (i.e., Fall, Spring, Summer). Although required courses in the first two years of a plan of study are often offered every semester, upper-level in-major courses may be offered less frequently, often due to less faculty teaching those courses and thus more limited availability for when and how frequently courses can be offered.

As currently constructed, the curricular complexity framework assumes that all courses are offered all semesters in a plan of study. However, transfer students who may arrive in a Fall semester and need a course that is not offered until the Spring (or is a course only offered on odd years) will be forced to wait for one, two or even three semesters. With this reconceptualization of the timing factor, we hope to better capture the nuance within course (in)flexibility that engineering transfer students encounter.

To operationalize our timing factor and capture how inflexible a curriculum is, we consider a useful measure already explored by Heileman et al. [1] in their original paper, i.e., the curriculum's *degrees-of-freedom* – denoted by z_c as the *degrees-of-freedom* for curriculum c . This measurement is the number of unique ways a curriculum may be reordered term by term given the existing prerequisite structure. The *degrees-of-freedom* is related to the *delay factor*; if vertices in the graph have high *delay factors*, then there are fewer ways to reorganize courses in a way that does not violate the existing prerequisite structures.

For example, Figure 3 shows a curriculum on the left with a strict prerequisite chain composed of five courses. Because we cannot reorder these courses given the three-term structure, the *degrees-of-freedom* is 1 – i.e., there is one way to represent that curriculum. If we freed one of the courses like v_{23} from enforcing a prerequisite, then it is feasible for v_{23} to be moved from the third semester to the first or second semester (the second of which would be more likely) – giving us two options. Moreover, the v_{22} course could be delayed to the third semester. This movement introduces three scenarios, v_{22} could be taken concurrently with v_{23} , v_{22} could be taken in the first semester with v_{11} , or v_{22} could be taken in the second semester with v_{12} . Thus, there are six ways to organize the curriculum, including the original arrangement, i.e., $z_c = 6$.

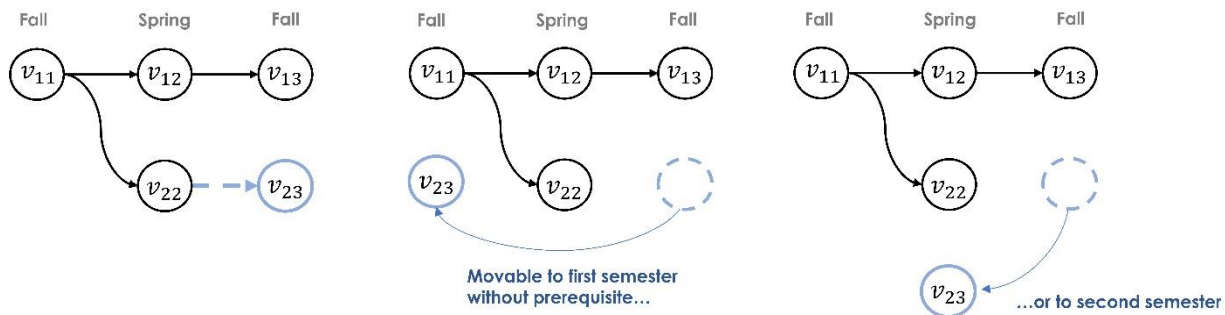


Figure 3. Demonstration of *degrees-of-freedom*, the curriculum on the left has $z_c = 1$ but removing the prerequisite from v_{22} to v_{23} allows flexibility for v_{23} to be taken in the first or second semester (along with other rearrangements when v_{22} is moved to semester 3).

Now, what if v_{23} was only offered in the Fall? This constraint would make the second semester infeasible because it is a Fall term, reducing the *degrees-of-freedom* from 6 to 4. Unlike courses earlier in the students' plan of study, which are offered more frequently, courses offered later in the curriculum, such as electives or capstone courses, are more likely to be offered in a specific semester – or even alternating years in a specific semester. Students need to wait until the appropriate semester to take the course with these timing conflicts, increasing their time-to-degree. Therefore, we contend that the *degrees-of-freedom* measure could be a useful indicator for our conceptual model.

Heilemen et al. [1] do not offer the precise calculation of *degrees-of-freedom*, though they provide a clue that the *degrees-of-freedom* is related to the number of *weakly connected components* in the graph. A *weakly connected component* is a sub-network where all vertices have a degree of at least one, meaning they are connected in some fashion and each vertex is reachable from another vertex. Because weakly connected components are independent of one another, we can find all valid rearrangements of the courses in them individually. Therefore, for the i th sub-network, consider how many ways its vertices can be moved between semesters while obeying the prerequisite structures and course time offers – call this value m_i .

Note that further restrictions can be placed on the rearrangements. For example, consider a set of six 3-credit-hour electives (18 total credit hours) students must take that fulfill general education requirements. It would be unreasonable for a student to take all of these elective courses in the same term in addition to three 3-credit-hour major courses in the same semester (27 total credit hours). So, we could neglect permutations of courses that result in a semester having more than a *reasonable* amount of credit hours (i.e., more than 19 credits). As we place more and more restrictions on the rearrangements, we doubt the m_i 's can be expressed using a *nice* formula and these quantities would need to be calculated using an algorithm.

Once we find all the m_i 's, we can use the fact that the *weakly connected components* are independent once again. Because we calculated the number of ways the vertices in the *weakly connected components* can be permuted independently of one another, we can use the *fundamental counting principle* to find the number of ways one could reconfigure the entire

curriculum graph. Therefore, we contend the calculation for the *degrees-of-freedom* is simply the product of the m_i values:

$$z_c = \prod_i m_i$$

However, implementing the measure is laborious and may fine-tune our calculations to a level that does not practically exist by considering implausible rearrangements. Moreover, this value tends to grow quickly due to the calculation's factorial nature, so we opted for a slightly different calculation.

In our operationalization, we still consider how the curriculum can be reorganized term-by-term, but we examine how many semesters of flexibility each course has in the prerequisite structure and add these values together. For courses at the end of a prerequisite chain or isolated, we can determine how many semesters of flexibility they have using the idea of *weakly connected components*. We approached this iteratively by first finding all isolated courses, finding how many semesters they can move, then deducting the semesters not allowable because of timing. Next, we calculate the delay factors of all the courses and subtract them from the number of terms in the plan of study as codified, n_t . Because the delay factor is the longest prerequisite chain flowing through the course, it tells us the number of semesters the course *could* move.

If the delay factor is equal to the number of terms, we classify the course as “stuck.” We then check for corequisites. If they are associated with a “stuck” course, they also are classified as “stuck” because corequisites are intended to be taken together. Finally, we check each “moveable” course by finding their prerequisites and postrequisites. Then, we compare the range of the terms these requisites span and compare them to an alternating string of Fall and Spring to deduce how many terms are ineligible for the course to be moved – this value is given by $u(v_{ij})$. Therefore, we contend *degrees-of-freedom* can be calculated as:

$$z_c = \sum_{\substack{v_{ij} \\ j \leq t_e}} n_t - d(v_{ij}) - u(v_{ij})$$

Note that we will exclude the courses beyond the expected time-to-degree because we are measuring the flexibility *within* the expected pathway. This would mean only calculating the values for the community college courses up to the expected time-to-degree there as well.

Finally, now that we have a metric describing how *flexible* a curriculum is, we can capture how much of the curriculum is *inflexible* by considering the maximum possible *degrees-of-freedom* the curriculum could have, z_c^* , and forming a ratio. This maximum can be found using the following formula:

$$z_c^* = n_c(n_t - 1)$$

where n_c is the number of courses in the plan of study. To calculate our metrics, we must treat the networks as two pieces: one for the community college and the other for the four-year

institution. Otherwise, a metric like the *degrees-of-freedom* would treat one of the four-year courses as movable to the community college. Therefore, let z_{cc} refer to the community college *degrees-of-freedom* and z_{fy} refer to the four-year institution's *degrees-of-freedom*. Then, we can form the *inflexibility factor*, I_f , as follows:

$$I_f = 100 \left(\frac{z_{cc}^* - z_{cc}}{z_{cc}^*} + \frac{z_{fy}^* - z_{fy}}{z_{fy}^*} + d_{cc} + d_{fy} \right)$$

We multiply by 100 for ease of interpretation.

An annoying numerical issue with I_f is that it decreases when transfer students are faced with the issue of a course not being available at the community college. This increase occurs because a course in a prerequisite chain can now move within the semesters at the community college, artificially inflating the *degrees-of-freedom*. Thus, we introduce penalty terms d_{cc} and d_{fy} .

These penalty terms account for the situation we described by adding the fraction of the number of required courses not offered at the community college, d_{cc} , and the fraction of the four-year's courses that must be taken beyond the expected time-to-degree d_{fy} .

We contend this measure aggregates the different effects on timing and availability that could impact transfer students, while adjusting for false increases in flexibility. With our other factors, this *inflexibility factor* measure will capture how inflexible the curriculum is for a transfer student, including the extent they can delay or pull courses to earlier semesters to fulfill credit hour requirements for financial aid packages.

Sequencing causing more semesters

There is extensive evidence that transfer students take longer than FTIC students to graduate in engineering (e.g., [7],[8],[9]). Few prior studies have considered how curricular complexity contributes to this extended time-to-degree for engineering transfers. In our previous work [6], we compared the most expedient pathways to graduation for 56 transfer pathways with 14 FTIC pathways in engineering at Virginia Tech – including pathways as a result of curricular change in the Electrical and Computer Engineering degree programs [10]. We found that, by design, 93% of FTIC pathways could be completed in 4 years (8 semesters) compared with only 11% of transfer pathways across engineering disciplines. Despite the additional semesters, the aggregate curricular complexity scores for the transfer pathways were lower than FTIC pathways, indicating that curricular complexity's current operationalization fell short in capturing how course sequencing is extending transfer students' time-to-degree, which has significant financial implications for transfer students and completion implications for engineering programs. Our reconceptualization seeks to remedy this issue.

To account for course sequencing, we draw upon the existing measure, *delay factor*, to build an indicator for our conceptual model of transfer student structural complexity. Recall that the *delay factor* is the longest prerequisite chain through a given course. Like the *degrees-of-freedom* measure, long prerequisite chains reduce the flexibility of the curriculum. For transfer students, high delay factors involve unnavigable prerequisite chains extending their time-to-degree by additional semesters. We can capture the extension to the students' time-to-degree by examining

the subgraph, including any courses beyond the intended time-to-degree, which is often eight semesters. Within the subgraph, we can find the sum of the *delay factors* for the courses extending the time-to-degree. This sum provides a measure of the prerequisite chain density leading up to the courses responsible for the extra semesters. We call the sum of the delay factors for courses that extend the expected time to degree the *transfer delay factor*, T_d . The *transfer delay factor* can be readily expressed using the delay factor notation. Let t_e be the *expected time-to-degree*, then:

$$T_d = \sum_{j>t_e} \sum_i d(v_{ij})$$

This formula involves selecting the vertices v_{ij} (i.e., i th course in j th semester) that have $j > t_e$ and adding all their delay factors, $d(v_{ij})$, together.

To illustrate how the *transfer delay factor* would work, consider the six-semester example in Figure 4. Assume the intention of the curriculum is for the student to graduate after the fourth semester, but the student needs to take v_{15} , v_{25} , and v_{26} due to issues with transfer credit. The subgraph we would form includes any courses that are listed as prerequisites for v_{15} , v_{25} , and v_{26} . First, v_{25} is a prerequisite for v_{26} . Then, we consider any courses listed as prerequisites for v_{15} and v_{25} , which are v_{14} and v_{24} . Next, we see which courses are listed as prerequisites for v_{14} and v_{24} , so we include v_{13} and v_{23} . Then, v_{12} and v_{22} are included because they're prerequisites for the previous pair. Finally, we incorporate v_{11} because it is a prerequisite for both v_{12} and v_{22} . Any other courses not related through the prerequisite chain are excluded – which are v_{21} , v_{32} , v_{33} , and v_{34} . From here, T_d can be calculated by finding the longest prerequisite chains leading to v_{15} , v_{25} , and v_{26} . Including v_{15} , v_{25} , and v_{26} themselves, the delay factor for v_{15} is five because there five courses in its prerequisite chain. On the other hand, v_{25} and v_{26} have delay factors of 6. Therefore, $T_g = d(v_{15}) + d(v_{25}) + d(v_{26}) = 5 + 6 + 6 = 17$.

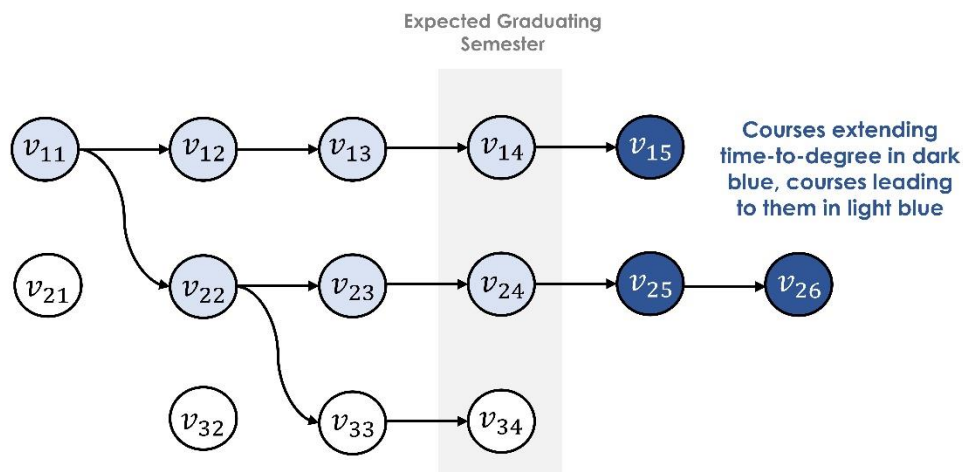


Figure 4. Example of calculating the transfer delay factor, v_{15} , v_{25} and v_{26} extend the time-to-degree by two semesters (dark blue), so the subgraph focused on those courses include any preceding courses (light blue) in the prerequisite chain

This approach punishes longer prerequisite chains that extend multiple semesters beyond the expected time-to-degree by the way the delay factor is calculated. Because v_{26} is preceded by v_{25} , the delay is magnified. We contend the punishment is appropriate because we can capture curricula that extend time-to-degree by multiple semesters with dense prerequisite chains.

Credit loss for major courses

Finally, we seek to address the issue of credit loss that is not currently included in the curricular complexity framework. Credit can be accounted for in three ways: 1) credits that *do not transfer* to the 4-year institution; 2) credits that are *transferred* wherein the student earns credit, but they count toward elective credits that do not advance degree progress; and 3) credit that was *applied* toward courses required for students to complete their degree at the 4-year institution. Our indicator captures (1) and (2).

Of all our metrics, credit loss is likely the most crucial because the loss of credits has been found by Monaghan and Attewell [11] to decrease the likelihood of graduation. The loss of credit is widespread; Simone [12] reported that students who began college in 2003-2004 lost 13 credits on average when transferring from one institution to another, and about 40 percent of students had no credits that transferred. Moreover, credits counted toward electives are particularly unhelpful. Kadlec and Gupta [13] describe courses that are transferred with the elective designation as an “academic graveyard where students essentially bury all those courses that transfer but do not meet any specific requirements in the new institution” (p. 7). In essence, students could bring in several credits that do not advance them in any practical way. Therefore, we contend that credit loss should be considered part of the transfer student conceptualization of curricular complexity because it is an issue that transfer students regularly encounter, particularly in engineering and other highly sequenced disciplines.

Our indicator for *credit loss*, which we will denote by C_l , tallies the number of credits the student did not have applied to their major courses. We can directly edit the curriculum graph by deleting vertices where credit was applied for courses that did apply. Deleting vertices is especially necessary if the curriculum graph is made by concatenating the degree program from a different institution. For example, if a student earned credit for Calculus I, it would be redundant to include Calculus I at the second institution. Therefore, we can delete the repetitive offering and reconcile the associated prerequisite chain. This step completes our operationalization of credit loss. We summarize the differences between *credit loss* and *credit applied* in Figure 5.

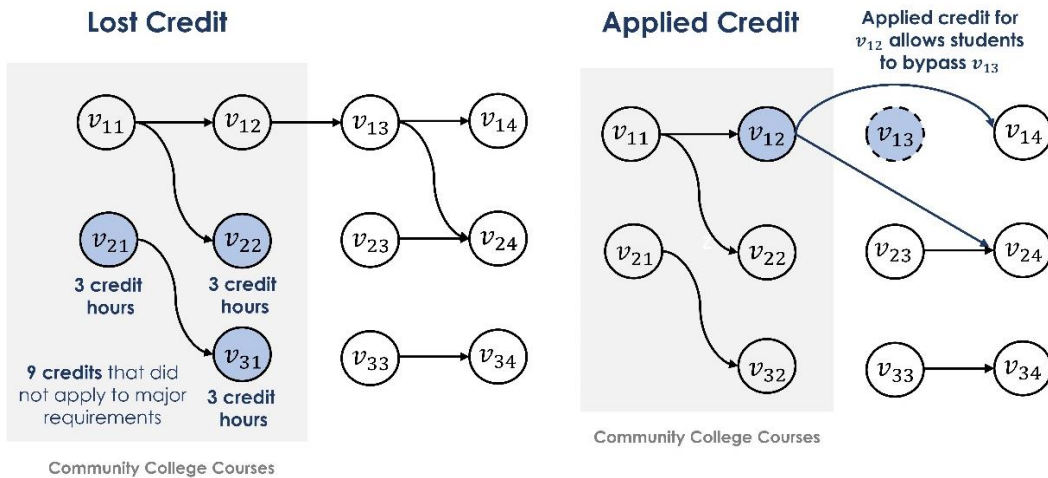


Figure 5. On the left, three courses worth three credits each are *transferred* into the next institution but do not count toward any major courses; on the right, v_{13} is excluded from the prerequisite chain because credit from v_{12} is *applied* to v_{13}

Summary of our operationalization

Now, to revisit our original conceptual framework and replace the constructs with the metrics we outlined. Our complete model is shown in Figure 6. We contend we can construct a measure of *transfer student structural complexity* using our combination of variables to account for the timing courses are offered (*inflexibility factor*), the sequencing of courses that lead to semesters beyond the expected time-to-degree (*transfer delay factor*), and credits lost during transfer (*credit loss*).

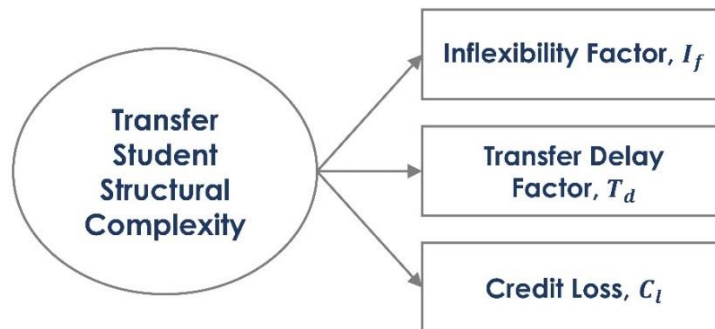


Figure 6. Conceptual model with metrics from the curriculum graphs as indicators

We created an R package to handle the calculations of all our chosen metrics. We acknowledge the Curricular Analytics team has a toolbox in Julia ([on Github](#)) [14] and an increasingly robust [online tool](#). The purpose of translating the package to R was to create a version amenable to this research's needs because we desired the custom functionality; both authors are well-versed in R. Moreover, we aim to increase the original tool's visibility by porting it to other languages used by the community, including R.

Exploring How Our Conceptualization Functions with Test Cases

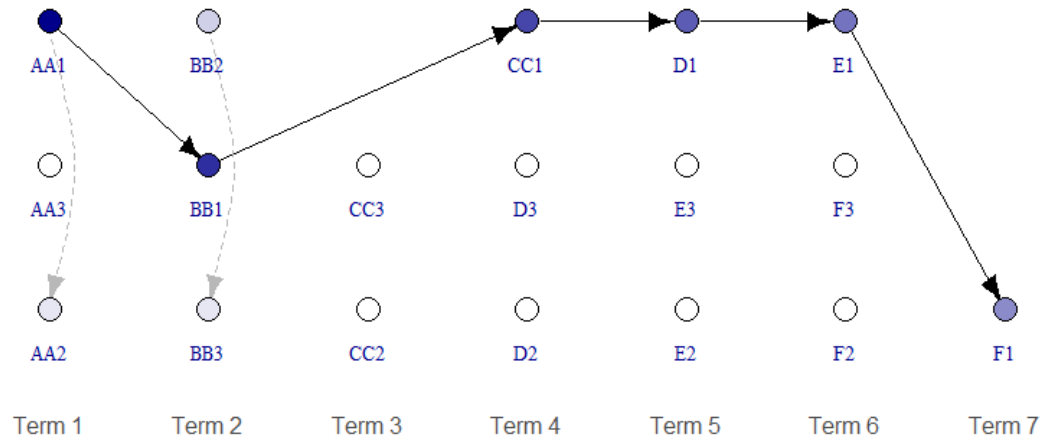
Because full plan of studies could make it difficult to parse out the individual effects for exploration in this conceptual argument, we created a series of test cases to showcase plausible situations transfer students could find themselves in at some point. Table 2 outlines a series of five test cases with two baseline cases to illustrate how our transfer student structural complexity metric applies to full plan of studies. In the plots, the nodes are ordered by outdegree, i.e., the number of edges leaving the node, from highest to lowest. Moreover, the darker the vertex, the greater that course's cruciality. Solid arrows represent prerequisites and corequisites are given by dotted arrows.

We assume a six-semester time-to-degree for these test cases, so $t_e = 6$ for simplicity. We use double alpha characters (e.g., AA, BB, and CC) to refer to community college courses and single alpha characters (e.g., A, B, and C) to denote the courses at the four-year institution. Finally, we will round the *inflexibility factor* to the nearest whole number as the amount of precision we need is unlikely to involve fractional results. An example calculation is given in Appendix B for test case 4.

Table 2. Test cases to illustrate our metrics for *transfer student structural complexity*

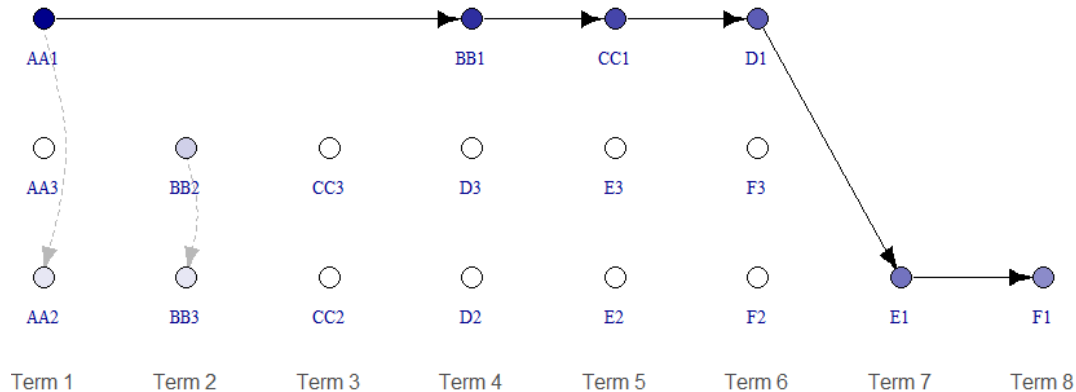
Situation and Metrics	Associated Network						Application to Transfer Students
(1) FTIC Pathway $I_f = 39$ $T_d = 0$ $C_l = 0$							This is the standard test case plan of study for FTIC students. The transfer pathways (below) will have the exact same pre- and corequisites, but will demonstrate visually how these three issues manifest for transfer students and how our proposed new metrics capture them.
(2) CC Pathway $I_f = 78$ $T_d = 0$ $C_l = 0$							This is the standard test case plan of study for community college transfer students where course with two alpha characters (e.g., AA) denote a community college course and single alpha courses denote a university course. The pre- and corequisites are otherwise identical to the FTIC pathway above. Note the I_f metric changed because we must calculate the <i>inflexibility factor</i> in two parts, once for the community college courses and once for the four-year courses.

(3) Missing One Prerequisite (CC1) at CC that delays time-to-degree
 $I_f = 83$
 $T_d = 6$
 $C_l = 0$



In this test case, students do not have access to a course (CC1) in the long prerequisite chain (AA1 to F1) while at the CC. The delayed access to the course, now taken at the University in Term 4, delays time-to-degree by one semester. Here, T_d increases to 6 because of the extra semester. Moreover, the inflexibility slightly increases because of the delayed access to CC1.

(4) Missing Two Prerequisites (BB1 & CC1) at CC that delays time-to-degree
 $I_f = 100$
 $T_d = 12$
 $C_l = 0$



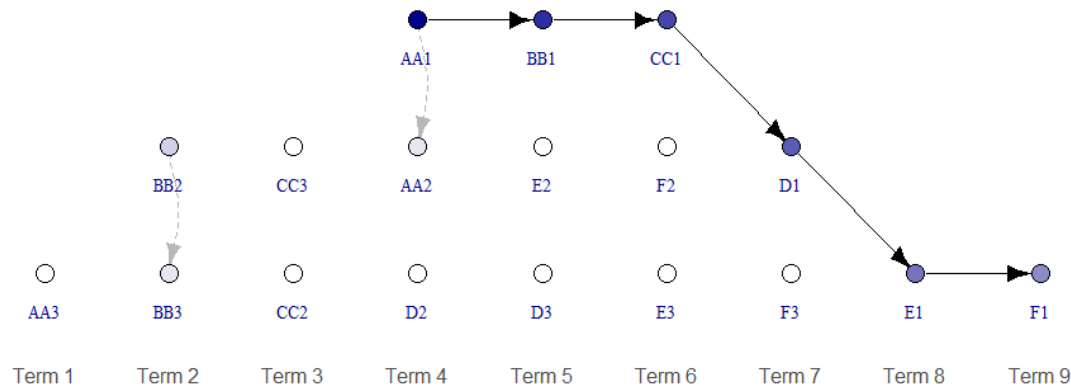
Extending the previous test case, students do not have access to two courses (BB1 & CC1) in the long prerequisite chain (AA1 to F1) while at the community college. The delayed access to the courses, now taken at the University in Terms 4 & 5, delays time-to-degree by two semesters. This inflates the inflexibility factor yet again and doubles the transfer delay factor.

(5) Missing Three Prerequisites (BB1 & CC1) at CC that delays time-to-degree

$$I_f = 167$$

$$T_d = 19$$

$$C_l = 0$$



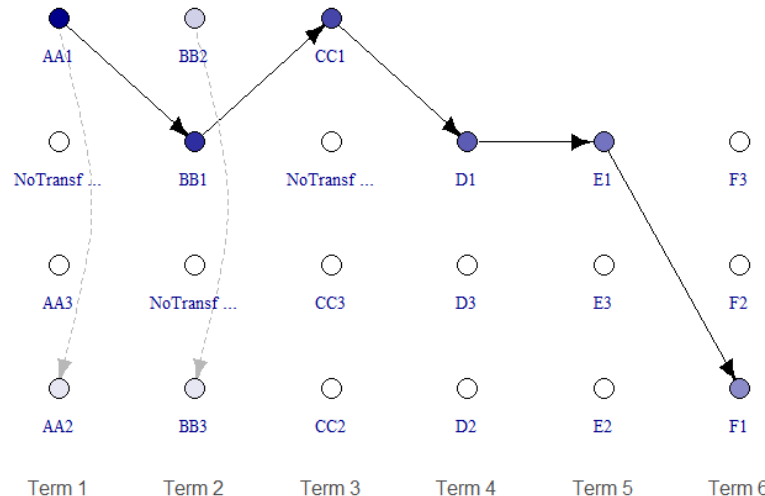
In the most extreme test case example demonstrating the potential impact of long-prerequisite chains on transfer pathways, students do not have access to any courses in the prerequisite chain (AA1 to F1) at the CC. This, unsurprisingly, extends time-to-degree by three terms beyond the expected 6 terms to degree. Because of the delayed courses, F3 is moved to the 7th term, which adds one more point to T_d after it tripled. Moreover, the empty spots in the community college plan of study make the inflexibility metric spike.

(6) Plan of Study with Lost Credits from CC that either do not transfer or are not applied to degree

$$I_f = 78$$

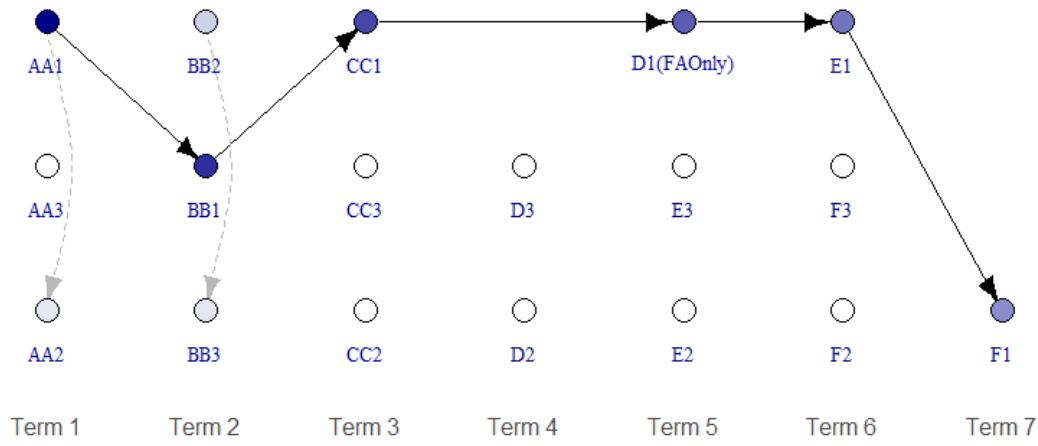
$$T_d = 0$$

$$C_l = 9$$



In this test case, students take 3 courses at the CC that either do not transfer to the University or are not applied toward students degree progress (NoTransf) because they are not core to the intended plan of study at the four year institution. Assuming each course was 3 credits, the student would lose 9 credits in total – hence, $C_l = 9$. Note that the NoTransf courses are not included in the inflexibility factor calculations because they are not courses in the official plan of study.

(7) Course in Prerequisite Chain is only offered once per year, which delays transfers degree progress
 $I_f = 89$
 $T_d = 6$
 $C_l = 0$



This test case captures a common scenario where students transfer to the University in a Spring semester (Term 4) but find the next course in the prerequisite chain D1(FAOnly) is only offered once per year in the Fall, which delays students' progress to degree. This causes a slight uptick in I_f , but will propagate as more courses are restricted to specific terms.

Discussion and Implications

With this paper, we reframed the curricular complexity framework within the context of vertical transfer students. We suggested three metrics that better capture challenges transfer students encounter in curricular sequencing in engineering and other highly sequential disciplines. Transfer pathways have been cited as critical to broadening participation in engineering [15] serve as a more affordable way to attain a degree in engineering. Yet, transfer students lag behind FTIC students in time-to-degree (e.g., [7]). The cost savings associated with the community college pathway are negated by the extended time-to-degree common for transfer students in engineering post-transfer.

In an effort to remedy this issue, states and institutions have made considerable efforts to streamline transfer, both within and outside of engineering, often through articulation agreements – formal partnerships that codify pathways between 2-year and 4-year institutions (e.g., [16]). However, this paper highlights a blind spot in these articulation efforts by operationalizing some common pitfalls mathematically in course sequencing that play a significant role in students' timely progress to engineering degrees via vertical transfer pathways. We organize this section by (1) discussing implications for articulation agreements that aim to streamline vertical transfer, (2) identifying a number of audiences and uses of transfer student structural complexity, and (3) acknowledging some of the assumptions of our metrics.

How our metrics may inform articulation agreements

The primary goal of articulation agreements at the state level and between 2-year and 4-year institutions is typically to preserve credits as they are transferred between institutions [17]. This is largely in response to evidence that credit loss contributes to lower transfer rates, lower completion and graduation rates, and extended time-to-degree (e.g., [11, 18]). Therefore, the inclusion of a credit loss metric to penalize credit loss during transfer is an essential addition to the transfer student structural complexity framework.

Credit loss is often retroactive due to transfer articulation processes and transcript review at the university post-transfer. Although some forward-thinking institutions offer transcript review pre-transfer [15,18], these are done on a case-by-case basis. Therefore, such reviews are expensive and time-consuming for university faculty and advisors, particularly if the transcripts being reviewed are only prospective or admitted students not yet committed to or enrolled at the University. Our proposed transfer student structural complexity metric may help to automate analyses of credit loss and remove the unpredictability of credit loss from transfer, and do so while reducing university labor costs of transcript review.

At the state level, policies aimed to preserve credits during transfer takes several forms including: 1) establishing a transferrable core of lower-division courses from 2-year to 4-year institutions; 2) requiring all courses follow a statewide common course numbering system shared by 2-year and 4-year institutions; 3) guaranteeing the transfer of an associate's degree across all state institutions, and 4) implementing statewide reverse transfer wherein students who transfer before completing an associate's degree have their university credits transferred back to the 2-year institution for conferral of a 2-year degree [20]. However, absent from all of these efforts is

any attention paid to streamlining how courses are sequenced in plans of study between institutions. Admittedly, the preservation of courses seems to be a logical precursor to establishing policies that streamline and preserve the sequencing of courses.

We also acknowledge that many disciplines are not as sequential as engineering and other STEM disciplines. However, we bring to light two metrics within transfer student structural complexity (i.e., timing of courses being offered and sequencing causing delays) that are common barriers to timely degree completion for transfer students in engineering. Efforts to preserve credits without thoughtful movement in aligning course sequences between 2-year and 4-year institutions will fall short of solving problems with time-to-degree for transfer students. This consideration is critical because an advantage of the vertical transfer pathway is cost savings for students as they complete community college coursework. Without improvements to course sequencing, our paper demonstrates, at least theoretically, how the cost savings evaporate as students' contend with extra semesters at the University to complete their 4-year degree. Efforts to streamline transfer for highly sequential disciplines must incorporate course sequencing between 2-year and 4-year institutions, and transfer student structural complexity could be of use.

Who could use these metrics?

Beyond articulation agreements, there are many other stakeholders and uses for the transfer student structural complexity metrics. Most prominently, transfer agents [21] at both 2-year and 4-year institutions who interface with vertical transfer students through pre-transfer admissions, advising, or other capacities can use these metrics and visualization capabilities to advise students purposively and intentionally in the courses they take and sequences they take them in. We contend that these transfer agents are already doing this but are doing it manually without visualization tools.

The metrics can also help these stakeholders communicate to institutional leaders the prevalence and effect of pre- and corequisite assignments to courses on transfer student success. There has been some prior research on how decisions are made to assign pre- and corequisites that establish a sequence (e.g., [22]). This existing work suggests that removing prerequisites may reduce the curriculum's structural complexity while not negatively impacting the staging of the content within the courses. We suggest our metrics are useful ways to contextualize the impacts of those decisions on transfer students and their ability to navigate curriculum in a timely manner.

Limitations

Finally, we want to address some key assumptions we have made in constructing the transfer student structural complexity metric. First, the test cases we included and the idea behind the metrics follow the logic of 2+2 vertical transfer models adopted by many states and most articulation agreements nationally (our tests used a six-semester form for simplicity). The 2+2 model, in ideal form, assumes that transfer plans of study include two years (roughly 60 credits) of courses at the community college that purposively align with the two years of curriculum post-transfer at the 4-year institution.

The ideals of the 2+2 model are crucial to consider, particularly for the sequencing of courses and timing of courses being offered metrics. There is some evidence that 1+3 models, which implies that students transfer after just one year at the community college, is a better model for engineering majors (e.g., [23]). The majority of the instances where sequencing of courses and timing of courses emerge are with courses within the second year of students' plan of study at the 2-year institution. For example, these include introductory courses for engineering sub-disciplines (e.g., Introduction to Aerospace Engineering, Introduction to Computer Engineering) and gateway math courses (e.g., Linear Algebra, Differential Equations). These courses are not consistently offered across 2-year institutions or may be offered but are not considered equivalent courses by the 4-year institution. By contrast, most of the courses within students' first year in engineering are offered by the 2-year institution. We do not take a position in this paper as to which model makes the most sense in engineering. However, we still acknowledge that these metrics largely occur in the 2+2 model that is most common nationally.

Conclusion

This theory paper presented our conceptualization of how the curricular complexity framework can be rethought to account for issues commonly encountered by transfer students. In our operationalization, we justified the content validity of our measures. We drew from Heileman et al.'s [1] work that outlined several metrics they considered in building the structural complexity measure in the framework. By mapping these existing metrics and customizations of our own to suggestions by [6], we outlined a conceptual model of how one could measure what we call *transfer student structural complexity* – a suggested refactored model of the *structural complexity* metric from Heileman et al. [1] designed for transfer student pathways. Our model includes measures that account for the timing of course offerings (*inflexibility factor*), sequencing of courses that cause an extended time-to-degree (*transfer delay factor*), and transfer credits that are lost to electives and not applied to degree requirements (*lost credit*).

Our future work will involve testing our conceptualization of the *transfer student structural complexity* with complete plans of study at several institutions drawing from the Multiple-Institution Database for Investigating Engineering Longitudinal Development (MIDFIELD) [24,25] as a sampling frame. We will correlate the associated metrics with students' time-to-degree in the dataset across disciplines. We also plan to examine common transfer student course-taking behaviors within these data and overlay those trajectories with our target conceptual model. With enough plans of study and estimates of credit loss, we can treat our conceptualization of *transfer student structural complexity* as a measurement model. From there, we can understand our measure's criterion validity, i.e., how well it correlates with other variables we expect them to correlate with, such as time-to-degree.

We contend our model can be useful to those studying transfer student pathways, transfer curricula, or issues with transfer credit. Moreover, those employing the curricular complexity framework to transfer students can use this approach to measure transfer-specific issues in their plan of study data.

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Appendix A: Notation and conventions for reference

Table A1. Notational conventions and definitions

Construct / Variable	Notation	Description
A course (vertex) in the curricular graph	v_{ij}	A course is given by a vertex in the graph indexed using i as an arbitrary label for the i th course in the semester and j as the semester number.
A prerequisite/corequisite (edge)	(v_{ab}, v_{cd})	Prerequisites and corequisites are given by edges, which are simply arrows pointing from one vertex to another. This prerequisite points from v_{ab} to v_{cd} .
Delay factor	$d(v_{ij})$	The longest prerequisite chain through v_{ij} .
Blocking factor	$b(v_{ij})$	The number of courses inaccessible to a student who fails the course v_{ij} .
Cruciality		This metric describes how central a course is to a curriculum. The sum of the blocking and delay factor for a course
Structural complexity		The sum of the crucialities in a curriculum graph, a big picture description of how interconnected/accessible a curriculum is.
Degrees-of-freedom	z_c	Heileman et al. [1] define <i>degrees-of-freedom</i> as the number of ways the courses in a curricular graph can be rearranged semester-by-semester while respecting prerequisite chains. The maximum degrees-of-freedom for a curriculum is given by z_c^* .
Weakly connected component		A subgraph of the curriculum graph where a path between each pair of vertices can be traveled. In other words, each vertex is reachable from all other vertices in the component.
Number of permutations of a weakly connected component's structure	m_i	This is a helper metric to calculate <i>degree-of-freedom</i> . The number of permutations refers to how many ways one can rearrange the courses in the weakly connected component while respecting certain constraints such as the prerequisite structure, timing of course offerings, and credit loads for each semester.
Number of terms	n_t	The number of semesters in a plan of study as codified.
Number of courses in plan of study	n_c	The number of possible courses in the plan of study; for the community college, this includes any courses delayed to the four-year institution.

Number of unavailable semesters for a course	$u(v_{ij})$	This metric is a count of how many semesters a course v_{ij} cannot be taken because it is only offered at certain times.
Inflexibility factor	I_f	A measure that quantifies how inflexible a curriculum is by examining the portion of the curriculum where courses are immovable because of prerequisite or corequisite chains, how many courses are inaccessible to students are certain times, and the extent to which these courses extend the intended time to degree.
Subgraph		A graph formed using the vertices and edges of a larger graph.
Time-to-degree	t_d	The number of semesters needed for a student to complete the degree program as specified in the graph.
Expected time-to-degree	t_e	The semester in which the student is intended to graduate, often taken to be eight semesters.
Transfer delay factor	T_d	The delay factors of the courses that extend the number of semesters beyond t_e when we only consider the prerequisites of those courses.
Credit loss	C_l	Sum of all credits not applied to major courses, these are courses that are <i>transferred</i> into the receiving institution but are not <i>applied</i> to courses that allow the students to progress.

Appendix B: Example Calculations for Scenario 4 – Missing Two Prerequisites

Because the inflexibility factor and transfer delay factor lend themselves well to visual representations, we will outline two example calculations based on one of our example plans of study scenarios in Figure A1. We'll use the fourth scenario outlined in Table 1.

First, we calculate the total possible degrees-of-freedom if all courses were free to vary in the community college half and the four-year institution half up until the expected time-to-degree – which gives $z_{cc}^* = z_{fy}^* = (n_t - 1)n_c = (3 - 1)(9) = 18$. We count BB1 and CC1 in the course count for the community college, whereas the remaining single alpha character courses in the four year give us $n_c = 9$. Then, we find the actual degrees-of-freedom by finding the number of semesters each course can vary when we follow the prerequisite structures as shown in Figure 7 – which gives $z_{cc} = 14$ and $z_{fy} = 12$. Because two of the nine courses are now unreachable in the community college portion and extend two of the nine four-year courses outside of the expected time-to-degree, we add a penalty of $2/9 + 2/9$. Thus, the inflexibility factor is...

$$I_f = 100 \left(\frac{18 - 14}{18} + \frac{18 - 12}{18} + \frac{2}{9} + \frac{2}{9} \right) = 100$$

The transfer delay factor is found by summing the delay factors of the courses extending beyond the time-to-degree. Both E1 and F1 are beyond the expected graduating semester, and they both belong to a prerequisite chain of length six, so the *transfer delay factor* for this configuration is 12.

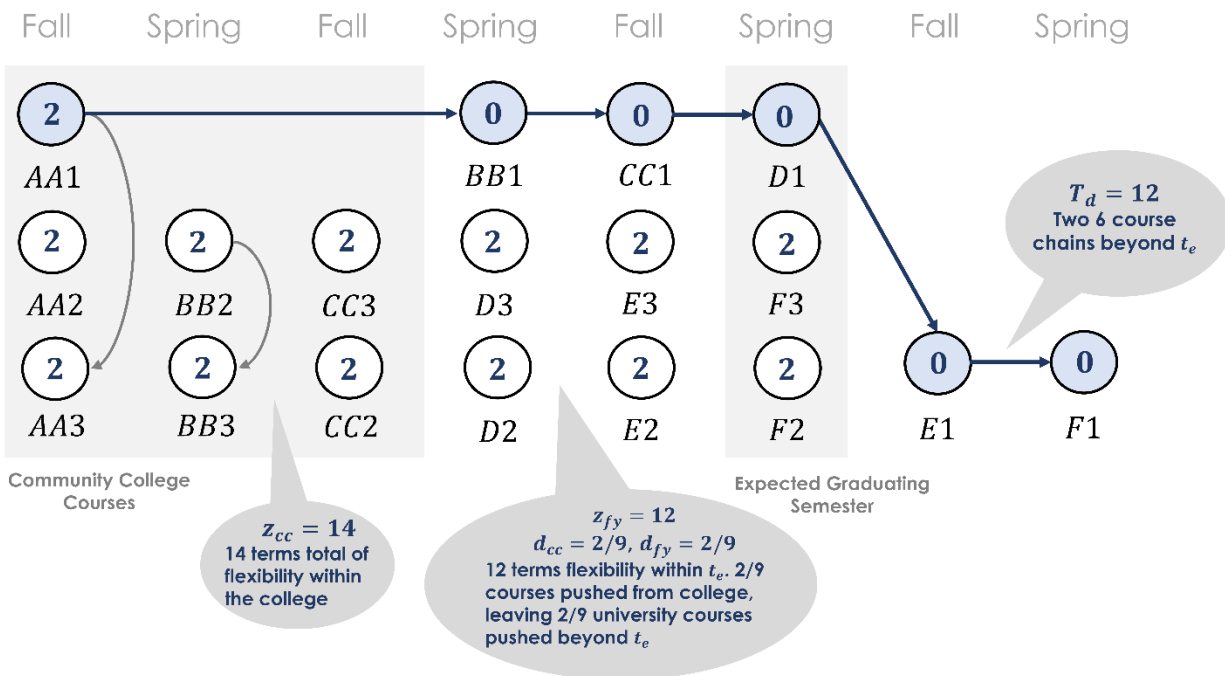


Figure A1. Example calculation of the inflexibility factor and the transfer delay factor; number of flexible semesters for each course given in their associated vertex; courses involved in the transfer delay factor calculation in blue