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Retina Identification Based on Moment Invariant

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Abstract

This paper demonstrates the importance of topics in physics and mathematics courses such as matrix, eigenvectors, centroid, and moment of inertia in the pattern recognition applications.

Teaching advance topics in physics and mathematics is not an easy task. Students always ask this question: What is the use? The best practice for teaching these topics is to combine them with a real industry application. For example, computer hard disk reading is based on the concept of derivatives and the change in the magnetic fields. To demonstrate this type of teaching, the use of three dimensional moments in the pattern recognition is explained.

The three-dimensional moments may be used to detect different patterns in a digitally represented image. The combination of several moments for an object has an invariant property. The mathematical foundation of an invariant feature is related to the theory that images taken from different angles from the same object have the same set of moment invariants. Moments contain information of an image which can be used in calculating the location and orientation of an object. An algorithm for recognition of an individual identity based on a digitally represented image of the scanned retina is presented. The technology is based upon the fact that no two retinal patterns are alike.

In this paper, the nine parameters of an ellipsoidal shape fitted into a retinal image such as coordinates of the center of the ellipsoid, the length of major, minor, intermediate axes, and the direction of three axes will be calculated. For each individual, these parameters are unique.

Key Words: Ellipsoid, image processing, pattern recognition, parallel algorithm, retina, three-dimensional moments .

1 Introduction

Retina is a light-sensitive layer of tissue (Figure 1), located at the inner surface of the eye. The optics of the eye creates an image on the retina, similar to the film in a camera. Light striking the retina activates nerve impulses. These pulses are sent to various parts of the brain through the optic nerve. Retina scans require that the person removes their glasses, place their eye close to the scanner. A retinal scan involves the use of a low-intensity coherent light source, which is projected onto the retina . A retina scan cannot be faked and it is impossible to forge a human retina. Furthermore, the retina of a deceased person decays too rapidly to be used to deceive a retinal scan [17, 18]. The recognition of a three-dimensional retinal image and determining its position and orientation in three-space is an important problem in identity detection. In this paper, three-dimensional moment has been used as a feature in detecting individuals identity based on a digitally represented image of the scanned retina. In this paper Retinal is modeled as an ellipsoid. Ellipsoidal shapes are found in many environments. Many objects have ellipsoidal form which when viewed in two-dimensional appears to be elliptical. Most of the existing techniques for pattern recognition consider two-dimensional images [13, 14]. In order to detect a pattern in an image i.e., circle, line, or an ellipsoid, a feature from the image is selected. This feature must be independent of the size, position, and orientation of the image-also called the invariant feature [1, 2, 3, 5, 6, 9, 11, 12].

In this paper three-dimensional moment of an image has been used as a feature for identity detection. Moments contain information about an image which is uique and can be used in calculating the location and orientation of that image.

The mathematical foundation of an invariant feature is based on the theory that deals with the calculus of algebraic invariant [10]. This theory deals with algebraic functions of a certain class which remain unchanged under some specific coordinate transformations. Hu [1] has derived results showing the algebraic invariant of two-dimensional moments. Alt [4] applied Hu's results for the recognition of letters and numerals. Dudani et al. have applied moment invariants to identify an aircraft [2]. Reddi [8] proposed a simpler construction method using radial and angular moments. Teague [3] extended Hu's idea by introducing the orthogonal moment set to recover images from moments. This idea is based on the theory of orthogonal polynomials. Teague introduced

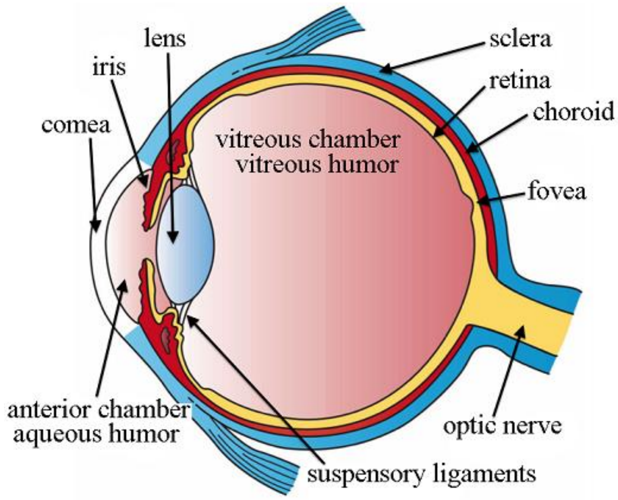

Figure 1: Retina

Zernike moments which allow for easy construction of independent moment invariants for high order. Abu-Mostafa and Psaltis [7] introduced the notion of complex moments and derived moment invariants. From the point of view of pattern recognition, moment invariants are considered reliable features if their values are insensitive to the presence of image noise. Sajadi and Hall [11], Lo and Don [6] have considered the extension of two-dimensional to three-dimensional moment invariants.

This paper considers an implementation of a parallel algorithm on a pyramid architecture (Figure 3) for identity detection based on the three-dimensional moments. Section 2, summarizes three-dimensional moment calculation. In Section 3, moments for ellipsoid (Figure 2) and their properties are described. Section 4, describes an algorithm for ellipsoid detection using pyramid architecture Section 5 presents simulation program, and Section 6 presents the conclusion.

2 Three-Dimensional Moments

Given a three-dimensional density distribution function $f(x, y, z)$, the *(p+q+r) order* moments are defined in terms of the Riemann integral as:

$$m_{pqr} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} r_x^p r_y^q r_z^r f(x, y, z) dxdydz$$

where r_i is the normal distance to axis i, $i = x, y, z$, and $p, q, r = 0, 1, 2, ...$

The integration extends over the domain of f. For an object with limited volume in the x, y, z space, the integration extends over the volume of the object. The second order moments about x,y, and z axes, i.e., $p = 2$, $q = 0$, $r = 0$ or $p = 0$, $q = 2$, $r = 0$ or $p = 0$, $q = 0$, $r = 2$ are called *moment of inertia* and are defined as follows:

$$m_{200} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (y^2 + z^2) f(x, y, z) dxdydz$$

$$m_{020} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x^2 + z^2) f(x, y, z) dxdydz$$

$$m_{002} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x^2 + y^2) f(x, y, z) dxdydz$$

for $p = 1$,$q = 1$, and $r = 1$ the moment is called *product of moment of inertia* and the distances r_x, r_y, and r_z are measured with respect to $y - z$, $x - z$, and $x - y$ planes respectively. This moment is calculated as follows:

$$m_{111} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} xyz f(x, y, z) dxdydz$$

Definition 1: The center of gravity, *centroid*, of an object has coordinates $\overline{x}$, $\overline{y}$, and $\overline{z}$ that are calculated as follows:

$\overline{x} = m_{100}/m_{000}$, $\overline{y} = m_{010}/m_{000}$, and $\overline{z} = m_{001}/m_{000}$. m_{000} is the volume of the object.

Central moments are given as:

$$\mu_{pqr} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - \overline{x})^p (y - \overline{y})^q (z - \overline{z})^r f(x, y, z) dxdydz$$

A uniqueness theorem states that if $f(x,y,z)$ is piecewise continuous and has nonzero values only in a finite section of the x,y,z coordinates, then the moments of all orders exist and the moment m_{pqr} uniquely determines $f(x,y,z)$. This theorem states the detection power of moments in pattern recognition. Only a small set of low-order moments is used to distinguish between different patterns.

One representation of a digital image is the collection of pixels and the intensity of each pixel in that image . In industrial applications, images are in general transformed to their binary representation by preprocessing techniques such as: region segmentation or edge detection. For a binary image where each pixel can be represented by 0 or 1, the triple integral for moment calculation is approximated by a summation. Therefore, the $(p+q+r)$ order moments is approximated as:

$$m_{pqr} = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{o} x_{ijk}^{p} y_{ijk}^{q} z_{ijk}^{r}$$

x_{ijk}, y_{ijk}, and z_{ijk} are the coordinates, and $m * n * o$ is the total number of pixels.

Central moments are approximated as:

$$\mu_{pqr} = \sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{o} (x_{ijk} - \overline{x})^{p} (y_{ijk} - \overline{y})^{q} (z_{ijk} - \overline{z})^{r}$$

When summation is used to compute moment invariants, certain variations are expected because of discreteness of the input data. The amount of variations depends on the size of the rotation, translation, and scale change.

3 Moments Calculation for Ellipsoid

In this section, we consider some of the properties of moments in an ellipsoidal object. Figure 2 shows an ellipsoid with major, minor, and intermediate axes of length $2a$, $2b$, and $2c$ respectively. Volume V of an ellipsoid is given by $\frac{3}{4}\pi abc$. An important property of an ellipsoid is described in Lemma 1.

Lemma 1: In an ellipsoid the second moment is minimum along the major axis and maximum along the minor axis; these moments are given by $\frac{4}{15}\pi abc(b^2+c^2)$, and $\frac{4}{15}\pi abc(a^2+c^2)$ respectively.

Proof: An ellipsoid is represented by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. This formula can be rewritten as follows:
$\frac{x^2}{a^2} + \frac{z^2}{c^2} = \frac{1}{b^2}(b^2 - y^2)$
$\frac{x^2}{(\frac{a}{b}\sqrt{b^2-y^2})^2} + \frac{y^2}{(\frac{c}{b}\sqrt{b^2-y^2})^2} = 1$
$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ where
$A = \frac{a}{b}\sqrt{b^2 - y^2}$, $B = \frac{c}{b}\sqrt{b^2 - y^2}$

The elliptic disk $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$ has the area of πAB. Thus the second moment with respect $x - z$ plane, m_{xz}, can be calculated as follows:

$$m_{xz} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} y^2 dxdydz = \int_{-b}^{+b} \pi ABy^2 dy =$$

$$\frac{2ac\pi}{b^2}\int_{-b}^{+b}(b^2 - y^2)y^2 dy = \frac{4ab^3c\pi}{15} = V\frac{b^2}{5}$$

V is the volume of an ellipsoid. Similarly moments with respect to $y - z$ and $x - z$ planes are:
$m_{yz} = V\frac{a^2}{5}$
$m_{xy} = V\frac{c^2}{5}$

The second moment m_{200} for an ellipsoidal object about x axis is the sum of moments about $x - z$ and $x - y$ planes and are calculated as follows:

$$m_{200} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (y^2 + z^2) dxdydz = m_{xz} + m_{xy}$$

$m_{200} = \frac{4}{15}abc\pi(b^2 + c^2) = V(b^2 + c^2)$

By interchanging a with b, m_{020} is given by $\frac{4}{15}abc\pi(a^2 + c^2) = V(a^2 + c^2)$. Since $b < a$, then $m_{200} < m_{020}$.

Definition 2: The angles between major, minor, and intermediate axes of an ellipsoid and positive direction of the x, y, and z axes, α, β, and γ, are defined as the *orientation* of an ellipsoid with respect to a given coordinate system.

3.1 Parameters of Ellipsoid

In this section, nine parameters of an ellipsoid in terms of different order moments are calculated. We assume that, the origin of coordinate system is

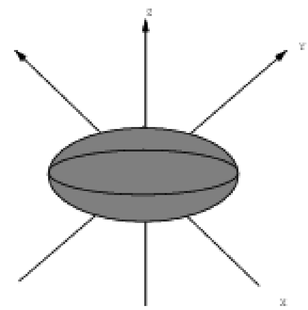

Figure 2: An Ellipsoid

located at the centroid of an ellipsoid (see Figure 2).

The following three-dimensional second order moments J_1, J_2, and J_3, proved to be invariants [11].

$J_1 = \mu_{200} + \mu_{020} + \mu_{002}$

$J_2 = \mu_{020}\mu_{002} - \mu_{011}^2 + \mu_{200}\mu_{002} - \mu_{101}^2 + \mu_{200}\mu_{020} - \mu_{110}^2$

$$J_3 = \begin{bmatrix} \mu_{200} & \mu_{110} & \mu_{101} \\ \mu_{110} & \mu_{020} & \mu_{011} \\ \mu_{101} & \mu_{011} & \mu_{002} \end{bmatrix}$$

The second moment J_3 is invariant under rotation. That is:

$J_3' = R^T J_3 R$

Where J_3' is the rotated second moment J_3 when coordinates rotate with direction cosines $\cos\alpha$, $\cos\beta$, and $\cos\gamma$. R is given as follows:

$$R = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix}$$

From Lemma 1, we infer that the second moments are minimum along major axis and maximum along the minor axis. The minimum value of J_3' is the smallest *eigenvalue* of matrix J_3. The value of R for which J_3' assumes its minimum can be found by determining the *eigenvector* corresponding to this eigenvalue.

To find the direction vector that minimizes J_3', first, the eigenvalues will be calculated as follows:

$$J_3 - \lambda I = \begin{bmatrix} \mu_{200} - \lambda & \mu_{110} & \mu_{101} \\ \mu_{110} & \mu_{020} - \lambda & \mu_{011} \\ \mu_{101} & \mu_{011} & \mu_{002} - \lambda \end{bmatrix}$$

Where I is a diagonal unit matrix. The characteristic equation is as follows:
$f(\lambda) = \lambda^3 - J_1\lambda^2 + J_2\lambda - J_3$
If λ_1, λ_2, and λ_3 are three eigenvalues of J_3 then we have:
$\lambda_1 = \frac{b^2+c^2}{5}$
$\lambda_2 = \frac{a^2+c^2}{5}$
$\lambda_3 = \frac{a^2+b^2}{5}$
Solving for a, b, and c, we have the following:
$a = \sqrt{\frac{5}{2}(\lambda_2 + \lambda_3 - \lambda_1)}$
$b = \sqrt{\frac{5}{2}(\lambda_1 + \lambda_3 - \lambda_2)}$
$c = \sqrt{\frac{5}{2}(\lambda_1 + \lambda_2 - \lambda_3)}$

The eigenvector R gives the orientation of ellipsoid. To determined the eigenvector corresponding to an eigenvalue, i.e., λ_1, we must solve a set of homogeneous linear equations:

$$\begin{bmatrix} \mu_{200} - \lambda_1 & \mu_{110} & \mu_{101} \\ \mu_{110} & \mu_{020} - \lambda_1 & \mu_{011} \\ \mu_{101} & \mu_{011} & \mu_{002} - \lambda_1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = 0$$

$r_1 = (\mu_{020} - \lambda_1)(\mu_{002} - \lambda_1) - \mu_{101}(\mu_{020} - \lambda_1) - \mu_{110}(\mu_{002} - \lambda_1) + \mu_{011}(\mu_{101} + \mu_{110} - \mu_{011})$

$r_2 = (\mu_{002} - \lambda_1)(\mu_{200} - \lambda_1) - \mu_{110}(\mu_{002} - \lambda_1) - \mu_{011}(\mu_{200} - \lambda_1) + \mu_{101}(\mu_{110} + \mu_{011} - \mu_{101})$

$r_3 = (\mu_{200} - \lambda_1)(\mu_{020} - \lambda_1) - \mu_{011}(\mu_{200} - \lambda_1) - \mu_{101}(\mu_{020} - \lambda_1) + \mu_{110}(\mu_{011} + \mu_{101} - \mu_{110})$

4 Moments Calculation on a Pyramid Architecture

In this section an algorithm for parallel calculation of three-dimensional moments implemented on pyramid architecture are considered. In this algorithm a continuous image is approximated by a digital image, and moments calculations are approximated by summations. Following is the description of pyramid architecture.

4.1 Pyramid Architecture

A pyramid computer with the base size of n is an SIMD machine that is constructed from $(1/2)\log n + 1$ levels of a two-dimensional mesh connected processor array, where the Lth level, $0 \leq L \leq (1/2)\log n$, is a two-dimensional mesh connected processor array of size $\frac{n}{4^L}$. A mesh connected computer of size n is a collection of n processing elements arranged in a $\sqrt{n} * \sqrt{n}$ grid, where each processing element except for those along the border, is connected to its four neighbors. Each processing element at level L is connected to its neighbors at level L and four children at level $L-1$ $(L > 0)$ and a parent at level $(L+1)$, $(L < (1/2)\log n)$. Thus, each internal processing element has nine connections. Figure 3 illustrates a pyramid with a 4-by-4 base configuration. Circles represent processing elements and lines represent communication paths. All of the processing elements in the pyramid operate in a strict SIMD mode under the direct control of a single node. Each processing element has its own memory and all of the communication links are bidirectional. The total number of processing elements is given by:

$$\sum_{L=0}^{(1/2)\log n} \frac{n}{4^L} = \frac{4n}{3} - \frac{1}{3}$$

The pyramid topology has been proposed as an architecture for high-speed image processing where its simple geometry adapts naturally to many types of problems [15, 16]. The pyramid architecture can be projected down onto a plane with a simple configuration providing a regular geometrical characteristic ideal for VLSI design. Pyramids are more attractive than meshes because they provide the potential for solving problems with logarithmic time complexity.

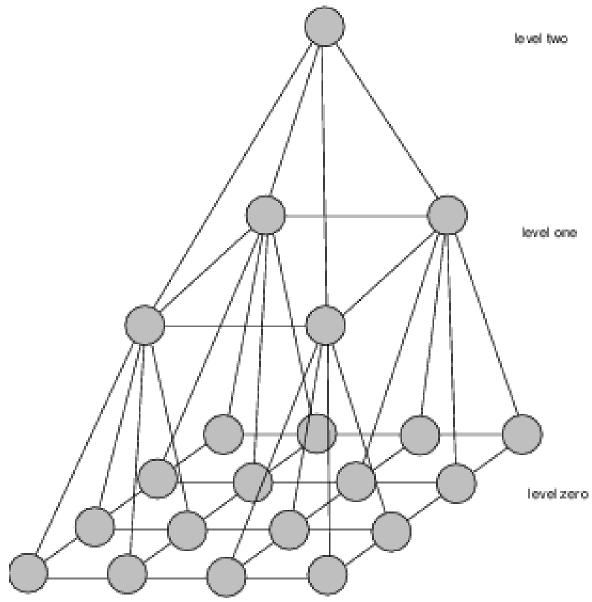

Figure 3: A 4 by 4 base Pyramid Architecture

4.2 Three-Dimensional Moments Algorithm

In the following algorithm we assume number of pixels in an image is equal to the number of processors at the base of the pyramid (i.e., n). Each processor at level zero (base) stores the following information in its local memory:

x_{ijk}: represents the x coordinate of the pixel p_{ijk}.

y_{ijk}: represents the y coordinate of the pixel p_{ijk}.

z_{ijk}: represents the z coordinate of the pixel p_{ijk}.

The three-dimensional Moments Algorithm (TDMA) uses both mesh and tree connections of pyramid and has four phases. During the first two phases moments, m_{100}, m_{010}, m_{001}, m_{110}, m_{101}, m_{011} will be calculated.

$$m_{pqr} = \sum_{i=1}^{\sqrt[3]{n}} \sum_{j=1}^{\sqrt[3]{n}} \sum_{k=1}^{\sqrt[3]{n}} x_{ijk}^{p} y_{ijk}^{q} z_{ijk}^{r}$$

where $p, q, r \in \{0, 1\}$

In the phase three m_{200}, m_{020}, and m_{002} will be calculated:

$$m_{200} = \sum_{i=1}^{\sqrt[3]{n}} \sum_{j=1}^{\sqrt[3]{n}} \sum_{k=1}^{\sqrt[3]{n}} (y_{ijk}^2 + z_{ijk}^2) \quad m_{020} = \sum_{i=1}^{\sqrt[3]{n}} \sum_{j=1}^{\sqrt[3]{n}} \sum_{k=1}^{\sqrt[3]{n}} (x_{ijk}^2 + z_{ijk}^2)$$

$$m_{002} = \sum_{i=1}^{\sqrt[3]{n}} \sum_{j=1}^{\sqrt[3]{n}} \sum_{k=1}^{\sqrt[3]{n}} (x_{ijk}^2 + y_{ijk}^2)$$

The method of calculation is similar to phases one and two, that is processors at the base calculate x_{ijk}^2, y_{ijk}^2, and z_{ijk}^2 and send the results to the next level for addition and this process is repeated for higher levels in a pipeline fashion.

In Phase four nine parameters of an ellipsoid will be calculate. In this phase, apex processor will find the center of the image by calculating $\overline{x}$, $\overline{y}$, and $\overline{z}$. Using the equations derived in Section 3, direction of the axes and the length of the major, minor, and intermediate axes (i.e., $2a$, $2b$, and $2c$) are calculated next.

4.3 Algorithm

A program written in C language finds the best fit of an ellipsoid in a retina. The algorithm implemented on a pyramid architecture. The bitmap file of the image is called portable bitmap(pbm). Figure 4 illustrates a 16 by 16 image created by *bitmap editor*. In this pbm file 1 means *black* and 0 means *white*. The simulation program consists of two major parts: *Simulation Controller* and *Algorithm Controller*. The Simulation Controller is used for scheduling. It offers a menu to the user to select the size of a pyramid. The program has a global clock called *Simulation Clock* and incremented whenever the processors at the same level terminate execution. Therefore, at the end of the program, Simulation Clock denotes the number of times that processors activated during execution of the program. Table 1 illustrates the computer simulation results for different size binary images.

5 Steps for the identification method

Step 1- Retina is scanned.
Step 2- Scanned image is converted to the bitmap.

```
0000000000000000
0000010000000000
0000100000000000
0000100000000000
0001000000000000
0001000000000000
0010000111000000
0000001000100000
0000010000010000
0000010000010000
0000010000010000
0000001000100000
0000000111000000
0000000000000000
0011111111111100
0000000000000000
```

Figure 4: A 16 by 16 image created by bitmap editor

Image size	*No. of Processors*	*No. of Levels*	*Simulation Clock in TDMA*
8 *by* 8	85	4	968
16 *by* 16	341	5	7,808
32 *by* 32	1365	6	84,432
64 *by* 64	5461	7	522,176
128 *by* 128	21845	8	3,428,800
256 *by* 256	87381	9	14,759,488
512 *by* 512	349525	10	99,300,288

Table 1: Image size, Pyramid size, and Simulation clock for ellipsoid detection algorithm

Step 3-The best fit ellipsoid will be detected.
Step 4- Moment invariants J_1, J_2, J_3 are calculated.
Step 5- Lengths of the major, minor, and intermediate axes are calculated.
Step 6- Coordinates of the center of the best fit ellipsoid are calculated.
Step 7- Angles between major, minor, and intermediate axes of the best fit ellipsoid and positive direction of the x, y, and z axes, (i.e. α, β, and γ) are calculated.
Step 8- For each individual, these nine parameters are unique.

6 Conclusion

The best practice for teaching advance topics in physics and mathematics courses is to combine them with a real industry application. This paper emphasizes the importance of the topics in physics and mathematics courses in the real world applications. To demonstrate this assertion we presented a procedure for retina detection through the moment invariant concept. Retina biometrics systems are suited for environments requiring maximum security including, but not limited to the military and banking sectors. The unique features of a retina are derived by the principal of moment invariant in this paper. Retinal scans require that the individual to remove his/her glasses and place their eyes close to the scanner. A retinal scan involves the use of a low-intensity coherent light source, which is then projected onto the retina. A retinal scan is secure and cannot be altered.

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