AC 2012-4827: REVAMPING DELTA DESIGN FOR INTRODUCTORY ME-CHANICS

Ms. Michelle Marie Grau, Stanford University

Michelle Grau is a junior in mechanical engineering at Stanford University, and was one of the students in the first revision of ENGR 14, Introduction to Solid Mechanics. Her research interests include engineering education, robotics in space applications, and using robots to introduce engineering to middle school students. She is passionate about the FIRST Robotics program, in which she coaches teams and volunteers at competitions. She also does wushu and gymnastics.

Dr. Sheri Sheppard, Stanford University

Sheri Sheppard, Ph.D., P.E., is professor of mechanical engineering at Stanford University. Besides teaching both undergraduate and graduate design and education-related classes at Stanford University, she conducts research on weld and solder-connect fatigue and impact failures, fracture mechanics, applied finite element analysis, and engineering education. In addition, from 1999-2008, she served as a Senior Scholar at the Carnegie Foundation for the Advancement of Teaching, leading the Foundation's engineering study (as reported in Educating Engineers: Designing for the Future of the Field). Sheppard's graduate work was done at the University of Michigan.

Ms. Samantha Ruth Brunhaver, Stanford University

Samantha Brunhaver is a fourth-year graduate student at Stanford University. She is currently working on her Ph.D. in mechanical engineering with a focus in engineering education. Brunhaver completed a B.S. in mechanical engineering from Northeastern University in 2008 and a M.S. in mechanical engineering with a focus in design for manufacturing from Stanford in 2010.

Revamping Delta Design for Introductory Solid Mechanics

Abstract

The Delta Design game was developed by MIT Professor Louis Bucciarelli for college-level engineering education. The game's main goal is to demonstrate that designing in teams is as much a social process as it is a technical one, and that compromise is a key part of creating a successful design. To play the game, four students, each with a different role, form a design team and are tasked with developing a structure that meets the different sets of constraints posed by each role. The four roles are the structural engineer, thermal engineer, project manager, and architect. Each team must design a residence in an imaginary world which they build with red and blue triangles on a diamond grid. The red and blue tiles mean different things to each player. For example, the thermal engineer sees the red triangles as heat-producing elements, while the project manager sees them as a representation of cost. Each player also has different constraints that they are trying to optimize, e.g. the structural engineer calculates moments and safety factors to make sure the design is safe, the project manager keeps track of cost and time, and the architect makes sure that the end result is aesthetically pleasing to the future inhabitants. It is a complex game, requiring the students to be trained in their roles beforehand, and takes at least three hours to complete. In addition, instructors can make the game more or less challenging by changing the values of certain constraints such as the cost or by adding complexities like sudden shifts in gravity.

We redesigned the Delta Design game for a sophomore-level solid mechanics course. The goals of this redesign were to emphasize the role of solid mechanics in the design process and reinforce concepts taught in the class. We also sought to make the game shorter so that it could be played in a class period and to maintain the focus on teamwork. Examples of the modifications include removing the positions that were unrelated to the course (e.g., the thermal engineer and the architect) as well as adding design constraints to the two remaining roles to keep the nature of the game complex. The redesigned game was piloted in the summer and fall of 2011 with nine students, most of whom had recently graduated from or were currently enrolled in an Introduction to Solid Mechanics course at Stanford University. In this paper, we fully describe our decision-making process to redesign the game as well as the actual design changes. We also summarize the feedback we received during the pilot games and describe our next steps in terms of further changes. Ultimately, we hope that the game can be used to increase students' engagement and conceptual understanding in learning solid mechanics, and to help students draw connections between the course material and real-world applications.

Introduction

The Introduction to Solid Mechanics class (ENGR 14) at Stanford University has undergone significant changes in the course structure over the last year. The changes aim to make the class more interactive in order to help the students gain a firmer understanding of and appreciation for solid mechanics, as suggested by Mazur (2003)^[1] and Smith, et al.^[2]. The principal method for accomplishing this was to make class time more interactive. First, more practice problems were worked into the class period, allowing students to test their comprehension of the material they had just learned by attempting a problem themselves. While students work on the problem, the

course instructor and teaching assistants answer questions and check students' work, thus providing students with immediate feedback and guidance. Another way the course was made more interactive was by integrating mini hands-on lab exercises during class time. These exercises are advantageous because they help students develop a more intuitive feeling for mechanics concepts that come up regularly in both homework and real-world problems.

The Delta Design game was considered as one possible lab exercise for the class. MIT Professor Louis Bucciarelli (1999) developed the Delta Design game for college engineering education in order to give students first-hand experience with the challenges of designing in a team where each member has different expertise and constraints^[3]. The game demonstrates that the engineering design process is as social as it is technical, and that compromise is a key skill for an engineer to have. Games such as Delta Design are useful in teaching certain aspects of professional practice because they level the playing field in a classroom of diverse students, giving everyone a common experience ^[4]. However, the original version of Delta Design is complex and time-consuming, requiring the students to be trained in their roles beforehand and taking about three hours to design a structure that satisfies all of the design constraints. In this paper, we describe the steps taken to make the Delta Design game conducive for use in ENGR 14. We also present initial student feedback collected during pilots of the game as well as our plans for further changes.

Delta Design Background

In the original Delta Design game, teams of four students are tasked with designing a residence for inhabitants of the imaginary planar world called Deltoid Plane. The game is comprised of four roles: the Structural Engineer, the Project Manager, the Thermal Engineer, and the Architect. Each student is given the same initial introduction to the design task, as well as role-specific constraints to meet depending on his or her individual role; the specifics of these constraints are initially unknown to the other team members.

The team creates the residence by assembling red and blue tiles called deltas on a diamond grid board. The red and blue deltas mean different things to each person. The thermal engineer sees red deltas as heat-producers, and there are limits on how many red deltas can be adjacent to one another. The project manager perceives red deltas as more expensive than blue deltas, and is wary of joining red deltas to blue deltas because this incurs the highest cement cost. The architect is concerned with the overall aesthetics of the structure and must also ensure that the number of blue deltas does not exceed sixty percent of the total number of deltas. Finally, the structural engineer cares about the stability of the structure, e.g. whether the two points anchoring the structure will be able to support its weight, whether the cement bond between each pair of deltas will be strong enough to resist the moment applied by the rest of the structure, and whether it can survive sudden shifts in gravity. In summary, each player is tasked with optimizing different constraints, whether temperature, cost, safety factor, or interior-to-exterior wall ratio.



Figure 1: Example Residence

Figure 1 depicts a sample residence. The door must be oriented such that the residents fall into the house. The points "A" and "B" mark the two chosen anchor points, used in the structural engineer's calculations. Complete details of the original game can be found in ^[3].

The original Delta Design game has been implemented successfully in several academic settings. The Industrial Engineering department at the University of Pittsburg incorporated the Delta Design game in an introductory sophomore-level class that teaches students how to model as well as solve open-ended problems and work in teams. The addition of the game received positive feedback from the students ^[5]. The game was also used in a study on the impact of faculty-mentored learning versus online learning conducted with freshmen at MIT ^[6].

At the graduate level, the Delta Design game has been used as a tool to teach graduate students reflective practice. Instead of using a real problem, instructors chose to use the Delta Design game to because it is easier to control the amount of training each student receives and levels the playing field since no student has outside knowledge of the challenge. Additionally, the instructor can control the focus of the game such that if the students are having difficulty creating a viable structure, he or she can draw their focus back to reflective practice by changing the values of constraints to make the task easier ^[4].

Details of the Redesign

The Delta Design game in its original form was not conducive for use in ENGR 14 because it took too long to play in a single class session. The goal of the redesign was to scale down the game such that it would fit into one class period, while maintaining Bucciarelli's original intent, to emphasize both teamwork and the challenges of designing with several different sets of constraints. Although proficiency with moment calculations was not one of Buciarelli's primary learning objectives, since the game was to be played in an introductory solid mechanics course, we wanted to include statics concepts and to demonstrate how these concepts apply to the design process.

Another important consideration in the redesign was obtaining a balance between simplicity and complexity. We wanted game play to be challenging enough such that the first design the team proposes should not meet all of the constraints, thereby emphasizing the iterative nature of

design work and building teamwork skills. As such, we experimented with many different values for the constraints until we reached ones that provided this balance.

In addition to creating a game that was more focused on structural evaluation in the context of a larger project and teamwork, a major aim of the redesign was to make the game last one hour and to cause teams to go through at least three iterations of the design.

We made changes to the rules of the game in several different ways. We removed roles from the game, and provided more constraints in the overall design task. We also added constraints to the game and modified some of the existing rules to help achieve an appropriate level of complexity. These changes are described below, and are summarized in Table 1 (along with the rules for the original game). Instructions on how to play the new game are shown in the Appendix.

	Original Delta Design	Delta Design Revamped
Overall Design Goals	Four different roles balance four different sets of constraints to design	Two different roles balance two different sets of constraints to design
	one structure that meets all of the	one structure that meets all of the
	constraints below	constraints below
Key parts of Structural Engineer	Concerned with the structural	Same as the original Delta Design
role	stability of the design:	but added a required safety factor on
	• Can the anchor points support	all joints.
	Are the joints structurely	
	sound to resist the moments	
	applied to it by the rest of the	
	structure?	
Key parts of Project Manager role	Concerned with the logistics of the	Same as the original Delta Design
	design including:	but added a maximum size limit to
	Budget	each module and a new cost based
	Modular Construction Construction Time	on labor.
Key parts of Thermal Engineer	Concerned with the thermal aspects	Fliminated this role, but kept the
role	of the design:	restriction that more than two red
	• Will the structure be too hot	deltas cannot be joined together;
	or cold for the residents?	added to the problem statement so
		that everyone has knowledge of it.
Key parts of Architect role	Concerned with the aesthetics of the	Eliminated this role, but kept the
	design:	restrictions about the percentage of blue deltas allowed and the interior
	• Are there too many blue deltas?	to exterior length ratio: added to the
	 Is the interior to exterior 	problem statement so that everyone
	length ratios within the	has knowledge of it.
	acceptable range?	
Overall Design Task	Discussed the following constraints:	Added constraints (as mentioned
	Minimum interior area	adove) from the thermal engineer
		Also added the constraint of a
		maximum and minimum door width.
Units	Imaginary units (e.g., wex, lyn)	Standard units (e.g., year, meter)
Play Length	3-4 hours	1 hour

Table 1: Summary of Original and Revamped Delta Design Games

Adjusting the Number of Roles

Because they were the least relevant to ENGR 14, we removed the roles dealing with thermal engineering and architecture from the game. We did however recommend that students still work in teams of four (i.e. two project managers and two structural engineers each) to maintain the feeling of working in a team rather than in pairs.

Adding Constraints

One caveat to removing the thermal engineer and architect roles was that the game became seemingly too easy since nearly every design met the constraints without the need for modifications. Thus the most difficult challenge of the redesign process was changing and adding constraints in order to reach an appropriate level of complexity. This challenge was resolved by changing a few of the constraints, playing the game, testing how much time and how many iterations it took to reach a satisfactory design, and evaluating which constraints were posing problems either because they were too easily met or too difficult to meet. We began by arbitrarily picking constraints to change, such as increasing the interior area and decreasing the budget. Sometimes it was too difficult to meet a constraint, such as adhering to the decreased budget when extra deltas were needed to increase the interior area, while other constraints, such as not exceeding the maximum weight an anchor can support, seemed to be met automatically, no matter what was tried. Thus, in the next iteration, we increased the budget and decreased the weight an anchor can hold.

After experimenting with different values for the existing constraints, we were still not satisfied with the game's level of complexity. To make the game more complex, we introduced additional constraints as well. One way that we did this was by retaining some of the constraints from the thermal engineer's and architect's instructions. From the thermal engineer's role, we kept the constraint that there cannot be more than two red deltas next to each other, and from the architect's rules, that there can be no more than 60 percent blue deltas and that the interior-to-exterior length ratio of the structure must be greater than one. The constraints of the number of adjacent red deltas and the total number of blue deltas ensured that there would need to be more red-blue joints, thus making the budget a tougher constraint to meet because the cement needed to join red and blue deltas is the most expensive. The interior to exterior ratio also provides an interesting optimization challenge that often causes teams to iterate on their design repeatedly. These rules were added to the overall design task, making all team members responsible for them, because at this point we did not wish to make the individual roles any more complicated.

Another way that we added constraints was to invent new ones. A number of rules were invented to address certain issues or to make specific parts of the game more challenging. First, there can be no more than six deltas in a module, defined by horizontal breaks in the structure; module joints cost extra and increase the time needed to build the residence. This change was made because with only a few large modules, the budget constraint is met very easily. Since not exceeding the budget is the project manager's main responsibility, we wanted it to be a challenging constraint. Second, the door, in addition to having to face a certain direction in relation to gravity, now has to be between 3 and 5 meters. Previously, there was no specific size

for the door, and we discovered that large doorways decrease the number of deltas needed to meet the internal area constraint, which in turn makes it easier to meet all the other constraints because fewer deltas means a lower weight and cost. Third, there is a new labor cost based on construction time, further reflecting real-world engineering constraints. To accommodate this change, we also increased the total budget to \$1500. Fourth, the value of K (an overhead factor applied to the cost of labor and raw materials) is 1.5, though its value can be changed at the instructor's discretion to make the game more or less challenging, specifically for the project managers. Lastly, there is a recommended safety factor of 1.3 on all of the joints connecting each pair of deltas, showing how safety factors, an important concept of real world engineering emphasized in introductory solid mechanics, are used in practice.

Changing Original Rules

In addition to adding new constraints, we also made changes to the original game rules. One of the first such changes we made was to update the unit system. In the original game, the units of length (lyn), time (wex), and currency (zwig) are fictional. Because in this situation, we thought that it would be more useful for students to become familiar with traditional engineering units, the fictional units were changed to the standard metric units of measurement. However, because the diamonds on the game board are each two quarter-deltas long, determining the interior area was simpler to do in terms of quarter-deltas rather than square meters. Thus, using quarter-deltas as the unit of area was retained because it made more sense.

Overall, the process of making changes to the game was iterative, requiring many trials to get the constraints to what we believed was an appropriate level of difficulty. As a consequence, we experimentally changed some constraints, only to decide they were more effective at their original values. One example of this is we had originally planned to prohibit quarter-and three-quarter length joints between deltas, allowing only full side or half joints, to make calculations easier, but this was too limiting in the creation of successful designs. We also considered increasing the internal area requirement from 100 quarter-deltas to 150. However, a larger area would require more deltas and thus make it difficult for the entire class of twenty teams to play at once, resource-wise.

Writing New Instructions

The last change we made to the game was writing a new set of instructions (see Appendix) that incorporated all of the changes that we made. We divided these instructions into three parts: the Design Task, instructions for the Project Manager, and instructions for the Structural Engineer. Each of these documents was made as short and easy-to-read as possible. Many portions remained the same as in the original game, but with more step-by-step explanations, worked-out examples, and diagrams to facilitate understanding. The Design Task was also updated to include the many constraints that all team members must now consider.

Pilot Feedback

In the course of creating the new game, we conducted two preliminary trials, making modifications after each one. Even though we preferred to have four students on a team, we

were only able to test the game with pairs. Nonetheless, each trial was still valuable because we were able to collect feedback on the fundamental mechanics of the game.

First Trial

The main purpose of the first trial was to solicit feedback about the clarity of the instructions. The first trial game was played by a female sophomore mechanical engineering major and a male junior history major. For the first fifteen minutes, the students worked independently to try to meet all of the design constraints. Once they began communicating their constraints to one another and placing deltas on the board, however, they discovered the task to be easy to accomplish as long as they worked together and were open to changing the design. The students took one hour to create a successful design. Afterwards, they provided constructive feedback on the instructions, and their suggestions to include more examples and diagrams were incorporated and used in the second trial.

Second Trial

The second trial game was played by two sophomore engineering students, one male and one female. The female had taken ENGR 14 the previous quarter. This pair of students started the game by explaining their constraints to each other and then built their structure. In total, it took them exactly one hour to create a design that met all of the constraints. These students found the instructions to be clear but long. When asked if they would have preferred to work in teams of four, these students agreed, saying that it would have been helpful to work with someone who had the same set of constraints.

Formal Pilot

For our formal pilot of the redesigned Delta Design game, we organized a focus group with five engineering students who had just finished ENGR 14 in order to gain more insight on how useful the game is and where the game could potentially fit within the course. The students were divided into two teams. Two males formed one team, while two females and one male formed the other team. Each team was comprised of at least one student in the project manager role and one student in the structural engineer role, and the team of three had two students in the structural engineer role. Rather than having the students silently read the instructions, we (two members of the author team) explained the design task instructions to all of the students before separating them into two groups based on their roles and teaching them their specific instructions. This modification from the previous two trials was found to be more time-efficient, leaving more time for asking and answering questions as well as playing the game itself. Both teams constructed acceptable designs within an hour. Each team began their game by explaining their constraints to one another and doing some basic strategizing. For example, the cement needed to join a red delta to a blue delta is the most expensive, so one team decided to minimize red-blue joints. Likewise, the other team decided to use exactly 60 percent blue deltas to minimize the cost (since blue deltas are less expensive than red ones). However, both teams prioritized the structural stability of the residence above the project cost.

After playing the game, each student filled out a questionnaire asking which statics concepts from the class were used and if they thought the game would have been beneficial if played earlier in the quarter. They were all able to identify a few concepts such as forces, moments, and safety factors, though the project managers noted that they did not really use them, as their role was not concerned with the structure. Responses varied about the usefulness of the game; while one student indicated that playing the game was "not necessary" for understanding statics, several thought it could have been useful either as an introduction to of the reinforcement of the moments concepts introduced midway through the courses and two noted that the real use of the game lay in the teamwork component.

The focus group elicited other useful comments from the students. Many of the students said that they liked that the game was more about working in groups than about doing the calculations, since the calculations were "simple" and "straightforward". The two students sharing the structural engineer role reported enjoying working together. They appreciated having someone to help with the calculations and to make sure that they were doing their job correctly. Every student agreed that, while playing the game in a group of two works, it would be more beneficial to play in groups of four such that everyone would have a partner. They also advocated for working in larger groups in general, as that would further emphasize the teamwork aspect of the game. The students reported learning that designing in a team requires give-and-take between players with different primary interests, which was one of the main goals of the exercise. One student also noted his appreciation of the safety factor constraint, because it is a constraint that only needs to be met, not exceeded. This idea of the safety factor not being exceeded is important in a bridge design project the students complete as part of ENGR 14; students are instructed to build a bridge that meets but does not exceed the safety factor to emphasize the tradeoffs between safety and cost.

The students also had feedback for improving the game board and pieces. They did not like how easily the pieces moved on the game board, recommending there be some way to attach the deltas to the board, such as with Velcro or tape. They also recommended numbering the grid lines on the board and marking the edges of the deltas at the ¹/₄, ¹/₂, and ³/₄ marks to make lining the pieces up to the board and to one another easier. One student recommended including a checklist with all of the constraints for each role so that players could be sure that they were not overlooking any. Another student (in the project manager role) was frustrated by the fact that the calculations had to be done every time a piece was moved and would have liked the option of using Excel. However, most of the students did not think it was necessary because the calculations were simple and stand alone, the outcome of one calculation not affecting other calculations.

With regards to whether the game might fit into ENGR 14, four students thought it would be a good addition. Only one student disagreed, stating that since they did far more complicated moment calculations in class, Delta Design would not greatly improve their understanding. (Not coincidentally, this same student wrote that the game was unnecessary on their questionnaire.) One suggestion from the group was to emphasize the teamwork aspect more and to incorporate the game as a precursor to the aforementioned bridge project, which lasts several weeks, features more constraints, and requires a greater level of team interaction. Another proposed idea was to

use the game to replace one of the guest lectures which might not appeal to as many students in the class.

Plans for Further Changes

With this latest round of student feedback, we are now working on the next iteration of the game. In addition to designing and fabricating new delta pieces using a laser cutter, we plan to laminate the game boards. The game boards are currently paper, so laminating them would make them last longer, would make them able to be written on with dry-erase markers, and would make it easier to secure deltas to them. We also plan to include laminated checklists of the constraints for each role at the end of their respective instruction documents.

As noted by at least one student, one potential option for expanding learning from the game is to have the students create Excel spreadsheets to allow them to more quickly do some calculations. This would require them to receive their roles a class period before, read the instructions, and create the spreadsheet before coming to class to play the game. If the students had their roles ahead of time, another possibility is to have them each create an initial design without talking to other group members so the group has references to start from when they play.

Another idea to make the game more engaging is to stage a competition between teams. Every team that we piloted with managed to create a satisfactory design in an hour. One way to raise the stakes between teams is to select one constraint that the teams must go back and optimize once they have a design that works; whoever comes closest to optimizing the constraint would be declared "a winner." For example, the teams could try to make the smallest, quickest, or cheapest residence that still meets the all of the other requirements. Implementing such a requirement would ensure that the teams keep iterating on their design even after finding a solution that works, further emphasizing the teamwork aspect of the game; it will also keep teams that finish the standard task busy while others try to finish the original task. Another option is to impose a shorter time limit to add pressure, as tough time constraints are often a reality in real engineering practice.

As far as integrating the game into the ENGR 14 curriculum, the leading idea proposed by the students was to have the students play after learning about moments but before starting the bridge project. The students could also work on the same team they would work on for the bridge project to help them to learn how to work together and to work through issues they might otherwise have later.

Going forward, we plan to administer pre- and post-game surveys to ensure that students meet the stated learning objectives. As previously mentioned, these include becoming more comfortable working in teams and dealing with problems in which they do not have all of the information upfront. The surveys will ask the students about their confidence levels with respect to these goals on a sliding scale; they will also contain indicator questions to test understanding of key statics concepts before and after play. We will implement these new ideas in future offerings of ENGR 14.

Acknowledgements

The authors would like to acknowledge the work of Louis Bucciarelli as well as the assistance of the nine student volunteers who participated in this study. In addition, two of the authors (MG and MW) were supported by the Stanford Vice Provost of Undergraduate Education (VPUE) Summer Undergraduate Research Program. Another author (SB) was supported by a Stanford Vice Provost of Graduate Education (VPGE) Graduate Fellowship.

Bibliography

[1] Mazur, E. (1996) Peer instruction: a user's manual. Upper Saddle River, NJ: Prentice Hall.

[2] Smith, K., Sheppard, S. D., Johnson, D., Johnson, R. (2005). "Pedagogies of engagement: Classroom-based practices" *Journal of Engineering Education*, 94 (1): 87-102.

[3] Bucciarelli, L. L. (1999). *Design delta design: Seeing/seeing as*. Presented at the Design Thinking Research Symposium 4, Boston, MA, April 23-25.

[4] Prudhomme, G., Boujui, J. F. O., and Brissaud, D. (2003). Toward reflective practice in engineering design education. *International Journal of Engineering Education*, 19 (2): 328-337.

[5] Shuman, L. J., Atman, C. J. and Wolfe, H. (1996). *Applying TQM in the IE classroom: The switch to active learning*. Presented at the American Association for Engineering Education Annual Conference, Washington, D.C., June 26-29.

[6] Masi, B. (2003). *The impact of faculty-mentored versus web-guided engineering design experience on freshman skills*. Presented at the American Society of Engineering Education Annual Conference, Nashville, TN, June 22-25.

The Design Task

Introduction

Congratulations! You are now a member of an expert design team. Your collective task will be to design a new residence suitable for inhabitants of the imaginary Deltoid plane. These written materials, provided to help you prepare for this task, are organized in two sections. This section provides an overview of life on the Deltoid plane, DeltaP as it is known to the natives, your team, and your design task. A second handout provides the specific information you will need to perform the role you have been assigned within your team. Some people will be project managers and some will be structural engineers, and you will contribute different expertise to the project. All must work together for your team to create a first-rate design.

Life on DeltaP is quite different from what you have grown accustomed to here on Earth. First off, DeltaP is a plane, not a planet, so your team will be designing in two-dimensional rather than three-dimensional space.



Figure 1: Example Residence

In addition to lacking a z-axis, Deltoid space has unfamiliar relations between the x- and y-axes (see Figure 1). What we think of as "perpendicular" is hopelessly skewed to a Deltan, and vice versa. In our units, a right angle on DeltaP measures 60 degrees or $\pi/3$ radians. Thus all sides of an equilateral triangle form lines considered perpendicular to all others.

In this flat, angular world, residents construct their residences strictly with discrete triangular forms. Of these, the equilateral triangle -- with its three perpendicular sides (!) -- is considered the most pleasing. Accordingly, your team will design the residence by assembling into a cluster of equilateral components called "deltas." Deltas come in red and blue versions and always measure 2 meters per side. Four triangular units of area measure with sides of 1 meter (known as quarter-deltas or QDs), fit within a delta, as shown in Figure 2.



Figure 2: Quarter-deltas

As building components, deltas have more complex functional and aesthetic characteristics than their simple form and dimensions would suggest. Especially when assembled into a cluster, as you will be doing, they behave in interesting ways. All deltas are subject to DeltaP's gravity (which is itself subject to axial shifts during DeltaP's not-infrequent gravity waves). Three different kinds of cement are needed to join them together, and joint alignment with respect to gravity affects ease of production as well as structural integrity. Different colors and different quantities of deltas cost different amounts, and can be assembled in clusters that are either exceedingly ugly or very attractive to the Deltans. Your task will be to create a structurally sound design that meets prescribed goals for all these characteristics.

The Design Team

The design team is organized such that each of you will be responsible for a subset of the design goals. Two people will be **Project Managers**, whose main concerns will be with cost and schedule. They want to keep costs and time-to-build at a minimum, but not at the expense of quality. The other two people will be the **Structural Engineers**, whose main concern will be to see that the design is structurally sound under the prescribed loading conditions. You all have a common Design Task, but different additional constraints that you must work together to meet.

The Design Task

Your Deltan clients have cleared the space shown on the site map and come to your team with their need for the design of a new residential cluster. The cluster itself must meet the following specifications.

• The client wants the cluster to provide a minimum interior area of 100 QDs. The shape of this space, which can of course, exceed the minimum, is a matter of design.

- It's becoming more fashionable to have a smoother exterior and an angular interior, which means lots of nooks and crannies. The space all needs to be connected though, so no walls can entirely split the residence. One quarter-delta is an area in which three inhabitants could stand and talk comfortably to one another, but be aware of the need for air circulation. Thus, you don't want the nooks and crannies to be too small.
- Measure the interior and exterior wall lengths (in meters) and divide the interior measurement by the exterior measurement. Because a craggy wall will be longer than a smooth one, the higher the ratio is, the better you have met the client's desires. This ratio must be greater than 1.

Interior wall length

Exterior wall length

- There must be one and only one entrance/exit, and it must be aligned with gravity so that the residents enter the cluster in the direction of gravitational pull. This is so because Deltans are themselves subject to gravity. They have evolved so they are now able to move about the plane without conscious attention to the force field. However, the entrance is located so that the residents would fall into rather than out of the cluster if they were to lose this sense. This orientation is essential during the passage of a gravity wave. The doorway must be between 3 and 5 meters wide.
- The client is known to be color sensitive to blue; too much blue brings on the blues, so to speak. Blue deltas may not exceed 60% of the total number of deltas. For the same reason, no more than three blue deltas should be placed next to each other.
- For thermal considerations (red deltas have an odd property that they produce heat), you cannot have more than two red deltas next to one another.
- To meet building codes, the residence must be anchored at two points and two points only. Furthermore, there is a limit to the amount of force each anchor can support, as well as to the amount of internal moment each joint can withstand. Exceeding these limits would cause catastrophic failure and send the unwary residents tumbling into the void. Gravity waves, rare but always possible, should be considered.

All of this -- design, fabrication, and construction -- must be done under a fixed budget and within given time period (see Table 1). At your team meeting you are to develop a conceptual design that meets or exceeds all design goals.

/ I	
Functional Interior Area	100 QDs
Maximum Blue Deltas	60%
Maximum Load at Anchor Points	15 N
Overhead Factor K	1.5
Total Budget	\$1500
Interior-to-Exterior ratio	>1
Door width	3-5 meters

Table 1: Summary of Specifications

Delta Design: Structural Engineer Instructions

Introduction

As structural engineer, you are responsible for the physical integrity and robustness of your team's design. You should see to it that the two anchor points are appropriately chosen, that all joints can withstand their internal moment, and that the overall shape of the cluster does not violate sound structural engineering practice. You should also strive for an elegant and efficient design, one that provides the requisite strength and durability with minimum cost and materials.

When your team submits its final design, you will be asked to attest to its quality by explaining the location of the anchors, identifying the strongest and weakest joints and the factor of safety, and estimating, as a measure of robustness, the average factor of safety on all joints. You may be asked to predict what will happen to your design during the next gravity wave. This primer will give you the tools, essentially the methods of static equilibrium analysis, with which to do your work.

Basic Information:



- A uni-directional, gravitational force field acts on DeltaP. The direction of this force is parallel to the y-axis as shown in Figure 1.
- Each delta experiences a gravitational force of one Newton. Thus, for the example, in Figure 1 the structure has 24 deltas and therefore a total weight of 24N.
- The structure is kept stationary by reaction forces at the anchors, marked in the figure as points A and B.

Finding the center of gravity:

The first step in structural analysis is to locate the cluster's center of gravity. For our initial purposes we only need the center of gravity's x-coordinate, which gives us the line of action of the gravity force. We do not need to know the y-coordinate until we consider Delta's gravity waves. There are two things to keep in mind throughout your calculations. First, keep them as simple as possible by working only in integers and estimating distances, forces and moments to the nearest meter, Newton or Newton-meter, respectively. Second, keep in mind the peculiarities of Deltan space, where "perpendicular" describes an arc measuring only 60 degrees or $\pi/3$ radians and where distance measurements are made only along

lines parallel to the axes. This distinction is critical in the calculation of moments. As on Earth, moment is still the product of the force and its distance from the point, but the distance must be measured along the Deltan axes.



In static equilibrium we know that all moments around any given point will sum to zero. So, using this knowledge, finding the x-coordinate of the center of gravity of a structure is as simple as summing the x-coordinates of all the deltas in the structure and dividing by the total number of deltas. In Figure 2 we see the same structure from Figure 1 labeled with the x-coordinate of each delta. In this, the x-coordinates of all deltas sum to 102 m. Dividing by the total number of deltas in the structure (24) gives us 4 m, the location of the center of gravity's line of action of the force due to gravity.



Estimating support loads:

Each of the two anchors has sufficient strength to support a load of 15N. However, just as on Earth, it is important to include a factor of safety (we recommend 1.3) to guard against unforeseen circumstances. This means that each anchor should only be supporting approximately 11.5N. The anchors are frictionless pins meaning that they cannot resist any moment.

To estimate the support load of each anchor (see Figure 3) we must first find the total force due to gravity acting on all deltas in the cluster, which is equal to the number of deltas in the structure times 1N. We also know that, for equilibrium, the two anchor forces must equal the gravitational force in magnitude:

Total weight of structure = F_G = (number of deltas)*(1 N)

 $F_A + F_B = F_G$

In this case: $F_A + F_B = 24N$

Since we have two unknown variables, we need a second equation relating F_A and F_B , which we can get by calculating the total moment of the entire cluster. For this example, we will calculate the moment about anchor B. Since we know that, in equilibrium, the moment of F_A about anchor B must balance the moment of F_G about anchor B. So we have:

(distance of A to B)(F_A)- F_G (distance of center of gravity to B)=0

(distance of A to B)(F_A) = F_G (distance of center of gravity to B)

For this example: (10m)(F_A) = (24N)(3m) F_A =7N

Plugging this value into the first equation: $F_B = 24$ - F_A =17N

Since anchors should be supporting only approximately 10N, anchor A fully meets requirements but anchor B is unacceptable. In this case the anchors would need to be relocated and the total number of deltas reduced to create an acceptable design.

Internal Moments and Fastener Requirements:

Adjacent deltas are held together by cement. Cement is most important for counteracting internal moments caused by gravitational loading. Your cement has been certified at a strength of 20 N-m per m of contact. This relation is seen in the figure below:



Figure 4

Given this length-strength relationship, how do we estimate the actual internal moment that joint will experience? Simply treat each joint as the boundary of a sub-cluster and apply the same method as for the whole structure to this sub-cluster (see Figure 5). Here are the steps:



First, determine the sub-cluster's center of mass and gravity line of action using the same method that was applied to the whole structure when evaluating the anchor strength (see figure 6).



Next, identify the forces acting on the sub-cluster. This will be the force due to gravity applied at the center of mass and the force applied by the anchor, if an anchor is included in the sub-cluster.



Figure 7

Since the sub-cluster must be in static equilibrium, we know that the internal moment at a particular joint must be equal to the moments exerted by the force due to gravity and the anchor force. Example (see Figure 7):

-Fg(3m)+Fa(6m)=Moment at the indentified boundary Fg=11N, Fa=7N (as previously calculated) -11(3)+7(6)=9N-m

Since this particular joint is a ³/₄ connection, it can withstand a moment of 30N-m. Therefore the 9N-m of moment applied to the joint is fully within the joint's capacity to withstand. In fact, the joint is probably over engineered.

Gravity Waves

Finally, although the gravitation force across the Deltoid plane is forever constant in magnitude, the rare but inevitable gravity wave will, when it comes, instantaneously shift its direction from the y-axis to the x-axis. Gravity will then retain that direction until the next wave arrives and causes it to shift back. The last gravity wave passed through about 100 years ago, causing widespread destruction. The time period between waves has a mean arrival frequency of once every 200 years. Additionally, the validity of these statistics is questionable and time between gravitational waves has been known to be as low as 10 years. Is this a factor you need to worry about? To see how your structure would respond to a gravity wave, use the same method described above.

Delta Design: Project Manager Instructions

Introduction

As project manager, your main concerns are cost and schedule. You want to keep costs and time-to-build to a minimum, but not at the expense of quality. When your team submits its final design, you must report the cost and time you estimate will be required to build it.

Estimating Project Costs

The cost for the final structure includes 4 components:

- 1. Cost of the deltas
- 2. Cost of the cement to glue them together
- 3. Cost from the modular construction technique
- 4. Cost of labor (based on time to build the residence)

To estimate the cost of your team's design:

Figure the cost of the deltas used Figure the cost of the cement needed to join them Figure the number of modules and the cost to join them Estimate the time to build and multiply by the labor cost Sum all these up and multiply by the overhead rate (K)

To estimate how long it will take to construct your design:

Identify the separate modules Determine how long it will take to construct each one Determine how long it will take to assemble them at the site Sum these up

Delta Costs

The cost of deltas varies by color and quantity purchased, as shown in Figure 1. The price break for blue deltas is at 8 units: blues cost \$10 apiece if fewer than 8 are purchased, \$6 each for 8 or more. The price break for red deltas is at 12 units: reds cost \$8 if fewer than 12 are purchased, \$6 each for 12 or more.



Figure 1: Delta Costs

Cement Costs

You will need to purchase three different kinds of cement, at three different costs, to assemble deltas into your structure (see Table 1). Three types $-R^2$, B^2 , and RB - are required because different types of joints require different types of cement.

Table 1: Cement Costs		
Cement Unit Costs		
R ²	\$10/meter	
RB	\$20/meter	
B ²	\$5/meter	

Note that the cost of fastening one delta to another will be determined by the length of contact between elements as well as by their respective colors: the longer the joint, the more glue required (see Figure 2).



Module Joining Costs

Construction will proceed in two stages. In the first stage, individual deltas are joined into modules. This takes place at the factory, where the supplier firm has developed jigs and fixtures that simplify the task.

The individual modules into which a given structure will be decomposed and constructed at the factory are easy to identify because the boundaries between them are defined by the orientation of the joints relative to gravity. To an earthly eye, any intersection of two deltas that runs left to right, across the page, is a module boundary (see Figure 3). Since these modules have to be made in the factory and then shipped to the site, you can have no more than 6 deltas in a module. The figure below shows a partial design with three boundaries and thus four modules.



Figure 3: Module Breaks

When all modules are complete, they are transported to the site, joined together, and anchored to the plane. The on-site work is more difficult to cost out in advance, so the client will essentially have to pay whatever costs are incurred, not including the labor costs. Your experienced contractor, however, has told you that her rule of thumb for predicting them is to figure out the cost of the glue needed for the module-to-module joints and double it. These module-to-module costs are in addition to the cost of cement.

Labor Cost

First, estimate the time it will take to build (see below for how to do that!). Multiply the number of days by \$5.

Total Cost

The total cost to execute your design may be estimated by summing up the cost of the deltas, cement, and module joinery, and multiplying the result by an overhead factory, K. K = 1.5, which takes into account the cost of living on DeltaP.

Total cost = K x (delta cost +cement cost + module cost + labor cost)

Estimating Time-to-Build

Estimating time to build is inexact, at best, but again your contractor has supplied some rules of thumb.

For each module consisting of three deltas or fewer, allow 2 days. For each module consisting of more than three deltas, allow 3 days. For each module-to-module joint, allow 4 days.

Sum these up and double the result.