Seeing Mathematics for a Deep Understanding: Two Calculus Practices

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Abstract

Students must understand and be able to use mathematics in both their professional and personal lives. Nevertheless, the mathematics learned at school are considered by many to be irrelevant, unnecessary, and unrelated to the activities that the students will encounter, not only in their personal but also in their professional lives. These negative attitudes about mathematics can be changed if the students have a deep understanding of mathematics based on practices, as users of mathematics, that involve solving real life problems. In real life problems there are no given directions about what operations have to be performed; neither are there given the functions to have under consideration. Real life problems are introduced as a command: You have to design the cheapest container which satisfies these specifications of volume, shape, and materials! Or as a necessity: I have to find the center of mass of this machine part!

This paper presents two exercises to solve real life problems using calculus. The first activity consists of determining the volume of a plastic glass. Several glasses will be distributed among the listeners, and they will be asked to calculate its volume. After a couple of minutes of brainstorming about how to solve the problem, some guides will occur to solve it. The final stage happens when the glass is filled with colored water using an amount equivalent to the volume found, and this visually verifies the certainty of the answer. The second educational exercise consists of calculating the weight of a piece of wood with constant thickness but irregular shape. Some of these items will be distributed among the spectators as well as the information about the weight of a two-inch square sample of wood that has the same thickness. The final stage happens when the irregular piece of wood is placed on the digital electronic weighing scale to verify its weight.

Introduction

Society has many misconceptions about mathematics. The pervasive role of mathematics is underestimated in both the world of work and the world of everyday living. The mathematics learned at school is considered to be irrelevant, unnecessary, and unrelated to the mathematics students will encounter in their professional and personal lives. These false perceptions and regrettable attitudes about mathematics have a significant and negative impact on mathematics education. These attitudes about mathematics can be changed only when students become knowledgeable about mathematics and are prepared to become users of mathematics in both their personal and professional lives.

The goal of a mathematics course, especially a calculus course, must be to provide an opportunity for every student to learn significant mathematics with a deep understanding not only of the concepts but of the applications, as well. To achieve this goal, the mathematics course must be more than a collection of sets of memorized facts and rules combined with several books and computer exercises. In particular, the calculus course must be challenging and complemented with practices and activities about real life problems in engineering and science. In a calculus real life problem, there are not specific directions like: find the area of the region bounded by the function $f(x)$, the x-axis, the line $x = a$, and the line $x = b$. Real life calculus problems are introduced by a command: design the cheapest container which satisfies these specifications of volume and shape; or by a necessity: I have to find the center of mass of this machine part.

In real life and in engineering and science courses, students need to know and be able to use calculus facts and procedures as quickly as possible. However, research[1] makes it clear that how mathematics is taught is as important, or even more important than the mathematical concepts being taught. Teaching mathematics through laboratory activities is an important way for instructors to "measure" the understanding of the student's mathematics concepts by observing their behavior, attitudes, and skills in situations of real life problems.

The findings of learning for physics are generally applicable to mathematics, too. Research has demonstrated that a more active way of learning brings students to a more robust understanding of physics concepts than does the traditional lecture approach[2-6]. Other research has demonstrated that students in university physics programs that engage them in a hands-on, active construction of knowledge have been shown to outperform those in traditional lecture/demonstration classes, even on quantitative (as opposed to conceptual) assessments[7-9].

Unfortunately, there is also evidence that some groups of students may benefit more from interaction than others. One study of high school students in the Netherlands found that girls did not perform as well as boys when the activities were performed by the students themselves, rather than by an instructor[10-12]. Pricilla Laws also reports that college women in her Workshop Physics courses sometimes expressed frustration with the time and effort required for active learning, in spite of having done well in the courses[13].

The active learning method has been criticized for its lack of coverage[14] and lack of correspondence to social and cultural expectations[15]. Abstract algebra, in particular, is difficult to approach from a hands-on standpoint.

This paper shows, by presenting two activities performed by Sophomore Engineering Technology students in the class of Advanced Analytical Math, that the active learning method is particularly appropriate for the topics of the application of calculus. This study has shown that when activities, such as these described in this paper, are performed by students themselves, they are more motivated and become active participants in the learning process. Each activity encourages students to demonstrate their understanding of calculus concepts. It is suggested that each math educator consider the importance of including active-based exercises in their courses, even if it is with little frequency or with limited resources. It is also recommend to find real-world evidence of the uses of calculus in the community and then build into the curriculum a set of “case of studies” as evidence of calculus at work.

Activity 1: Volume of a Plastic Glass

Abstract

In this activity, students will demonstrate their knowledge of the calculation of volumes of a solid of revolution.

Prerequisites

Students should be:

- ✓ Able to take measurements in centimeters.
- ✓ Able to find the radius of a circle given its circumference.
- ✓ Able to graph the points of height vs. radius (scatter diagram), using the TI-83 calculator.
- ✓ Able to find the function of the curve that models (best fit) the scatter diagram, using the TI-83.

✓ Able to find the volume of solids of revolution.

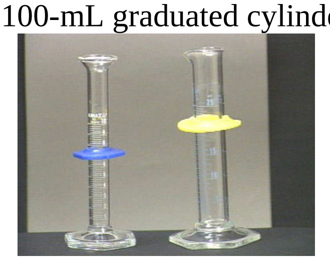
Materials

- Truncated conical plastic glass (or flower vase).

- One-half meter string.
- Ruler.
- Tape.
- TI-83 calculator.
- A gallon of water (colored water is better).
- A 250-mL plastic chemistry beaker.

- A 100-mL graduated cylinder.



Every group will receive a truncated conical plastic glass (or flower vase), string, tape, and a ruler. Be sure that each group has a TI-83 calculator. Ask the students to find the volume of the glass (or vase). One member of the group will present their result to the instructor.

Instructions

These directions may be given or omitted at the instructor's discretion.

Truncated Conical Glass

If the group receives a truncated conical glass, take measurements of the radius of the base (r), the radius of the top (R), and the height (h). Those measurements are list below:

r = 1.1 in,
R = 1.5 in, and
h = 5.64 in

The equation of the line passing through the points (0 , 1.1) and (5.64 , 1.5) is as follows

$$y = \frac{0.4}{5.64}x + 1.1$$

The volume of the glass is now calculated

$$V = \pi\int_0^h [f(x)]^2\,dx = \pi\int_0^{5.64}\left[\frac{0.4}{5.64}x + 1.1\right]^2 dx = 30.18 \quad in^3$$

Since 1 in = 2.54 cm, and 1cm^3 = 1 mL, then

$$V = 30.18 \times 16.39 \quad cm^3 = 494.7 \quad mL$$

This amount is compared versus the real volume of the glass, which is

$$V = 495 \quad mL$$

The difference is very small and is due to errors in measurements.

Flower Vase.

If the group receives a flower vase, by using the string, take measurements of circumferences at the base and at each 0.25 in interval from the base to the rim of the top. Let is assume that the height of the vase is h. Calculate the radius related to each circumference.

Create a scatter plot with this data. On the TI-83 calculator press the following keys: STAT, 1. Now, in L1 enter the intervals 0, 0.25, 0.5, … , h; and in L2 enter the corresponding radius value to each interval value. Press the following keys to make the scatter plot: 2nd, Y =, 1, ENTER, ZOOM, 6. Press the following keys to find the

equation of the polynomial of fourth degree $f(x)$ that best fits the data set: STAT, ▶, 7, ENTER.
Calculate the volume of the vase by solving the following definite integral

$$V = \pi \int_0^h \left[f(x) \right]^2 dx$$

Checking

The instructor will verify the answer presented by each group by filling the graduated cylinder with water to the desired level (1 cm^3 = 1 mL) and transferring the entire contents of the cylinder to the glass (or vase). If the glass (or vase) overflows or does not fill completely, the group has to check its calculations and measurements to look for mistakes.

Activity 2: Calculating the weight of a wood piece

Abstract

In this activity, the student will calculate the weight of a piece of wood that has a constant thickness but curved shape.

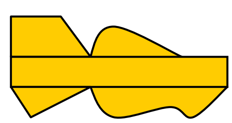
Prerequisites

Students should be:

- ✓ Able to take measurements in inches.
- ✓ Able to find proportions.
- ✓ Able to find areas of geometric figures (rectangles, triangles, circles,...).
- ✓ Able to do a scatter diagram, using the TI-83 calculator.
- ✓ Able to find the function of the curve that models (best fit) the scatter diagram, using the TI-83.
- ✓ Able to evaluate definite integrals.

Materials

- ➢ Irregular piece of wood (make your own design) .



- Graph paper.
- Ruler.
- TI-83 calculator.
- Digital scale

Every group will receive a piece of wood, a sample 2”×2” block of the same wood with a label on it indicating its weight in grams, and a ruler. Ensure that each group has a TI-83 calculator. Ask the students to find the weight of the irregular piece of wood. One member of the group will present their results to the teacher.

Instructions

These directions may be given or omitted at the instructor discretion.

Weigh the square sample of wood and find its density. Divide the piece of wood into rectangles, triangles, circles, etc. Take the necessary measurements to find the area of each geometric figure. Draw the curved shapes of the piece of wood on the graph paper. Take the necessary measurements to set up a scatter diagram. Find the equation of the curve that best fits the data set. Evaluate corresponding definite integrals to find the area of these curved regions. Add up all the areas found. Multiply the total area by the density of the wood. The resulting number is the weight of the piece of wood.

Checking

The instructor will verify the answer presented by each group by putting the piece of wood on the digital scale. Only the instructor will read the dial on the scale. If the weight is correct, the instructor will tell them that they got the right answer and then show them the display on the scale. Otherwise, the instructor will tell them that they have an incorrect answer. The group has to check its calculations to find the mistakes.

Conclusions

The above practices were designed to illustrate sophomore students, in Engineering Technology, the applications of calculus. Particularly, these practices were implemented in the MATH 2413 Calculus I course (3 lecture and 2 laboratory). This course covers integration with applications. Its prerequisite is Pre-calculus. Such practices were about finding the volume of a truncated conical plastic glass and about estimating the weight of an irregular shape piece of wood. The reality is that one would just weigh the object to find its weight and put water in to find the volume of the plastic glass. The reality is also that the actions described before cannot be performed if the physical model of the region or the solid is not given. Thinking of the professional activities of an engineering technology student, one can conclude that he needs to learn how calculate areas of regions and volumes of solids from drawings. This is exactly the learning objective of such practices.

The authors of this paper suggest that calculus teachers at college, or even at high school, implement these practices. The reward will be a class more motivated in learning Calculus.

A manual of Calculus applications involving practices to calculate the areas of regions, the volumes of solids, the lengths of curves, the amount of work it takes to pump liquids from below ground, the forces against floodgates, the center of mass, etc. will be of great benefit to Calculus teachers. The design of this manual, and the success of its implementation, will generate the necessity of a Calculus Laboratory room. These are the future contributions of the authors.

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