

# Shape Optimization Design and Material Selection for a Fitness Equipment

Ruoxu Jia<sup>1</sup>, Junling Hu<sup>1</sup>, Xingguo Xiong<sup>2</sup>, and Linfeng Zhang<sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, University of Bridgeport, Bridgeport, CT, USA

<sup>2</sup> Department of Electrical Engineering and Computer Engineering, University of Bridgeport, Bridgeport, CT, USA

**Abstract**—Engineers face constant challenges to design products with optimal geometry, dimensions, and select optimal materials and manufacturing process for an optimal design to enhance product performance and reduce cost. This paper designed a bar structure for a light and cheap pull up bar. Ashby's material selection method was used to select material for the two conflicting objectives of minimizing weight and cost and also satisfy the structure constraint. An FEM model was built in ANSYS for stress analysis of the bar structure and for geometry optimization. The optimized geometry was found to have a 30% weight reduction.

**Keywords**—bar structure; ANSYS; material selection

## I. INTRODUCTION

Engineers face constant challenge to design a product or system with low weight, low cost and good performance. A key objective of mechanical engineering design is to define the dimensions of a component and the materials from which it is made so that it can perform a function acceptably and economically [1]. Optimum design of a product is the selection of the geometry, material and manufacturing process to meet design requirements and maximize its performance and minimize its cost [2,3].

There are three aspects to consider in the geometry design of structure a product: (i) topology, which concerns the number and connectivity of members; (ii) shape, which pertains to the location of structural joints; and (iii) sizing, which involves defining member cross-sections [6]. The specification of each aspect of the structure typically corresponds to the three major stages of the engineering design process as defined by Pahl and Beitz [7]: conceptual, embodiment (design development) and detail. The topology of the structure is typically identified during conceptual design based on the functional requirements and architectural aesthetics, whereas the structure's shape and member sizing are determined during the design development and detailed design phases, respectively.

Materials and process information is needed at every design stage. Material identification at the early design stage need approximate data for all materials and processes and material selection at the final detail stage need to consider precise and detailed data for one or a few materials and processes [2,3].

There are tens of thousands of materials and hundreds of manufacturing processes to be chosen to shape, join and finish for a product. Mechanical engineers either assume a material before optimizing the geometry or select the best material for an existing geometry of a structure, but neither approach guarantee the optimal combination of geometry and material [1]. Many optimization methods have been developed to integrate geometry design and material selection. However, these methods are only for simple systems.

Extensive research has been devoted to develop various material selection methods. Ashby etc. [3] have developed materials strategies for materials and processes. They presented four steps to choose materials and processes for design requirements: (1) translating design requirements into a specification for material and process; (2) screening out those that cannot meet the specification; (3) ranking the surviving materials and process and identifying those have the greatest potential; (4) searching for supporting information about the top ranked candidates, such as case studies of their use to know their strengths and weaknesses. The key part of the material selection process is screening and ranking of solutions. There is an increasing use of computer tools to help manage the large amount of information and to implement selection strategies, particularly for multi-criteria decision making [8].

In this project, Ashby's material selection strategy is used to choose material and manufacturing processes for a wall mounted pull up bar structure. A computer aided material selection software package CES Edupack is used to select the material and process for the bar structure. The structural analysis of the bar structure is carried out in ANSYS Workbench.

## II. PROBLEM STATEMENT

Pull-up is a popular exercise to build up upper-body muscles with pulling motions. This project is to design a pull-up bar for every day pull-up exercise. There are hundreds pull-up bars available in the market because its popularity [9]. There are four different types of pull-up bars, including door frame leverage, telescopic door way bar, wall mounted bar, and ceiling hung bar. These bars have various structure

designs and provide different user weight support and space. Wall/ceiling mounted bars in the market support more weight than other two types. Wall mounted bars are easy to install compared to ceiling mounted bars and have no restrictions on door sizes compared to door frame or door way bars. Besides the function of the pull-up bars, price and weight of the bars are very important factors for user to select a pull-up bar.

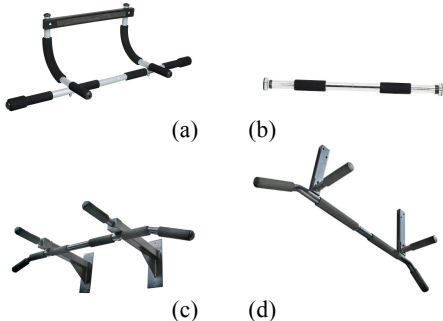


Fig.1 Four types of pull up bar: (a) door frame leverage bar, (b) door way bar, (c) wall mounted bar, (d) ceiling mounted bar [9]

The objectives will be met through an optimal structure design and selection of the best material and manufacturing process. The wall mounted pull-up models in the market have various structure designs, supporting a weight in a range of 200lb to 500lb, and providing a distance from wall in a range of 19 in to 30 in. This project is aimed to design an inexpensive light pull-up bar for a person with an average weight.

The project adopted a wall mounted pull up bar model from Ultimate Body Press [10], as shown in Fig. 2. The bar structure consists of two reinforced heavy duty beams with a grip at each end and a pull up bar with four grips. As the most important component of this structure is the supporting beam, this paper presented the structural design and material selection of a supporting beam which is designed to support a body weight of 250 lb applied at the grip handle 20 in away from wall.

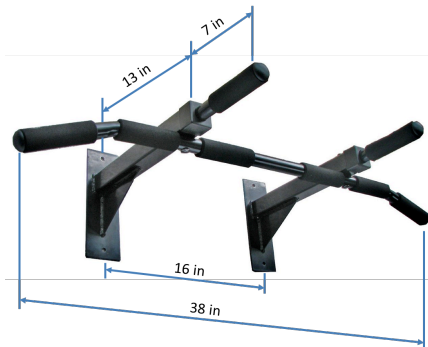


Fig.2 The wall mounted pull up bar model adopted for this design [10]

### III. SELECTION OF MATERIAL

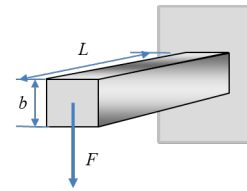


Fig.3 Beam model of a pull up bar

The supporting beam can be modeled as a cantilever beam with a standard squire cross section of thickness  $b$  and length  $L$ , as shown in Fig. 3. The design requirements are summarized in Table I.

TABLE I. TRANSLATION OF DESIGN REQUIREMENTS

Function	Beam to resist bending load
Objective	Minimize mass and material cost
Constraint	Load $F$ is specified Length $L$ is specified as
Free variables	Material choice Cross section shape

The objectives for the pull up bars are to minimize mass and minimize cost. The mass and cost of the bar can be expressed as:

$$m = AL\rho \quad (1)$$

$$C = C_m AL\rho \quad (2)$$

where  $A$  is the cross sectional area,  $\rho$  is the density, and  $C_m$  is the price of the material of the beam. The beam will need to support a maximum bending load at the fixed end,  $M = FL$ .

$$M \leq Z\sigma_y \quad (3)$$

where  $Z$  is the section modulus of the cross sectional area and  $\sigma_y$  is the yield strength of the material of the beam. There are other cross-section shapes, such as hollow rectangular section and hollow circular section, and they are more efficient in resisting bending load. A plastic bending shape efficiency factor is introduced to compare the bending shape efficiency of a cross section with a standard solid squire cross section of same area.

$$\phi_B^f = \frac{6Z}{A^{3/2}} \quad (4)$$

Substitute Eq. (3) into Eq. (4) to obtain a lower limit for the cross section area  $A$ .

$$A \geq \left( \frac{6M}{\sigma_y \phi_B^f} \right)^{2/3} \quad (5)$$

Substitute Eq. (5) into Eqs. (1) and (2) to obtain the expression of mass and cost in the combination of geometry constraints and load constraints

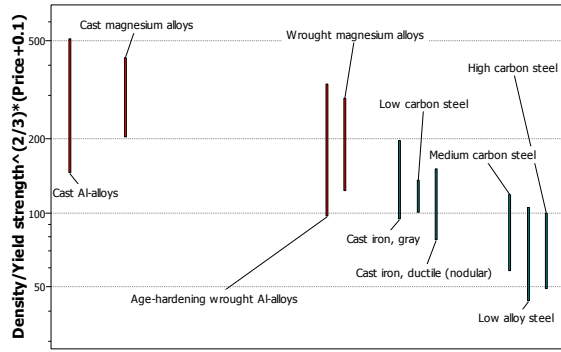
$$m \geq \rho L \left( \frac{6FL}{\sigma_y \phi_B^f} \right)^{2/3} = \frac{\rho}{(\sigma_y \phi_B^f)^{2/3}} L (6FL)^{2/3} \quad (6)$$

$$C \geq \rho L \left( \frac{6FL}{\sigma_y \phi_B^f} \right)^{2/3} = \frac{\rho C_m}{(\sigma_y \phi_B^f)^{2/3}} L (6FL)^{2/3} \quad (7)$$

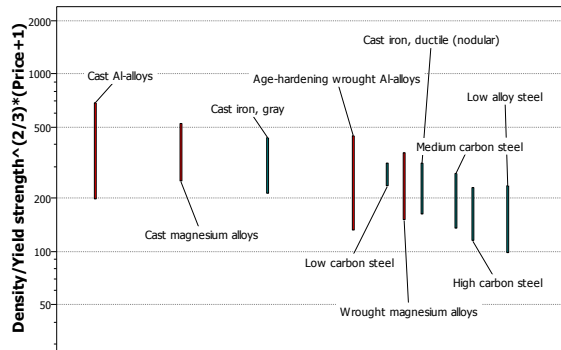
The penalty method was applied to combine the two objective functions. For a specified load  $F$  and the beam length  $L$ , the mass and cost of the beam change only with the material properties and cross-section shape. Therefore, the load  $F$  and length  $L$  are dropped out the equation for material selection. A penalty function is constructed for the optimization with two conflicting objectives.

$$Z_p = \frac{\rho}{(\sigma_y \phi_B^f)^{2/3}} (C_m + \alpha) \quad (8)$$

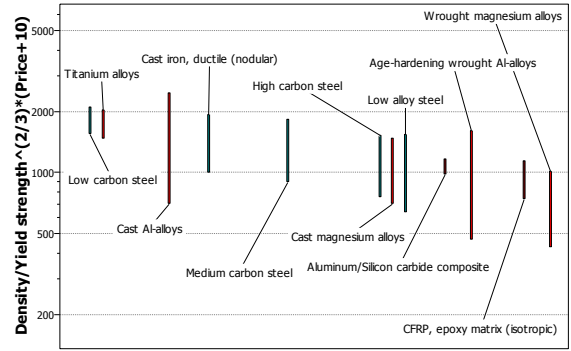
where  $\alpha$  is the exchange constant, which measures the value of performance. Three exchange constants are used to represent three cases, 0.1\$/kg for weight is a less concern compared to cost, 1\$/kg for weight and cost are equally important, and 10\$/kg for weight is more important than cost. The penalty functions for the three cases are plotted in CES and the best sets of materials are shown in Fig. 4. The plots does not include shape factor in the penalty function. As it can be seen in Fig. 4 that the best material for a cheap pull up bar is high carbon steel, for a light and cheap bar is low alloy steel, and a light bar is wrought magnesium alloy or CFRP.



(a)



(b)



(c)

Fig. 4. Best materials with three exchange constants: (a)  $\alpha = 0.1$ \$/kg, (b)  $\alpha = 1$ \$/kg, (c)  $\alpha = 10$ \$/kg

As the wall mount pull up bars do not need to be portable, cost is equally or more important than weight. Therefore, low alloy steel is chosen as the material for the pull up bars. Further selection with CES level 3 materials gives the best material as AISI 9255 low alloy steel. The properties of this steel is listed in Table II.

TABLE II PROPERTIES OF LOW ALLY STEEL

Properties	Symbol	Value	Unit
Density	$\rho$	7800 - 7900	kg/m <sup>3</sup>
Price	$C_m$	0.54 - 0.60	\$/kg
Young's modulus	$E$	206 - 216	GPa
Poisson's ratio	$\nu$	0.285 - 0.295	
Yield strength	$\sigma_y$	1840 - 2260	MPa

### III SELECTION OF SHAPE AND SIZE

The maximum achievable plastic bending shape factor for structural steel is 13. It can be achieved by using effective section shapes such as hollow tubes. The shape efficiency factors for circular and rectangular hollow tubes are listed in Table III. For the same tube thickness, a circular hollow section is more efficient in torsion and less efficient in bending than a rectangular section. Therefore, a hollow rectangular section is chosen for the pull up bar and a hollow circular section is chosen for the bar handle.

TABLE III SHAPE EFFICIENCY FACTORS FOR HOLLOW TUBES [3]

Section Shape	Bending Factor $\phi_b^c$	Torsional Factor $\phi_t^c$	Bending Factor $\phi_b^r$	Torsional Factor $\phi_t^r$
	$\frac{3}{\pi} \left( \frac{r}{t} \right) (r \gg t)$	$1.14 \left( \frac{r}{t} \right)$	$\frac{3}{\sqrt{2\pi}} \sqrt{\frac{r}{t}}$	$1.91 \sqrt{\frac{r}{t}}$
	$\frac{1}{2} \frac{h}{t} \frac{(1+3b/h)}{(1+b/h)^2} (h, b \gg t)$	$\frac{3.57b^2 \left(1 - \frac{t}{h}\right)^4}{th \left(1 + \frac{b}{h}\right)^3}$	$\frac{1}{\sqrt{2}} \sqrt{\frac{h}{t}} \frac{\left(1 + \frac{3b}{h}\right)}{\left(1 + \frac{b}{h}\right)^{3/2}}$	$3.39 \sqrt{\frac{h}{bt}} \frac{1}{\left(1 + \frac{h}{b}\right)^{3/2}}$

The pull up bar is designed to support a load of 250lb and applied on a handle of 20 inches away from the fixture. The sectional size of the hollow rectangular bar is 0.8 inch  $\times$  0.8 inch. The steel thickness is 0.16 inch (14 gauge). The plastic

bending shape factor is calculated to be 5 according to the formula in Table II. The safety factor is calculated to be 2.35.

#### IV. FINITE ELEMENT MODEL

In order to get an accurate stress when a load is used, the static structure analysis is conducted in ANSYS Workbench. The finite element model includes three components bonded together: a fixture fixed to wall, a hollow square beam joined to the fixture, and a circular cross bar connected to the square beam. The outer dimensions of the cross-sections of these three components are shown in Fig. 4. The fixture has a size of 3 inch  $\times$  3 inch  $\times$  0.1 inch. The hollow square beam has a size of 0.8 inch  $\times$  0.8 inch  $\times$  13 inch. Its thickness is 0.09 inch. The circular bar has a radius of 0.35 inch and a length of 7 inch.

The surfaces of the fixtures connected to wall have a fixed boundary condition. A 250 lb. load is applied on the bar. As shown in Fig. 5, the maximum equivalent (von-Mises) stress is found to be  $5.02 \times 10^8$  Pa (502MPa) at the square beam. The safety factor is calculated to be 3.66.

The simulation results show the maximum Equivalent (von-Mises) stress is at the junction of end of the bars. Therefore, an optimization is further carried out in ANSYS to optimize the size of the square bar. The thickness and width of the hollow square bar are set as two parameters to optimize in order to find the optimal cross section dimension for the hollow square bar to achieve minimization of mass. Taking a safety factor of 1.5, the maximum equivalent stress was targeted to be 1.24 GPa.

The tradeoff plots of the parameters are shown in Figs. 6 and 7 for equivalent stress and mass, respectively. It shows that equivalent stress decreases with the increases of bar size and bar thickness, while mass increases with bar size and bar thickness. The sensitivities of equivalent stress and mass to bar size and bar thickness are shown in Fig. 8. It is found that both stresses and mass are more sensitive to bar size than bar thickness.

The optimal cross section of the hollow square bar is found to have a width of 0.586 inch and a thickness 0.103 inch. The mass is reduced to 0.35 kg with a 30% reduction. The shape factor is calculated as 5.7. And the safety factor is 1.51. As a result, the bar thickness and bar size can be modified for a lighter structure.

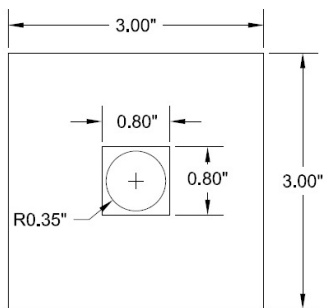


Fig.4 Dimension of bars

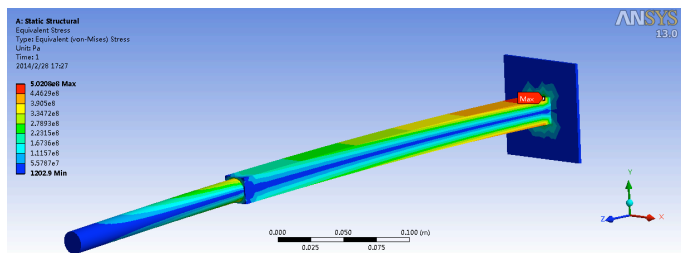


Fig.5 distribution of Equivalent (von-Mises) stress

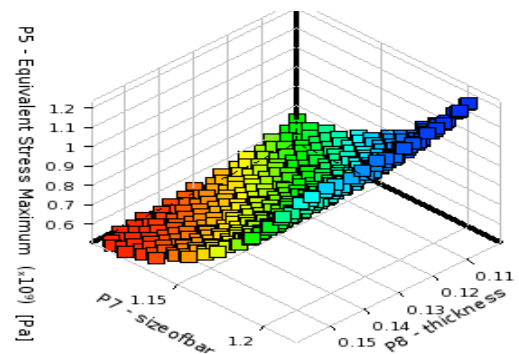


Fig.6 Tradeoff of Equivalent (von-Mises) stress

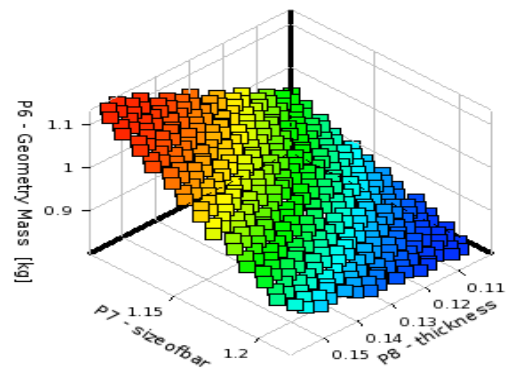


Fig.7 Tradeoff of mass

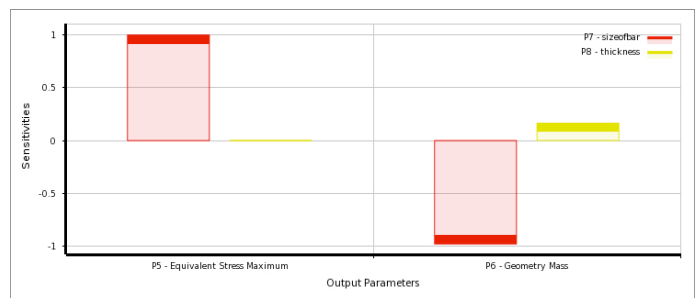


Fig.8 the sensitivities of bar size and bar thickness

#### V. CONCLUSIONS

This paper designed a light and cheap pull-up bar to support a 250 lb people for upper body exercises. The support beam was designed and the material was selected. The material selection followed the four steps of Ashby's

materials selection method. The first step translated the design requirement in terms of functions, objectives, constraints and free variables. The material indices were derived to combine constraints and objectives for material selection in CES. The material indices and constraints were used to screen and rank materials in CES. The best material found to achieve for a light and cheap pull-up bar structure was low alloy steel. The best material was used to choose appropriate cross section shape and its design dimensions. A finite element model was built in ANSYS to calculate the maximum equivalent stress and also used for structure optimization. The optimized structure design was found to reduce weight and thus cost of material by 30%.

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