AC 2009-1086: SIMPLIFIED TEACHING AND UNDERSTANDING OF HISTOGRAM EQUALIZATION IN DIGITAL IMAGE PROCESSING

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Simplified Teaching and Understanding of Histogram Equalization in Digital Image Processing

1.0 Abstract

Histogram equalization is a widely used contrast-enhancement technique in image processing. This subtopic is included in almost all image-processing courses and textbooks. It is however one of the difficult image processing techniques to fully understand, especially for those encountering it for the first time. This is because of the complex nature of the mathematics used in standard and other textbooks in introducing histogram equalization. To alleviate these problems and to provide to those wanting to understand this topic with an insight into the process and operations involved, the author developed a simpler teaching/ learning framework and background (methodology), a simple and clear theory and the necessary derived equations, a clear histogram equalization process, and a MATLAB GUIDE® based GUI tool for visual demonstrations. Because of these developments, it was possible to easily explain and teach histogram equalization clearly at a very high level of rigor than was otherwise possible.

2.0 Introduction

Some images contain significant amounts of details which are many times not visually apparent, and thus these images may not be suitable for any serious visual analysis or even viewing pleasure. One reason for this may be is that these images are poorly contrasted, i.e., they have a poor dynamic range in their pixel intensities. What is desirable in such situations is that the dynamic range of these intensities (gray levels) in the images are made much higher, and thus provide improved visual contrast for greatest contrast clarity (meaning that their intensity distributions be made much more spread across the full intensity range). Because of this need, various contrast-enhancement techniques[1-9] are applied to an image in such situations.

One subclass within such contrast-enhancement techniques is known as image contrast-stretching. This is a pixel (point) processing class of technique in which pixel intensities in an image are remapped to appropriate values based on a desired visual end result. A very important contrast-enhancement/ stretching technique with wide applications is an automatic procedure known as histogram equalization[1-9]. It is called an automatic procedure because it does not require any user control parameters for its application to an image.

Because of the importance of histogram equalization and its potential wide applications, this subtopic is included in almost all serious image-processing courses and textbooks[1-9]. However, it is one of the difficult image processing techniques to understand and implement especially for those encountering it for the first time (except when using a canned function to perform its operation). The reason for this difficulty is because, though an image in this regard has nothing to do with probabilities and probability distributions as such in general, the formulation and presentation of the background and theory for histogram equalization in almost all standard and other textbooks[1-9] are based on the above "advanced" background and theory with the additional use of integral calculus further confusing and complicating the background and theory needed.

Because of this, many students (especially the undergraduate students and others who are encountering it for the first time) begin to think of histogram equalization as something marred with complex mathematical theory. This also makes it difficult for those who do not have a good background in these areas to fully understand histogram equalization and its underlying theory and mathematics in order to have a good conceptual feel and knowledge about the underlying mechanisms when using it. In addition, the final equations or the technique for histogram equalization stated in the different textbooks[1-9] seems somewhat varied and not very clear. The students are therefore confused and avoid trying to gain a good background or understanding of this topic except for trying to somehow get over with it losing motivation. It also makes it difficult porting this subtopic to a lower level class in order to bring in newer learning material as the technical field rapidly advances, until such relatively advanced background is covered in a mathematics (or related) course ahead of it.

Intuitively however, all these complications do not seem natural or needed for teaching or learning something as simple as histogram equalization in the author's belief. Thus, to alleviate the above problems and enable histogram equalization to be even taught to anyone with only a basic mathematics background but interested in image processing, the author developed (on his own, after having looked at what had existed[1-9]) a simpler teaching/ learning framework and background (methodology), a simple and clear theory and the necessary derived equations, a clear process for histogram equalization, and a MATLAB GUIDE® (Graphical User Interface Development Environment) based GUI (Graphical User Interface) tool for visual demonstrations, for teaching and learning histogram equalization and its technique. Note that MATLAB® and GUIDE® are trade marks of MathWorks Inc.

This process of teaching histogram equalization provided the students a good understanding of the concepts and the underlying mechanisms involved with good insights (based on student comments). It made teaching and learning this subtopic rather straightforward, and surprisingly with a higher (if not the highest) level of rigor than was otherwise possible. It also opened the possibility of teaching this topic in a lower level (than even a senior level) course while also providing all the necessary mathematical underpinnings in a clear and simplified manner with no downgrading of the content or rigor as such.

In the rest of this paper, I will provide and discuss the simpler framework and background (methodology), the theory and the necessary derived equations, the histogram equalization process, and the MATLAB GUIDE based GUI tool that was subsequently developed and used by the author for visual demonstrations. All of these were developed (very different and simpler but of greater rigor than the background and theory found in the textbooks) to simplify the teaching and understanding of histogram equalization and its technique. Using this methodology, theory, and visual demonstrations, it was relatively easy to clearly explain and teach histogram equalization to the students encountering for the first time. It was also possible for the students to master the concepts that too at a much higher level of rigor than was otherwise possible. Because of the clarity, there was no need to skim over material or avoid complex homework assignments.

3.0 Teaching Framework/ Methodology, and Theory and Derivations

The teaching flow of histogram equalization was as follows. The details of the teachings as how it was derived and taught by the author are provided after briefly introducing here the overall teaching flow.

In order to lead into histogram equalization, intensity histogram and cumulative intensity histogram were initially defined and taught (Section 3.1), and various MATLAB-based homework exercises were then given.

This was followed by teaching intensity transformation as a point-processing transformation by which image pixel intensities are transformed into a different set of intensities based on input-out mappings. Intensity transformation was taught as being viewed as, directly or indirectly performing a lookup table (LUT) based mapping (Section 3.3) on the input pixel intensities of an image to produce a set of output intensities and thereby producing a modified image. Through this, the concept of lookup tables (LUTs) in image processing was introduced and taught as something aiding the process of intensity transformation. MATLAB-based exercises were then given to intensity transform images using various given LUTs. Image enhancement using contrast-stretching (Section 3.3) was thereafter introduced and taught as one possible image enhancement technique, and as one class of intensity transformation operations using LUTs. The students were then given MATLAB exercises to contrast-enhance various given images using various contrast stretching operations.

Histogram equalization (Section 3.4) was then introduced and taught as another but a special contrast enhancement technique under intensity transformation. It was thus explained that in these cases of intensity transformations, all that was required was to come up with the needed LUT and apply this lookup table to intensity transform the given image to obtain the needed intensity transformed image end result. What histogram equalization actually does and what it achieves were then taught. And based on these, the necessary background and theory was derived and developed (authors own derivation based on the overall objective of histogram equalization). This was developed in a clear and systematic manner using the simplest possible mathematics, and without any loss of rigor towards understanding the concepts and mathematics involved or the insight into what actually was happening. Based on the theory and derivations for histogram equalization, a summary of the theory and process was then provided.

MATLAB based m-functions were created by the author to visually demonstrate the various concepts and theory taught (Section 4.1). These m-functions were subsequently integrated into a MATLAB GUIDE based GUI tool (Section 4.2) developed by the author for quickly and visually demonstrating histogram equalization and its effects on a large variety of images (as later described in this paper). Homework assignments were then given (as later described in this paper) to ensure that the students understood what was taught and to test their learning and understanding of histogram equalization.

In order to make clear what all the above is and how all the above were essentially developed, explained and taught, they are provided in detail in the following subsections.

3.1 Intensity Histogram and Cumulative Intensity Histogram

An intensity histogram **H** of an image is a graph or table of all the possible intensity (gray level) values arranged in ascending order and the number of image pixels having the corresponding intensity values. Thus as a set, an intensity histogram of an image can be written as **H** = { h(k) | k = 0, 1, 2, …, L-1}. Here **H** is the composite histogram and h(k) is the histogram value at intensity "k" (which is the number of image pixels n(k) having the intensity value "k"), and "L" is the total number of different intensity levels that an image can have for the intensity-quantization in use. Thus, h(k) = n(k).

A cumulative intensity-histogram **C** of an image is a graph or table of all the possible intensity values arranged in ascending order and the number of image pixels having an intensity of up to and including the corresponding intensity value. Thus as a set, cumulative intensity histogram of an image can be written as **C** = { c(k) | k = 0, 1, 2, …, L-1}. Here **C** is the composite cumulative histogram and c(k) is the cumulative histogram value at intensity "k" (which is the total number of image pixels having the intensity value "k" or less), and, "L" is the total number of different intensity levels that an image can have for the intensity-quantization in use.

$$\text{Thus, } c(k) = \sum_{i=0}^{k} h(i)$$

3.2 Intensity Transformation

Intensity transformation in the context of digital image processing is a point (pixel) processing technique or operation in which pixel intensities in an image are transformed into (reassigned) different intensities based on some input-output intensity mapping. This is in order to modify the original (given) image to meet a desired end result of "enhancing" the original image in some way for the particular application. It should be noted that there are different sub categories within intensity transformation (point processing/ image enhancement) such as contrast stretching (or modification), histogram equalization, and histogram specification or matching to mention a few.

3.3 The Lookup Table and Intensity Transformation

All the above intensity transformation (point-processing) operations can be viewed as directly or indirectly performing a lookup table (LUT) based mapping on the input pixel intensities of an image to produce a new set of output pixel intensities for the corresponding pixels, and thereby producing a modified image. It should be noted that as the name implies, a lookup table is a table that contains a set of all possible (full range) input intensity values arranged in increasing order **R** = { r_0=0, r_1=1 r_2=2 …, r_k=k …, r_{L-1}=L-1}, and a corresponding set of output (mapped, reassigned) intensity values **S** = {s_0, s_1, s_2, …, s_k, …, s_{L-1}} into which the input intensity values are correspondingly mapped (Figure 1). Thus, each pixel-intensity in the original image is mapped into a new intensity by indexing into this table based on the input intensity and obtaining the corresponding reassigned (output) pixel intensity. This operation when applied to the entire image will result in the image becoming modified as dictated by the lookup table (Figure 2).

LUT	↑ s_k	s_0	s_1	s_2				s_{L-1} = L-1
	r_k	r_0 = 0	r_1 = 1	r_2 = 2				r_{L-1} = L-1

Figure 1. A Lookup Table (LUT)

The above lookup table operation (mapping) can be generally written as **Y**=*LUT*(**X**) where **X** is the input image and **Y** is the output (modified input) image with *LUT* being the table lookup functional operation. If it is otherwise desired to have the lookup table as a parameter (table) to a general intensity transformation operation instead of as a functional operation as above, this operation can be written as **Y**=*Intensity_Transform*(**X**, **LUT**) where **X** is the input image and **Y** is the output (modified input) image with **LUT** being the input-to-output mapping lookup table.

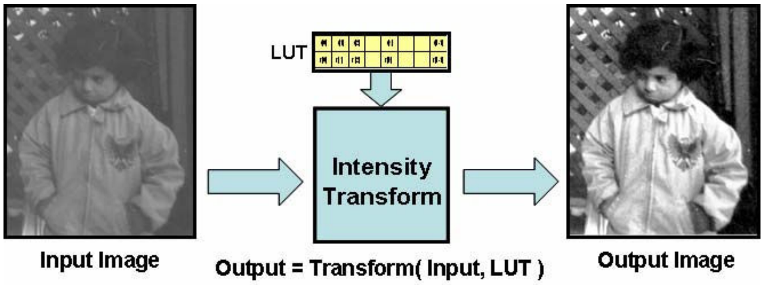


Figure 2. Intensity Transformation

Thus, based on the desired nature of the output intensities, the entire set of lookup table values need to be accordingly computed, and then applied to the input image to obtain the desired output (remapped) image (Figure 2). The question therefore becomes, "how do we proceed to determine/ compute and populate the lookup table mapping values, to thereby obtain a desired end result in an image?"

Along these lines of looking at intensity transformation (point processing) of images, the operation of "contrast stretching (modification)" (Figure 3) can be considered as obtaining the LUT based on a contrast stretching or modification function, and then performing the remapping of the image based on this lookup table. Thus, the need here is to determine/ compute and populate the lookup table based on a desired input-output intensity-mapping graph, function, or characteristic (Figure 3).

Likewise, the operation of "histogram equalization" (defined and discussed in detail later) (Figure 4) can be considered as obtaining the LUT based on "histogram equalized" end result, and then performing the remapping of the image based on this lookup table. More specifically, we can state that the main step in "histogram equalization" is to obtain the LUT that would result in the image histogram of the point processed input image becoming flat across its entire range of possible intensity values. Thus, the need is to obtain an LUT mapping that would correspond to the end result of having the histogram equalized. The question is, "how do we develop the needed mapping?" Once such an LUT is obtained, it is a matter of applying this LUT to the input image (intensity transform) to obtain the histogram equalized version of the input image.

The key in all these cases therefore is to know how we can obtain an LUT (the theory behind it) to meet a desired end result.

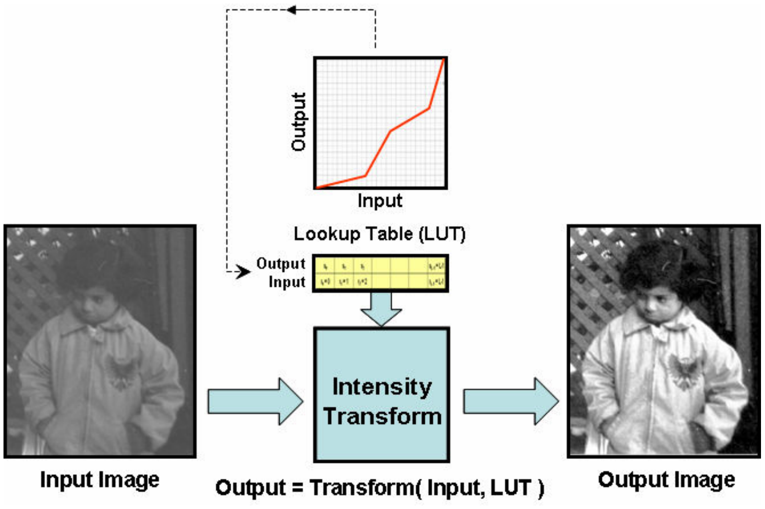


Figure 3. Intensity Transformation: Contrast Stretching

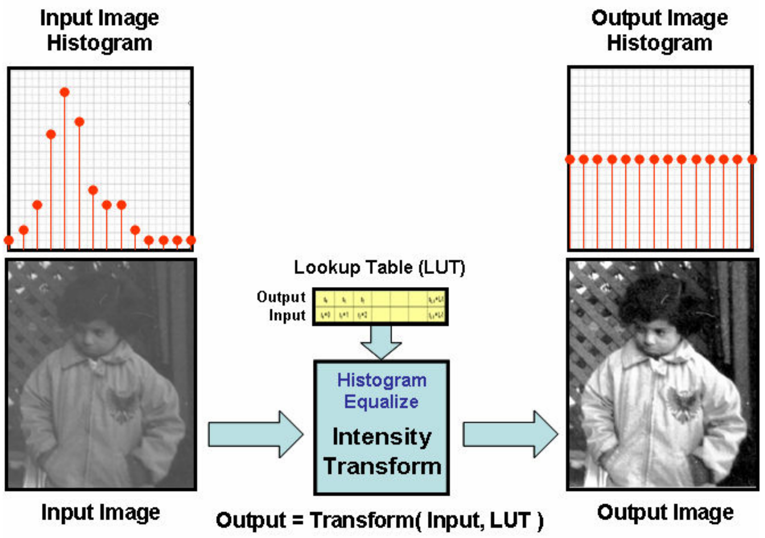


Figure 4. Intensity Transformation: Histogram Equalization

3.4 Histogram Equalization

We will look at histogram equalization by viewing it along the lines of the above teaching and development framework and derive and develop the necessary theory and steps to perform it.

The students found the above methodology to be a clean and direct technique of introducing and teaching this otherwise difficult subtopic. The theory and derivations (as discussed in this paper) too seemed very clear when either viewed in isolation as developed and taught by the author (but very different to other sources[1-9]) or when integrated and viewed within the teaching framework provided by the author (as discussed above). The MATLAB GUIDE based GUI tool subsequently developed and used by the author for visually demonstrating and teaching (discussed in this paper) too was of considerable help with respect to student-learning and understanding (conceptually or otherwise).

To begin with, let us first take a look at what histogram equalization is and then proceed to develop the theory/ equations in a simple (minimally complex) manner. This will enable explaining and teaching this topic to a much lower level class than otherwise possible, and also to students with not-so-good mathematics background at that time. Also, it is the author's belief that nothing should be made more complex than it ought to while maintaining the required rigor.

Histogram equalization can be said to be an important and automatic point processing technique for contrast enhancing images. It is called an automatic procedure because it does not require any user control parameters for its application to an image (unlike in the case with most other contrast enhancement techniques, if not all other) – i.e., there is one and only one "histogram equalized" result for a given image.

It is a sophisticated technique that can be used to improve the dynamic range and contrast of an image by non-linearly reassigning pixel intensity values in the image such that the resulting image has a uniform (or in a practical sense, close to uniform) distribution of the pixel intensity values across its entire range of values (i.e., a flat intensity histogram) (Figure 5). This is done while the intensity reassignments are subject to constraints that preserve image integrity> The image integrity is preserved by not allowing the reassignments to affect the intensity information structure (intensity ranking) with respect to the pixel geometry within the image under consideration. These intensity reassignments are performed by eventually employing a single-valued, monotonically increasing (for image integrity) nonlinear intensity mapping. This intensity mapping is derived from the image itself (without any user-control parameter) to meet the above image integrity and histogram equalization objectives.

The whole idea (or basis) behind histogram equalization is the belief (which may be right at times, and may be wrong at times) that the pixel intensities should be evenly distributed across the entire possible intensity range for better dynamic range of the intensities and contrast for a better image. Thus, it can be said that the aim of histogram equalization is to obtain a modified image that has a flat histogram, without affecting the intensity information structure (image integrity) of the original image.

Based on the above defined aim of histogram equalization, an expression and technique that could be used for achieving the objective was derived by the author in a simplified manner (without any loss of rigor). We will now take a look at this derivation developed by the author. In this connection, let us develop an expression that will let us know what any intensity level in the original image (any pixel having that intensity) should be mapped to (without affecting image integrity) in order for the modified image to have an equalized histogram.

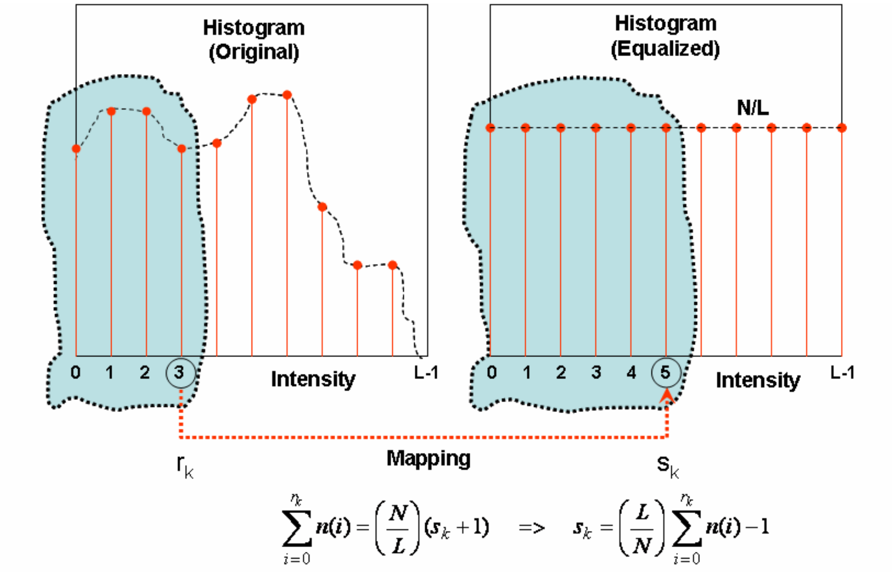
Let us start with the smallest possible intensity level in any image which is zero, and proceed through the highest possible intensity level (L-1). We will assume that the image is quantized (digitized) to one of “L” possible intensity levels. As an aside, if “B” bits are used for storing a pixel value, we have $L=2^B$.

Since we want the histogram of the modified image to be equalized, the number of pixels having any mapped intensity level (0,1,2,…,L-1) in the modified image should be the same, and equal to “N/L”. Here, “N” is the total number of pixels in the image and “L” is the total number of possible intensity levels. We will use the notation that **R** represents histogram or cumulative histogram (or related) properties of the original (given) image and **S** represents the histogram or cumulative histogram (or related) properties of the histogram equalized image. Further, let r_k (r_k=k; k=0,1,2,…,L-1) be an original intensity “k”, and s_k be the mapped intensity corresponding to this original intensity “k” (i.e., r_k) for histogram equalization.

Now, up to any intensity r_k ((r_k=k; k=0,1,2, …, L-1) in the original image, the total number of image pixels with intensities up to r_k should be the same as the total number of image pixels up to s_k in the mapped intensities for image integrity (Figure 5). As mentioned, for histogram equalization, the number of pixels at any mapped image intensity s_k (k=0,1,2,…,L-1) is the same, and equal to “N/L”. Thus we can write (in terms of the number of image pixels at each intensity level),

$$\sum_{i=0}^{r_k} n_R(i) = \left(\frac{N}{L}\right)(s_k+1) \quad : \quad k=0,1,2,3,...,L-1;\ r_k = k$$

(Note: “k” begins at zero, hence the term s_k+1 in the above equation).



$$\sum_{i=0}^{r_k} n(i) = \left(\frac{N}{L}\right)(s_k+1) \quad => \quad s_k = \left(\frac{L}{N}\right)\sum_{i=0}^{r_k} n(i) - 1$$

Figure 5. Histogram Equalization: Histogram Based

Another way for directly deriving the above equation is by noting that intensity mapping from r_k to s_k should be such that the cumulative histogram at r_k should be the same as that at s_k (for any “k” zero through L-1) for histogram equalization (Figure 6).

In the above, “N” is the total number of pixels in the given image, “L” is the total number of possible intensity (gray) levels (possible intensity levels are 0, 1, 2, …, L-1), n(i) is the number of image pixels in the (original) image with intensity “i” (here, “i” is a “running index” in the above equation), r_k is the original intensity “k”, and s_k is the mapped intensity corresponding to original intensity “k” (i.e., corresponding to r_k). Note that the notation “r” is used to imply the original (given) image and “s” to imply the modified image in the above.

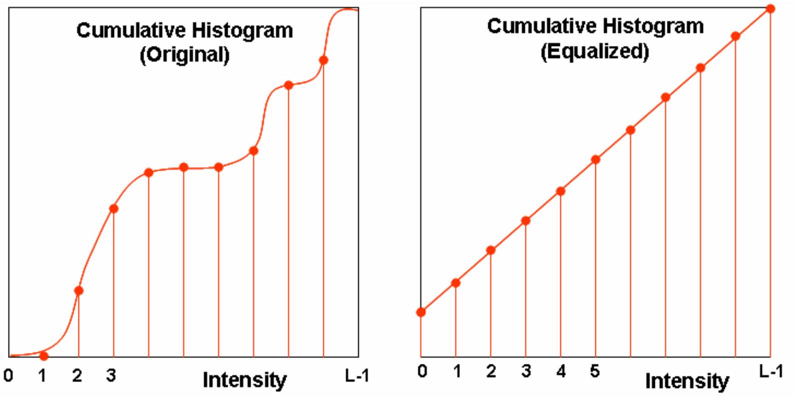


Figure 6. Histogram Equalization: Cumulative-Histogram Based

Thus for example, n(128) stands for the number of image pixels with an intensity of 128 (in the original image), r_{128} stands for intensity 128 in the original image (i.e., $r_{128} = 128$), and s_{128} stands for the mapped intensity of the original intensity 128 (which could be for example, intensity 147 in which case $s_{128} = 147$). This would mean that all pixels in the original image with a pixel intensity of 128 are mapped to an intensity of 147 in the modified image.

In the above equation, since n(i) is the number of image pixels having an intensity level of “i”, we have n(i) = h(i) by the definition of a histogram. Therefore, the above equation can be rewritten in terms of the histogram h(i) as,

$$\sum_{i=0}^{r_k} h_R(i) = \left(\frac{N}{L}\right)(s_k+1) \quad : \quad k=0,1,2,3,...,L\text{-}1;\ r_k = k$$

It should next be noted that intensity levels are discrete in digital images (they are integers in the range 0 through “L-1”). Further, in order to maintain “informational integrity” of the image, all image pixels with the same intensity in the original image cannot be assigned to different intensities but be mapped to the same new value en masse. Thus, remapping image intensities in an image for perfect histogram equalization will not be possible for digital images.

Thus, in a practical sense, what we would like to aim for is not a perfect equalization of the mapped intensities, but something that would be close. Therefore, what we want is to map intensity level r_k (r_k=k; k=0,1,2,…,L-1) to an intensity level s_k such that up to s_k (k=0,1,2,…,L-1) in the modified image, the error (i.e., difference) (Δ_k) between the histogram of the resulting mapped (modified) image and that of the equalized histogram is minimized. Of course in the ideal case (perfect equalization), Δ_k would be zero. Therefore, it can be said that we want to map a pixel intensity r_k (r_k=k; k = 0,1,2,…,L-1) in the original (given) image to an intensity s_k (to be determined) such that Δ_k in the equation below is minimized.

$$\Delta_k = \left| \left(\frac{N}{L}\right)(s_k+1) - \sum_{i=0}^{r_k} h_R(i) \right| \quad : \quad k=0,1,2,3,...,L\text{-}1;\; r_k = k$$

Here, |…| represents the absolute value of what is within. By factoring out N/L and rewriting the above equation, the above minimization can be easily modified to a minimization of the following:

$$\Delta'_k = \left| s_k - \left(\left(\frac{L}{N}\right) \sum_{i=0}^{r_k} h_R(i) - 1 \right) \right| \quad : \quad k=0,1,2,3,...,L\text{-}1;\; r_k = k$$

For any given r_k, it is always possible to determine s_k such that the above minimization is zero (ideal minimization). This could be done by solving for s_k after making Δ'_k zero. It would yield s_k as,

$$s_k = \left(\frac{L}{N}\right) \sum_{i=0}^{r_k} h_R(i) - 1 \quad : \quad k=0,1,2,3,...,L\text{-}1;\; r_k = k$$

However in the above, s_k (the mapped pixel intensity) is not guaranteed to be an integer (and, in general it will not be an integer). But for digital images, pixel intensities are constraint to be integers (0,1,2,…,L-1). Thus, it is self-evident from the minimization expression that for the best possible minimization with this constraint, the mapped pixel intensity s_k should be chosen as,

$$s_k = \text{round}\left(\left(\frac{L}{N}\right) \sum_{i=0}^{r_k} h_R(i) - 1 \right) \quad : \quad k=0,1,2,3,...,L\text{-}1;\; r_k = k$$

From the above equation it is evident that for intensity levels in the original image from zero up to the intensity level where the total number of image pixels with those intensities is less than "N/L", the expression would result in negatively mapped intensities (s_k) (or to be precise, this is up to the pixel intensity where the total number of pixels is less than "N/2L" when rounding is in use, and s_k would become -1 in this situation). However, for image intensities, s_k is constraint to be non-negative (intensities cannot be negative). Thus, s_k in the r_k–>s_k mapping for histogram equalization in terms of the intensity histogram of the original (given) image should be such that,

$$\left. \begin{array}{l} s'_k = \left(\frac{L}{N}\right) \displaystyle\sum_{i=0}^{r_k} h(i) - 1 \quad : \quad k=0,1,2,3,...,L\text{-}1;\; r_k = k \\ s_k = \begin{cases} 0 & : \; s'_k < 0 \\ \text{round}(s'_k) & : \; s'_k\,{}^{3}0 \end{cases} \quad : \quad k=0,1,2,3,...,L\text{-}1 \end{array} \right\}$$

By the definition of cumulative histogram, the summation in the above equation can be written as the cumulative histogram at intensity "k" (i.e., C_k). Thus, in terms of the cumulative intensity histogram of the original (given) image, the above can be written as,

$$\left.\begin{array}{l} s'_k = \left(\dfrac{L}{N}\right) C_R(k) - 1 \quad : \quad k=0,1,2,3,...,L-1;\ r_k = k \\ s_k = \begin{cases} 0 & : \ s'_k < 0 \\ \text{round}(s'_k) & : \ s'_k{}^3 0 \end{cases} \quad : \quad k=0,1,2,3,...,L-1 \end{array}\right\}$$

As a representational simplification, we can write the above mapping (either using the histogram or the cumulative histogram based expression) for histogram equalization as given below, but with the understanding that s_k is adjusted to zero if the expression on the right hand side evaluates to less than zero, and it is the rounded otherwise:

$$s_k = \left(\frac{L}{N}\right) \sum_{i=0}^{r_k} h_R(i) - 1 : \ k=0,1,2,3,...,L-1;\ r_k = k \quad \text{(w.r.t. Histogram)}$$

$$s_k = \left(\frac{L}{N}\right) C_R(k) - 1 \quad : \ k=0,1,2,3,...,L-1;\ r_k = k \quad \text{(w.r.t. Cumulative Histogram)}$$

The above equation(s) defines the mapping r_k–>s_k from an original intensity (r_k) (r_k=k; k=0,1,2, …, L-1) (i.e., **R**) to a mapped intensity (s_k) (i.e., **S**) for (best) histogram equalization. As we had noted, this will not result in a perfectly equalized histogram for the so modified image, but would be as close to an equalized histogram as possible that a digital image would permit while retaining image integrity (pixel ranking order).

LUT	↑ s_k	s_0	s_1	s_2				s_{L-1} = L-1
	r_k	r_0 = 0	r_1 = 1	r_2 = 2				r_{L-1} = L-1

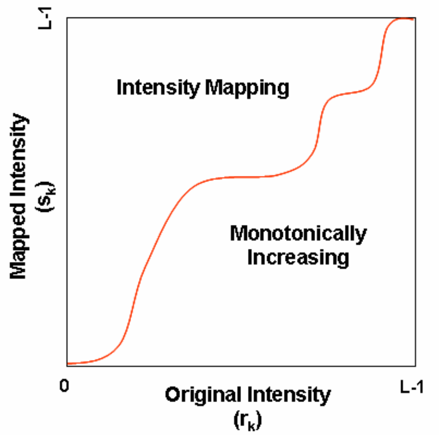


Figure 7. Histogram Equalization: Lookup Table and Input-Output Mapping

Also, it can be easily shown that for k = L-1, we get $s_k = r_k$; that is, the L-1 (max) intensity level will always map to L-1 (the max intensity level). Further, if $r_{k2} > r_{k1}$, then we get $s_{k2} \geq s_{k1}$ as it ought to be for any meaningful "image structure preserving" intensity mapping of an image. Thus as required and expected, the histogram equalization mapping is single-valued and monotonically increasing; it is also non-linear.

Based on the above, a lookup table (**LUT**$_{RS}$) for histogram equalization can be created. It could thereafter be used for mapping each pixel intensity value in the original image to a new intensity value (Figure 7, and Figure 4) for histogram equalization of the original image.

3.5 Histogram Equalization: Summary of Steps

The entire technique (steps) for histogram equalization (as discussed above) could be summarized and stated as follows:

Step 1: Compute the intensity histogram and/or cumulative intensity histogram of the original image. This will give us the number of pixels at each intensity level, or the cumulative number of pixels having an intensity up to or below any given intensity level in the original image. Also, determine the total number of image pixels in the given (original) image.

Step 2: Compute the (r_k–>s_k) lookup table (LUT$_{RS}$) that would histogram-equalize the original image. This can be done using either the histogram based equalization equation or the cumulative histogram based equalization equation given below (the histogram equalization mapping transformation – developed earlier), with the understanding that s_k is adjusted to zero if the expression on the right hand side evaluates to less than zero, and it is the rounded otherwise.

$$s_k = \left(\frac{L}{N}\right)\sum_{i=0}^{r_k} h_R(i) - 1 : \; k=0,1,2,3,...,L-1; \; r_k = k \quad \text{(w.r.t. Histogram)}$$

$$s_k = \left(\frac{L}{N}\right) C_R(k) - 1 \quad : \; k=0,1,2,3,...,L-1; \; r_k = k \quad \text{(w.r.t. Cumulative Histogram)}$$

Step 3: Using the above mapping transformation (**LUT**$_{RS}$), transform the original image to a histogram equalized image using the intensity transformation **S**=*Intensity_Transform*(**R**, **LUT**$_{RS}$) where **R** is the input image and **S** is the output (modified input) image with **LUT** being the input-to-output mapping lookup table

Step 4 (Optional): Display (i) the original image, its histogram, and its cumulative histogram, (ii) the modified image, its histogram, and its cumulative histogram, and (iii) the mapping function (as a graph).

4.0 A MATLAB GUIDE based GUI Tool for Teaching and Demonstration

4.1 The MATLAB m-Function Files for Demonstrations

Having developed the background and theory to be used for teaching histogram equalization, it was best to visually demonstrate and teach histogram equalization through demonstrating the various steps involved, and the effects of histogram equalization on a variety of images – all of these to supplement and complement the lectures and to aid understanding.

In order to accomplish this, various simple MATLAB m-function files were first modularly developed and used by the author for initial demonstrations. These m-functions were developed for the following purposes: (i) reading and displaying any given image, (ii) computing and displaying the given image's intensity histogram, (iii) computing and displaying the given image's cumulative intensity histogram, (iv) histogram equalizing the given image to produce a histogram equalized image, (v) displaying the histogram equalized image, (vi) computing and displaying the histogram equalized image's intensity histogram, (vii) computing and displaying the histogram equalized image's cumulative intensity histogram, and (viii) computing and displaying the mapping function that resulted by the histogram equalization function which brought about the histogram equalization of the given image. It should be noted that some of the above mentioned functions are actually combined into a single function with their differences incorporated as parameters to the respective functions (but is described as above for clarity). Special allowances were incorporated into these m-functions to suitably handle grayscale as well as color images (after having properly interpreted the meaning of histogram equalization under these circumstances).

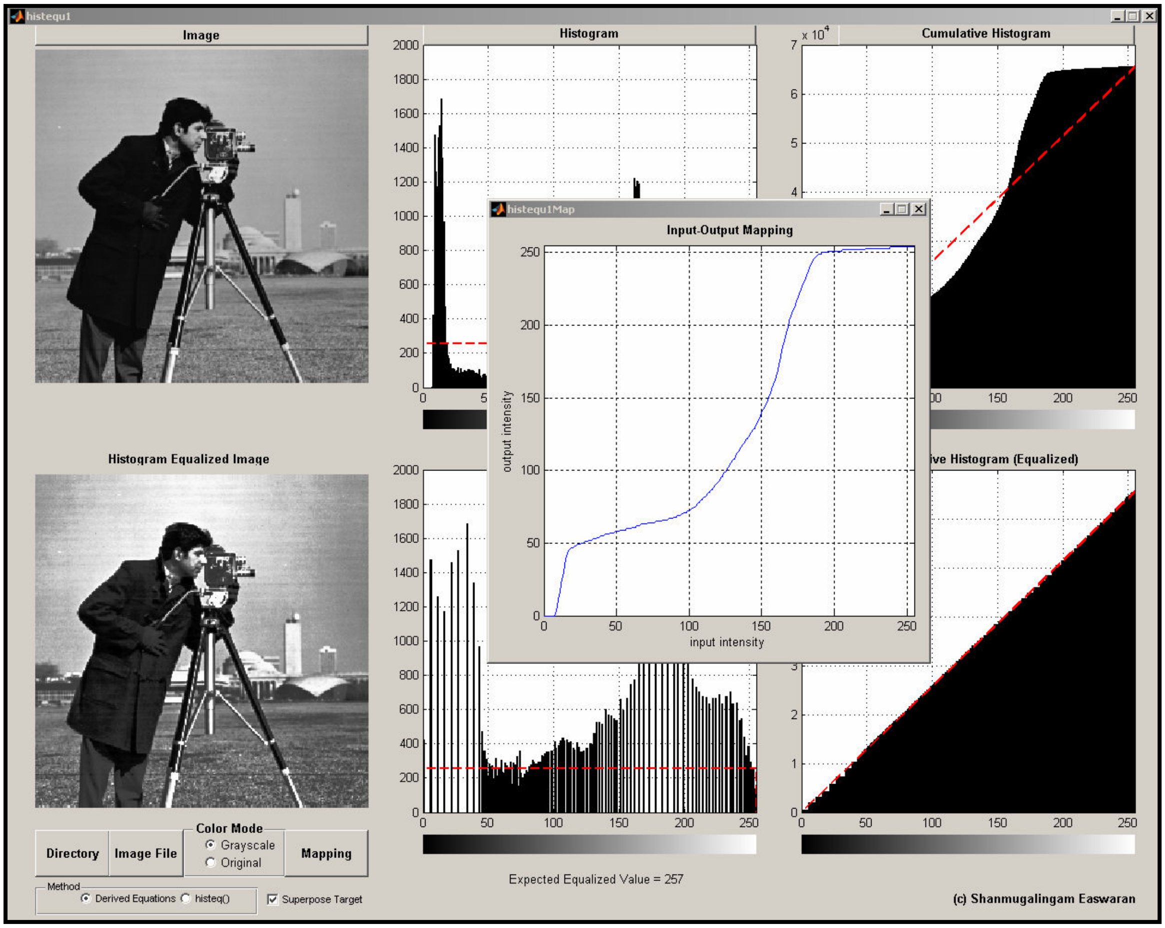
4.2 The MATLAB GUIDE based GUI for Demonstrations

A MATLAB GUIDE based GUI tool was subsequently developed by the author by quickly integrating into a common GUI these m-functions that were first developed and used for teaching purposes. This was to make the demonstrations quick and convenient. The user-interface of this GUI tool is shown in Figure 8. The interface and organization of this tool was based on making teaching, learning, and studying histogram equalization as painless as possible.

As can be seen (Figure 8), the main GUI window has a pushbutton to select the current directory (bottom left), and another to select an image file from this directory (bottom left). Upon clicking any of these two buttons, the respective popup window (not shown) appears to enable selecting the directory and the image file from that directory. Upon selecting an image file (with a file type supported by MATLAB), the following are immediately computed and displayed in their respective windows within this main GUI window: (1) (a) the original image, (b) its histogram, and (c) its cumulative histogram; and (2) (a) the histogram equalized image, (b) its histogram, and (c) its cumulative histogram.

In the histogram windows (of both the original and equalized), a horizontal line is plotted showing the expected (ideal) equalized value (N/L) for the image. Its numerical value too is given (bottom center). Similarly, in the cumulative histogram windows (both original and equalized), a linear graph is plotted showing the expected (ideal) cumulative equalized-histogram values. These are provided for comparison of expected and actual values of the histograms and cumulative histograms.

Clicking the "Image" pushbutton (top row, left of main GUI) will result in displaying the original image and the histogram equalized image in a separate full screen sub-window (for enlarged view of the images) (Figure 9). Similarly, clicking the "Histogram" button (top row, middle of main GUI) will result in displaying the histogram of the original image and that of the histogram equalized image in a separate full screen sub-window (for enlarged/ horizontally stretched view of the histograms) (Figure 10). Clicking the "Cumulative Histogram" button (top row, right of main GUI) will result in displaying the cumulative histogram of the original image and that of the histogram equalized image in a separate full screen sub-window (for enlarged/ horizontally stretched view of the cumulative histograms) (Figure 11). And, clicking the "Mapping" pushbutton (bottom left region of the main GUI) will result in displaying in a popup sub-window (shown) the computed input-output mapping (LUT) for histogram equalization (Figure 8).


Color Mode: ● Grayscale ○ Original

Method: ● Derived Equations ○ histeq()

☑ Superpose Target

Expected Equalized Value = 257

Figure 8: The Main GUI Window and Mapping Sub-Window of Tool

The GUI tool also has a "Color Mode" option box (two radio buttons, bottom left) to decide how we want to handle (and display) any color image (as opposed to grayscale image); that is, for deciding if we want to proceed as if any color image was a grayscale image (by internally converting it to a grayscale image and proceeding) (this is the default), or if we want to treat any color image differently as explained below:

Option 1 (default): If this option is selected, an image is converted to a grayscale image if it is not already a grayscale image by obtaining the "V" component (the "brightness equivalent") of the RGB (Red, Green, Blue) to HSV (Hue, Saturation, Value) converted image, and then proceeding to perform histogram equalization on this V-component (in the earlier described manner). The histogram and cumulative histogram are all based on the "V" component. With this option, the images displayed are the equivalent grayscale images regardless of their original chromatic condition.

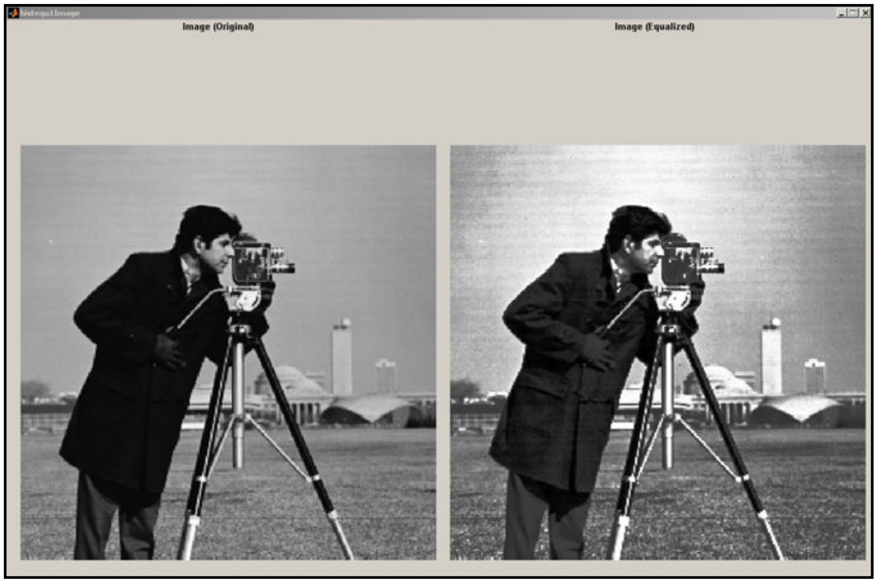


Figure 9: The Image Sub-GUI (popup) Window

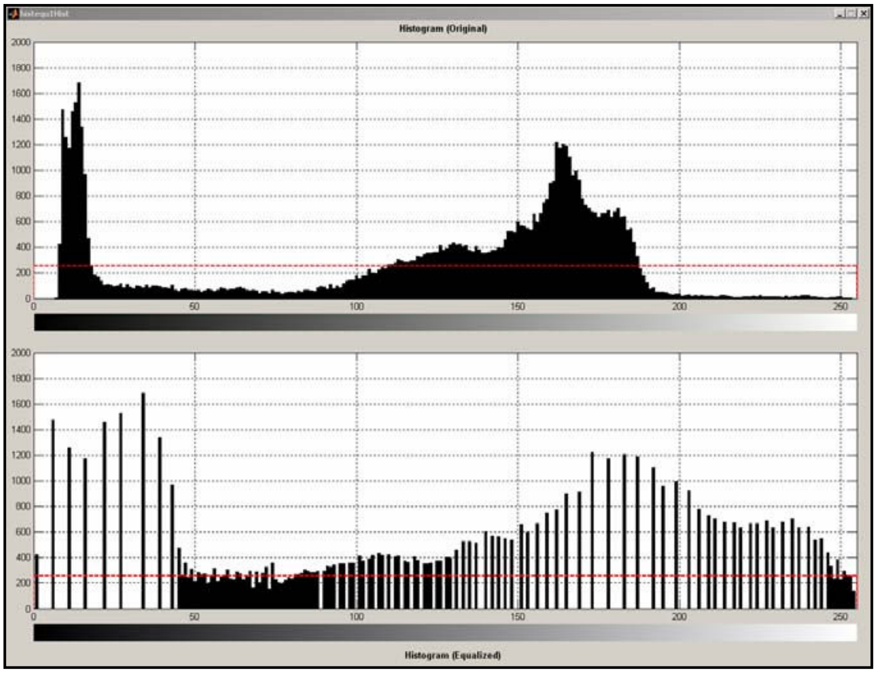


Figure 10: The Histogram Sub-GUI (popup) Window

<u>Option 2</u>: If this option is selected, in case of a color image the V-component of the RGB to HSV converted image is obtained, a histogram equalization (as above) is performed on this V-component (and only on the V-component), and then using this histogram equalized V-component and the original H and S-components the histogram equalized color image is

obtained via HSV to RGB conversion. The images displayed in this case are the original image (in color if the original was a color image) and the histogram-equalized image (in color if the original was a color image). That is, the images are displayed in their native chromatic form.

The GUI tool also has a "Matching Method" option box (two radio buttons, bottom left) that enables one to choose which method is to be employed in the histogram equalization process for comparing the results of histogram equalization using Option 1: "Derived Equations (by Author)", and Option 2: "MATLAB histeq() Function".

4.3 The Usage of the Demonstration Tools

Using this tool, in addition to supplementing and complementing the lectures as discussed earlier and demonstrating histogram equalization (on a variety of images), it was also possible to quickly demonstrate on a large variety of images the actual amount of equalization the histogram equalization technique would produce.

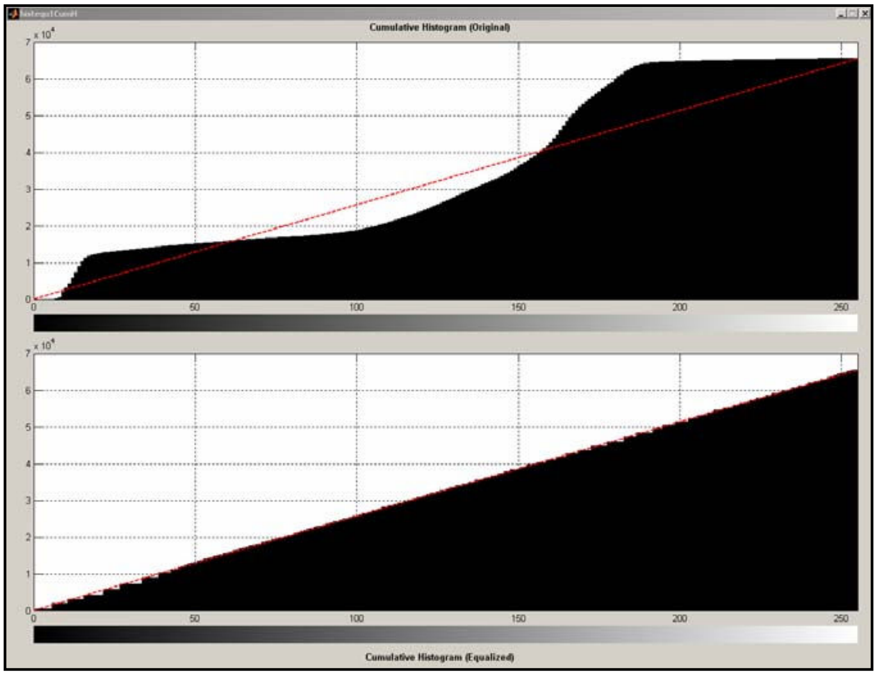

Cumulative Histogram (Original)
x 10^4
7
6
5
4
3
2
1
0
0
50
100
150
200
250
x 10^4
7
6
5
4
3
2
1
0
0
50
100
150
200
250
Cumulative Histogram (Equalized)

Figure 11: The Cumulative Histogram Sub-GUI (popup) Window

It helped demonstrate and explain why the equalized histogram may not look perfect due to the discrete nature of intensities (intensity quantization) in a digital image and the need for reassigning intensities en-masse for a any given intensity value to maintain image integrity. In order to further explain this, it was discussed that the cumulative histogram of an image would give a better indication of the equalization provided in a practical sense. That was why the cumulative histogram too was being displayed in all cases. It was mentioned to the students that the closer the cumulative histogram (C) is to a straight line C = (N/L).(intensity) + (N/L), the better is the image histogram ideally equalized. This is why the expected equalization and cumulative equalization values are also plotted in the respective histogram and cumulative histogram windows. These helped with understanding and appreciating histogram equalization in the real sense.

It should be noted that the histogram equalization results obtained using the equation derived and discussed by the author also matched very closely (hardly different if at all) to those obtained using the MATLAB function histeq() for histogram equalization.

5.0 The Effectiveness of the Teaching and GUI Tool

In order to evaluate the effectiveness of the provided teaching framework and methodology, theory and derived equations, the histogram-equalization process, and MATLAB GUIDE based GUI tool demonstrations, the following was done by the author. This was done for the spring 2006 semester with six computer-engineering as well as computer-science undergraduate students in the digital image processing class, and for the spring 2005 semester with nine computer-engineering undergraduate students in the digital signal processing class (which had an image processing a sub-area within the course).

In these courses, the students were initially taught histogram equalization in a general way (as described in the standard and other textbooks1-9 and not as described in this paper). It should be noted that it was the considerable difficulty I was having in teaching this subtopic to students as described in the standard and other textbooks (and their poor performance in corresponding homework assignments) that made me develop and use what was described in this paper.

It should also be noted that prior to teaching histogram equalization, the students had been taught intensity histograms and cumulative intensity histograms as applied to digital images, contrast enhancements through point processing using contrast stretching, and image intensity transformations using lookup tables (LUTs). They have also had MATLAB homework exercises based on these.

The students were also asked to refer to the course textbook and/ or other textbooks in image processing (most of which were available in the university library) to histogram equalize a given grayscale image. They were given two homework assignments to work on and submit. As always, they were allowed to discuss with the instructor finer points they had difficulties with. They were also asked to write down difficulties they have had while working through the homework assignments. The homework problems were essentially as described below:

(1) The students were given a simple artificial "image" (of size 8x8, and with eight quantization levels) and was asked to manually perform histogram equalization. In a different situation, they were told the number of pixels that had a certain intensity level for each of the possible eight intensity levels, i.e. its histogram, instead of giving them image intensity values). The idea behind this manual computation assignment was the hope of providing students with insights as they worked through the problem, and also to determine if they had understood the concepts involved. This was also to enable the students to subsequently write a MATLAB program to perform histogram equalization of a given real image. They were asked to show the histogram and the cumulative histogram of the original image, the histogram and the cumulative histogram

of the “histogram equalized” image, and the intensity mapping needed to go from the original image to the histogram equalized image.

(2) The students were asked to perform histogram equalization on a MATLAB provided image (image ‘pout.tif’, size 240 by 291, gray levels 256). They were asked to show the original image, the histogram and the cumulative histogram of the original image; the histogram equalized image, the histogram and the cumulative histogram of the “histogram equalized” image; and the intensity mapping to map the intensities from the original image to the histogram equalized image intensities. They were further asked to compare their results to that obtained using the MATLAB histeq() function.

After giving the students a week to work on these assignments, because of the difficulty all the students were having, they were taught histogram equalization as described in this paper. They were then given the same assignments to work on again and submit.

Before teaching histogram equalization as described in this paper, the students were frustrated that they could hardly understand the background/ theory and technique described in the textbooks or in the internet, and thus they were hardly able to proceed with the problem itself (none of the students were able to make progress with the problems in any meaningful way). They complained that the available materials[1-9] (textbooks, reference books, internet searches) were mathematically too advanced and complex to understand or was not clear, or that they do not have a clear background on probability/ probability distributions or integral calculus (all of the texts use these advanced mathematical concepts to discuss histogram equalization) to properly understand the process of histogram equalization, or that the procedure to follow was not clear or clearly explained in the texts.

However, after teaching histogram equalization in the manner described in this paper, they all had a clear understanding of histogram equalization. The student solutions, comments, survey results indicate that they understood histogram equalization quite well. They all said (at an individual interview with each student discussing their work) that they were able to completely understand the concepts, theory and technique behind histogram equalization.

In order to evaluate the effectiveness of the teaching of histogram equalization as stated above, the homework assignment scores of the students (on this subtopic) were scaled to 10 points and the statistics were computed for the 12 (of the total of 15) students who had attended all the lectures when this subtopic was taught, with and without the methodology described. The statistics for the pre and post-tests were follows: For the pre-test, the students were having so much difficulty in progressing with the assignments that their assignments were not graded as such. For the post-test, the average score was 8.85 with a standard deviation of 1.30.

The students were also given an evaluation form after they had been taught using the methodology described. Among other questions, they were asked how strongly they agreed (from “very strongly” = 5, to “strongly disagree” = 1) that their learning and understanding of this subtopic had considerably improved based on the methodology, developments and explanations, and demonstrations shown using the MATLAB-tool developed. Only the twelve students who had attended all the lectures when this topic was taught with and without the methodology described, were asked to provide their feedback: 10 strongly agreed and 2 agreed. Their comments too indicated similarly.

Though the results indicate significant improvements in student learning when taught using the methodology described in this paper, more student performance data and more careful testing and comparison strategies that are also practical may be needed for obtaining mathematically meaningful statistical measures and to draw statistically strong conclusions (if needed).

6.0 Summary and Conclusion

In this paper, the following that were derived/ developed/ implemented and used by the author for teaching histogram equalization in digital image processing were described: (i) a simpler teaching-learning framework and background (methodology), (ii) a simple and clear theory and the necessary derived equations, (iii) a clear process for histogram equalization, and (iv) a MATLAB GUIDE based GUI tool/ m-function files for visual demonstrations. Student performance before and after teaching this subtopic as described in this paper indicated that student learning and understanding was greatly simplified and improved. The teaching too was made considerably simpler at a greater level of rigor.

Bibliography

1. Gonzalez, Woods, "Digital Image Processing", Second Edition, Prentice Hall (2002)
2. Gonzalez, Woods, Eddins, "Digital Image Processing with Matlab", Prentice Hall (2006)
3. Castleman, "Digital Image Processing", Prentice Hall (1996)
4. Bose, "Digital Signal and Image Processing", John Wiley & Sons (November 2003)
5. McAndrew, "Introduction to Digital Image Processing with Matlab", Thomson Course Technology (2004)
6. Hall, "Computer Image Processing and Recognition", Academic Press (1979)
7. Petrou, Bosdogianni, "Image Processing The Fundamentals", Fourth Edition, John Wiley & Sons (April 2004)
8. Acharya, Ray, "Image Processing Principles and Applications", Wiley-Interscience (2005)
9. Nixon, Aguado, "Feature Extraction & Image Processing", Newnes (2002)
10. "MATLAB GUIDE User Manual", MathWorks Inc.