Simulation and Visualization of an Innovative Rotary Displacer Stirling Engine Operation

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ABSTRACT

Stirling engines (SEs) interact with a heat source and a heat sink, and operate by cyclic expansion and compression of a working fluid, generally a noble gas. SEs convert thermal energy into mechanical work, and can drive alternators to produce electricity. When SEs operate in a reverse manner, they can also function as coolers, to provide refrigeration at cryogenic temperatures. SEs are receiving renewed interest lately due to their unique advantages, in particular the ability to operate with any form of heat source (including external combustion, flue gases, biomass, solar, geothermal energy) even at low temperatures (<250°C), that provides SEs great flexibility for numerous uses. Several aspects of traditional SE configurations, e.g., complexity of design, high cost, and relatively low power-to-size/weight ratios, limited their applications to date.

This study focuses on a recently patented rotary displacer SE that features key benefits over traditional SEs, including reduced cycle energy needs, reduced friction, and fewer moving parts, and offers the potential to overcome current challenges. The study aims to develop simulation and visualization tools to (a) explain the operation of the subject SE in a clear, intuitive way, (b) enhance the curriculum on fundamental thermal-fluids sciences, and (c) increase the awareness of engineering students and general public on an emerging energy conversion technology. The simulation effort uses MATLAB® software for an idealized, thermodynamics-based analysis at various conditions representing potential applications, while the visualization effort uses PTC Creo® software for solid modeling and animation of the key engine components.

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1. INTRODUCTION

Stirling cycle machines were originally invented by Robert Stirling in 1816, and since then they have been subjected to many investigations and developments. Lately, Stirling engines (SEs) are receiving renewed interest due to their unique advantages, in particular the ability to operate with any form of heat source (including external combustion, flue gases, biomass, solar, geothermal energy) even at low temperatures (<250°C), that provides SEs great flexibility for numerous uses. However, some other aspects of the traditional SE configurations, including complexity of design, high cost, and relatively low power-to-size/weight ratios, have prevented their wide spread adoption to date.

Three related traditional SE configurations are commonly found in contemporary applications: the Alpha, Beta, and Gamma. All use a conventional piston, crankshaft, and cylinder. In their simplest form, each has four reciprocating parts and one rotary part per power cylinder. Actual commercial engines are more complex. Alpha engines use two power pistons, each with a separate cylinder and connecting rod. One piston and cylinder represent hot workspace, the other, cold. Connecting rods join at a common journal on a flywheel. Hot and cold workspaces are physically separated. Alphas require a close tolerance fit between piston and cylinder. The hot workspace piston and cylinder have persistent seal reliability issues. Beta engines have eliminated hot seal issues in Alphas. Betas use a power piston with a connecting rod, similar to the "cold" power piston of the Alpha, but the "hot" power piston is substituted by a displacer. The power piston and the displacer share a common cylinder and flywheel/crankshaft. Unlike a piston, the displacer fits loosely. Its function is to shuttle working fluid within the hot and cold workspaces. This insulates the power piston from high temperatures. The Gamma is similar to the Beta except the Gamma power piston does not share a common cylinder with the displacer; it employs two distinct cylinders. Free piston SEs are reciprocating but conceptually different; they use a flexible membrane seal which is unreliable in hot environments.

In 2013, Philip R. Foster introduced a new SE configuration which features a rotary displacer, instead of a common reciprocating displacer [1]. This innovative SE has three moving parts per power cylinder, two reciprocating and one rotary. The reciprocating parts are a power piston and connecting rod which are conventional. The rotary part is the displacer which also functions as a crankshaft/flywheel and incorporates a non-contact valve mechanism. The rotary displacer SE configuration has several major advantages:

- The rotary displacer never changes direction, and eliminates back and forth movement and the acceleration-deceleration related momentum losses.
- Rotary displacer SE has a simpler design with fewer components; the rotary displacer's physical mass is utilized as the flywheel/crankshaft.
- The rotary displacer and the interior of the displacer housing offer increased heat transfer area to the working fluid.
- The rotary displacer makes no physical contact with the interior of the displacer housing; no hot or sliding seals contact the displacer.
- All loads on the displacer (radial and thrust) are accommodated by ball and/or rolling bearings.

This study aims to develop simulation and visualization tools to (a) explain the operation of the subject rotary displacer SE in a clear, intuitive way, (b) enhance the curriculum on fundamental thermal-fluids sciences, and (c) increase the awareness of engineering students and general public on an emerging energy conversion technology. The simulation effort uses MATLAB® software for an idealized, thermodynamics-based analysis at various conditions representing potential applications, while the visualization effort uses PTC Creo® software for solid modeling and animation of the key engine components.

2. SIMULATION

A preliminary analytical analysis is done to gain insights on the operation parameters of the rotary displacer SE. The analysis is then used to predict the performance of several potential SE applications.

2.1 An Idealized Preliminary Analysis

The preliminary analysis adopts some of the assumptions from the well-known Schmidt analysis, and follows a similar approach to implement it for the innovative rotary displacer configuration. The Schmidt analysis was presented by Gustav Schmidt in 1871 [2] and describes the Stirling cycle's harmonic movement. Although Schmidt analysis is an ideal approach and cannot capture many of the realistic operation conditions, it still provides benchmark conditions and useful design guidelines as a first step towards optimization. This analysis [2] considers some assumptions:

- There is no heat loss in the system.
- There is no loss in the working fluid.
- The working fluid follows the ideal gas rule.
- The expansion and compression processes occur isothermally.
- There is no pressure loss in the cycle since the working fluid viscosity and friction are neglected.
- Pressure is the same across the entire enclosure.

Figure 1 shows the schematic design of the rotary displacer SE where the working fluid always remains in a closed enclosure. Therefore the conversation of mass equation can be written as follows:

$$m_{Total} = m_{Shuttled} + m_{Dead\ volume} + m_{Displaced} \quad \textbf{(1)}$$

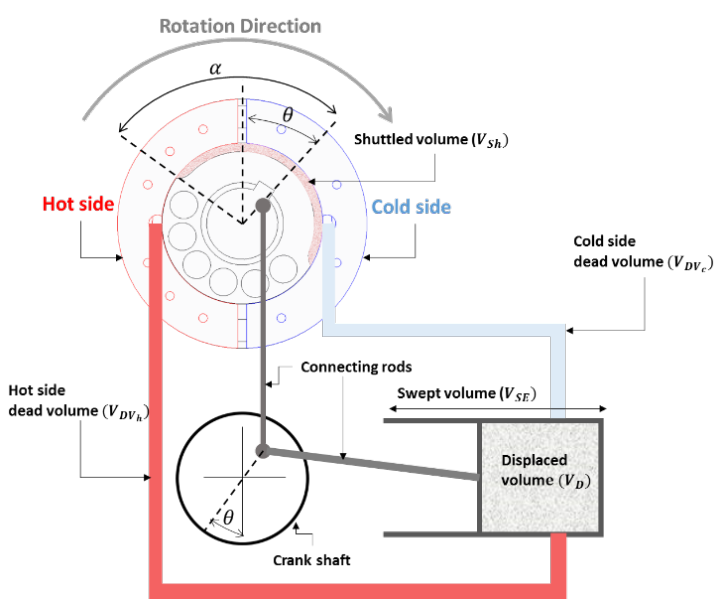


Figure 1. 2D schematic design of rotary displacer SE [3]

Where m is the overall mass of the working fluid inside the enclosure, m_{Sh} is the mass of working fluid inside the displacer housing (shuttled volume), m_{DV} is the mass of working fluid in the side connecting pipes, and m_D is the mass of working fluid in the power piston (displaced volume). The shuttled volume (V_{Sh}) remains constant during the entire cycle and consists of two varying volumes:

$$V_{Sh} = V_{Sh_c} + V_{Sh_h} \tag{2}$$

Where V_{Sh_c} is the volume faces the cold side, and V_{Sh_h} is the volume that faces the hot side. Considering the phase angle of α, the cold side shuttled volume (V_{Sh_c}) is described as a function of the crank angle (θ). (eq. (3))

$$V_{Sh_c} = \frac{V_{Sh}}{2}[1 + \cos(\theta - \alpha)] \tag{3}$$

And the hot side shuttled volume is obtained from eq. (4):

$$V_{Sh_h} = V_{Sh} - V_{Sh_c}$$

$$V_{Sh_c} = \frac{V_{Sh}}{2}[1 - \cos(\theta - \alpha)] \tag{4}$$

Also the displaced volume change can be explained as eq. (5).

$$V_D = \left(\frac{V_{SE}}{2}\right)[1 + \cos\theta] \tag{5}$$

In which V_{SE} is the maximum swept volume by the power piston. Then the overall volume V, is described in eq. (6).

$$V = V_{Sh} + V_D + V_{DV} \tag{6}$$

Where V_{DV} is the dead volume in the SE. In the Schmidt analysis, the fluid volume in each part is compared with the swept volume (V_{SE}) since, by definition, a displacement must occur to produce work in a thermodynamic cycle. In SE, the power piston does the work and if it does not move, there will be no work. The volume ratio is described as eq. (7).

$$X_{Sh} = \frac{V_{Sh}}{V_{SE}}, X_{DV} = \frac{V_{DV}}{V_{SE}} \tag{7}$$

Here, V_{Sh} is the shuttled volume, V_{DV} is the total amount of dead volumes, and V_{SE} is the swept volume. In the Schmidt analysis, it is assumed that the temperature distribution along each engine component remains constant. Besides, it is presumed that the fluid temperature in the connecting pipes and the power piston is equal to the average temperature of the hot and cold sides of displacer housing (eq. (9)).

$$T_{Sh} = T_c + \left(\frac{T_h - T_c}{2}\right)[1 - \cos(\theta - \alpha)] \tag{8}$$

$$T_D = T_{DV_c} = T_{DV_h} = \left(\frac{T_h + T_c}{2}\right) \tag{9}$$

T_h represents the displacer housing heat source temperature and T_c represents the displacer housing heat sink temperature. Another important variable in SEs is the temperature ratio (τ) as defined in eq. (10).

$$\tau = \frac{T_c}{T_h} \tag{10}$$

The pressure is then calculated from the following equation:

$$P = \frac{C(1 - \delta\, cos(\theta - \alpha))}{S - \sqrt{A^2 + B^2}\, sin(\theta - \phi) - \frac{\delta}{2} cos(2\theta - \alpha)} \tag{11}$$

Where C is defined as

$$C = \frac{P}{1 - \delta \cos(\theta - \alpha)} \left\{ S - \sqrt{A^2 + B^2} \sin(\theta + \phi) - \sqrt{A_1^2 + B_1^2} \sin(2\theta + \phi_1) \right\} \tag{12}$$

Therefore the pressure at $\theta = 0^o$ and $\theta = 180^o$ can be computed as following

$$P(\theta = 0) = \frac{C(1 - \delta\, cos\, \alpha)}{S - \sqrt{A^2 + B^2}\, sin\, \phi - \frac{\delta}{2} cos\, \alpha} \tag{13}$$

$$P(\theta = \pi) = \frac{C(1 + \delta\, cos\, \alpha)}{S + \sqrt{A^2 + B^2}\, sin(\phi) - \frac{\delta}{2} cos\, \alpha} \tag{14}$$

Finally, the amount of work per cycle is defined as

$$W = -\oint_0^{2\pi} P dV \tag{15}$$

Equation (15) cannot be solved analytically, thus the amount of work is calculated with numerical methods. Based on the given assumptions, the P-V diagram would look similar to the one in Figure 2. According to Figure 2, the pressure at $\theta = 180^o$ must be always greater than the pressure at $\theta = 0^o$.

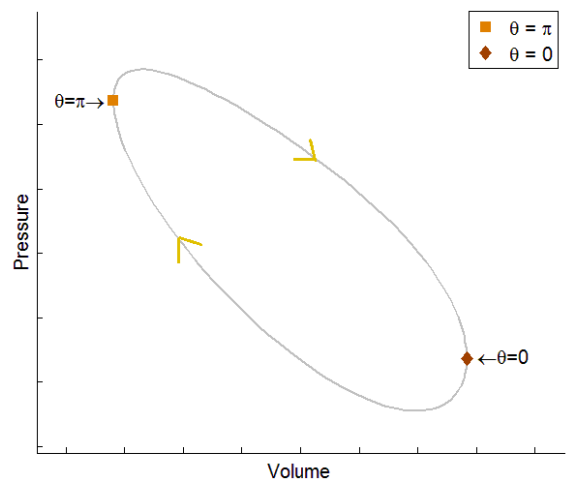


Figure 2. Pressure vs. volume schematic diagram [3]

Thus, eq. (11) results in:

$$\frac{P_{\theta=\pi}}{P_{\theta=0}} > 0$$

$$\Rightarrow \alpha_{max} = \pi - cos^{-1}\frac{1}{\delta}\left[-X_{Sh} + \sqrt{1 + X_{Sh}^2}\right] \quad \textbf{(16)}$$

Equation (16) suggests the optimum phase angle for rotary displacer SE configuration would be 90° [3].

2.2 Selected Cases for Potential Applications

There are many potential applications for the rotary displacer SEs. This study identifies four significant applications and analyzes them with the developed idealized model. Stirling cycle machines can be used as heat engines that converts heat input to mechanical work. When they operate in a reverse manner, Stirling cycle machines can be also be used as refrigerators that use mechanical work input to absorb heat at low temperatures. For each application case, a relevant heat source and sink temperature is defined and the amount of output work is calculated.

- Stirling Engine
  - Distributed power generation (High temperature difference)
  - Waste heat recovery (Low temperature difference)
  - Space power (High temperature difference)
- Stirling Refrigerators
  - Cryocooling (High temperature difference)

2.2.1 Distributed Power Generation

In this case, it is assumed that the SE is used as a heat engine in a residential building that provides a specific amount of power with the piped-in natural gas as the energy source. When the natural gas is burned in the combustion unit, the released heat is supplied to the hot side of the SE and electricity is generated through the coupled alternator. The hot exhaust gases are then routed to the heat exchangers

to supply space heating and hot water. During summer time, when cooling is needed, the waste heat still can be utilized to provide cooling via absorption cooling cycles. In this scenario, the hot side temperature (T_h) and the cold side temperature (T_c) are assumed as 1,000°C and 100°C respectively.

2.2.2 Waste Heat Recovery

Many industrial facilities, such as steel, glass, ceramics, and petro-chemical industries, have high temperature/quality, otherwise-wasted heat available on site. In such environments, part of the available exhaust heat can be recovered with the use of SEs. A typical exhaust temperature that can be assumed as the hot side temperature is 540°C [4]. In this case, the ambient air temperature (25°C) can be used as the heat sink temperature.

2.2.3 Space Power

Considering the fact that there is no air in space, the power generation has to rely on a non-combustion based process. However, the solar radiation can be used as the energy input. For instance, an exploration vehicle on Mars can be equipped with the SE to provide the required propulsion. When the hot and cold sides interface with the Sun and a shadow space, respectively, the corresponding hot and cold side temperatures can be taken as 455°C and -100°C [5].

2.2.4 Cryocooling

Cryocoolers are refrigerators which are used to reach cryogenic (very low) temperatures. One of the main applications for cryocoolers is the cooling of electronic sensors and optical devices. Considering that the cooler works at a room temperature ambient with a set temperature of -100° C, the hot side and the cold side temperatures can be assumed to be 20°C and -100°C, respectively.

All operating conditions for the selected cases are summarized in Table 1.

Table 1. Operating conditions assumed in this study

	T_L		T_H	
	(°C)	(K)	(°C)	(K)
Distributed power generation	100	373	1000	1273
Waste heat recovery	25	298	540	813
Space power	-100	173	455	728
Cryocooling	-100	173	20	293

2.3 Predicted Performance of Selected Cases

In order to predict the performance of the four selected cases, the values of the shuttled volume, the displaced volume and the dead volume are assumed to be constant as listed in Table 2.

Table 2. Assumed configuration for the SE used in this study

	Value (cm^3)	**Ratio over the displaced volume**
Swept volume (V_D)	8.39	1
Shuttled volume (V_{Sh})	45.57	5.43
Dead volume (V_{DV})	4.56	0.543

Besides, Helium is selected as the working fluid since it is the best, economically affordable working fluid for SEs [6]. Figure 3 shows the volume change as a function of crank shaft angle in a complete cycle. As it is indicated, the shuttled volume and the dead volume remain constant during the entire cycle while the displaced volume varies between zero and the full swept volume sinusoidally. Overall, the displaced and total volumes follow a harmonic motion.

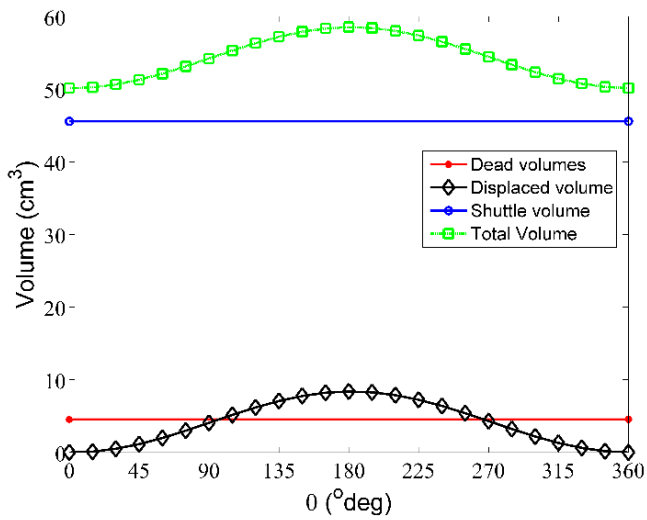
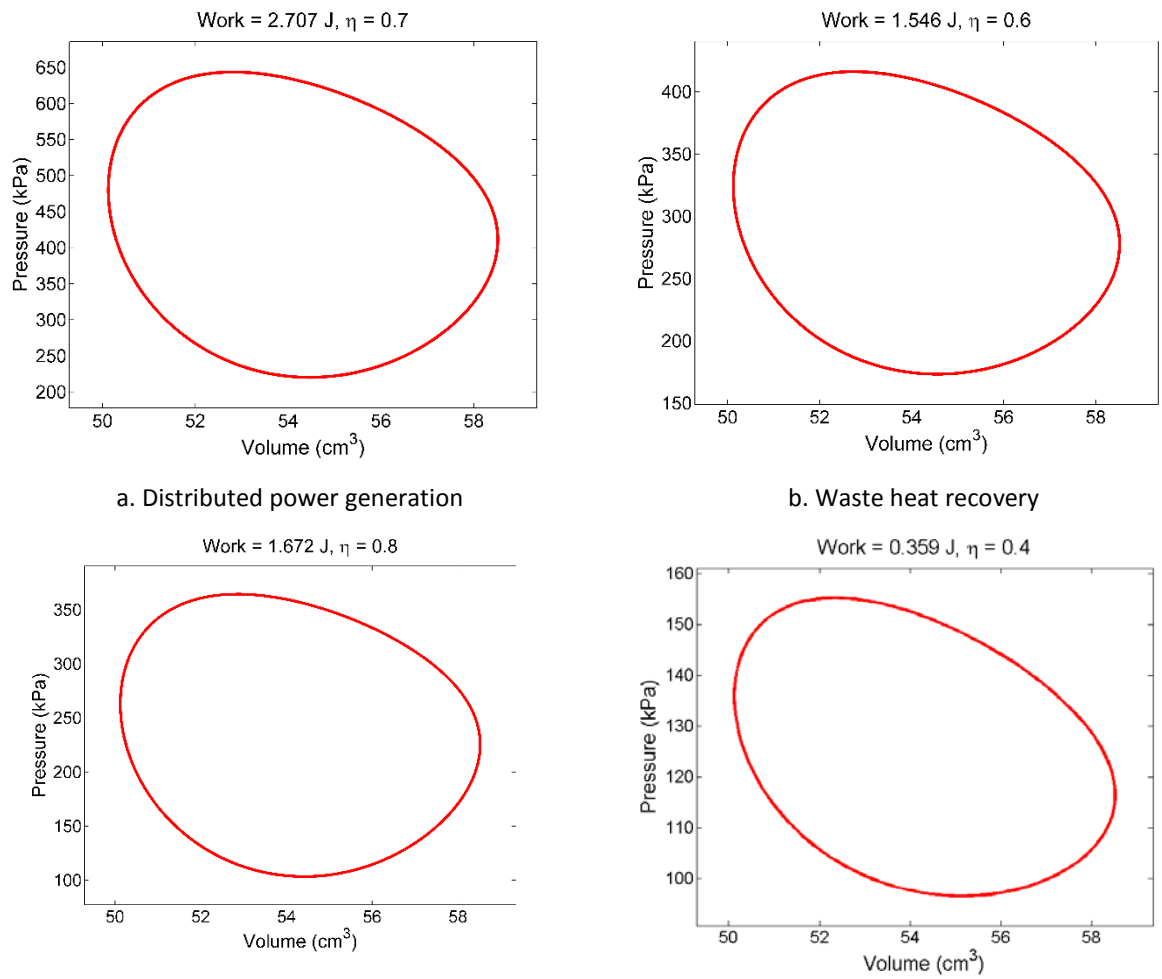


Figure 3. Volume vs. crank shaft angle

Our earlier study [3], based on the Schmidt analysis, predicts the amount of work per full cycle of rotary displacer SE. Considering a constant temperature ratio, the higher sink (cold side) or source (hot side) temperatures would result in greater amount of work. Figure 4 shows the generated work for each of the four cases, with distributed power generation (Figure 4a) being the highest, and cryocooling (Figure 4d) being the lowest.

Another critical SE design parameter is the amount of "dead volumes" with a significant effect on the overall performance. Dead volumes decribe the volume of connecting pipes or inevitable space that must be incorporated in the engine to complete the cycle of the working fluid. If the ideal situation is defined as having no dead volumes ($X_{DV} = V_{DV}/V_{SE} = 0$), effect of actual dead volumes on the performance can be clearly illustrated. Table 3 includes these comparisons for the generated work per cycle with actual vs ideal (zero) dead volumes. As expected, dead volumes lead to performance reduction since they do not exchange heat and thus do not contribute to the work generation.



a. Distributed power generation

b. Waste heat recovery

c. Space power

d. Cryocooling

Figure 4. Pressure vs. volume diagrams for four selected applications of SEs

Table 3. Comparison of generated work per cycle with actual vs. ideal dead volumes

	W (J)	$W_{X_{DV}=0}$ (J)	Ratio
Mass power generation	2.707	2.964	0.913
Waste heat recovery	1.546	1.695	0.912
Space	1.672	1.829	0.914
Cryocooling	0.359	0.395	0.909

3. VISULIZATION

Considering the unique design and working principle of the SEs, particularly the rotary displacer SE, understanding its operation could be quite challenging. Therefore, a visualization through a carefully created animation could be very helpful to emphasize the fundamental aspects of its operation and highlight the key components. In an effort to achieve such animation, the solid models of all the major components of the rotary displacer SE are created first, using a common commercial software, PTC Creo®, that has integrated capabilities to animate the assembled systems with defined parameters.

3.1 Displacer Housing

The displacer housing encompasses the rotary displacer, the innovative component of the subject engine, and exchanges heat with the source and sink. In contrast to traditional configurations that have displacers with a linear motion (Figure 5), the rotary displacer SE has a displacer that never changes direction and shuttles the working fluid inside the housing through the hot and cold sides in a rotary motion. This displacer also functions as a crankshaft/flywheel.

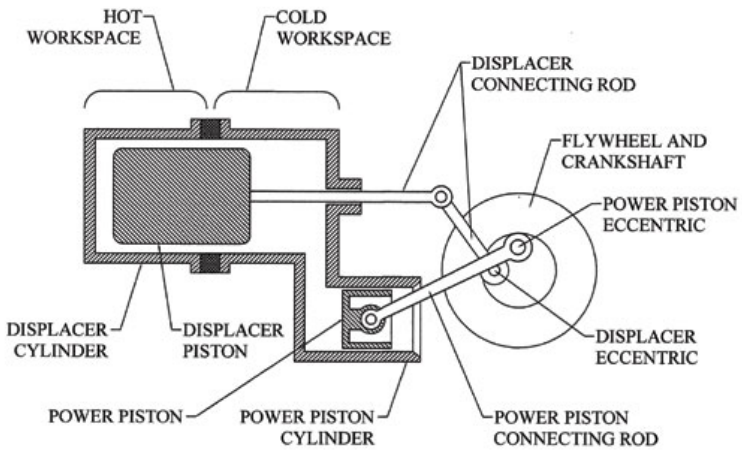


Figure 5. Schematic of a traditional Gamma-type SE with linear displacer [7]

The displacer housing consists of two symmetrical half cylinders, the hot side and the cold side, with identical inner surfaces (Figure 6). There is a significant thermally-induced load on the displacer housing since the hot side is exposed to heat source at high temperature, and cold side is exposed to heat sink at low temperature simultaneously during the operation. Therefore, a separator (Figure 7) made of polyetheretherketonein (PEEK) with high thermal resistance [8] is used as an intermediary insulator between the hot and cold sides to avoid a potential structural distortion.

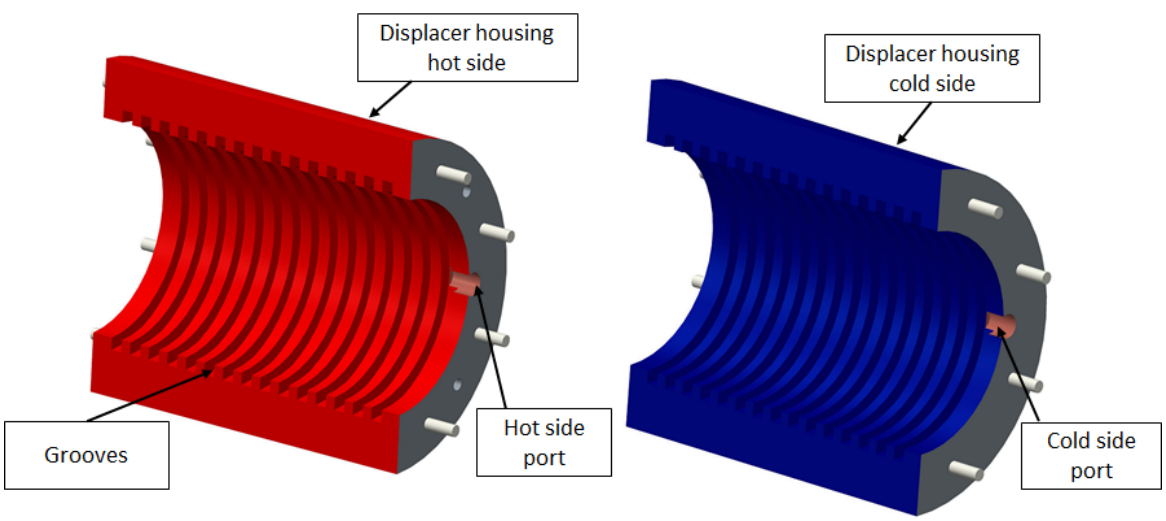


Figure 6. Displacer housing hot side and cold side

Figure 7. Upper and lower longitudinal seals

Inside the housing, there are displacer segments (Figure 8) that are coaxially placed side by side to build the rotary displacer unit. The segments have a missing volume in order to accommodate the working fluid that is shuttled in a rotary motion within the housing without touching to the internal walls. They also feature six holes, filled with low-density foam, that balance the missing volume during high speed spinning and eliminate vibration. A total of 16 segments are mounted on a crank shaft (Figure 9) which is then centrally placed inside the displacer housing between to the right and left hand displacer housing ends (Figure 10). The housing ends cap the hot and cold sides of displacer housing and contain bearings and provisions for liquid cooling. The displacer housing end seals (Figure 11) are installed between the housing ends and the hot and cold sides of the displacer housing to seal the working fluid and minimize heat loss. Another unique design feature is a rotary valve integral with the first displacer segment and the adjacent portion of the displacer housing end. This non-contact valve mechanism effectively directs the working fluid by alternately blocking and unblocking hot and cold ports.

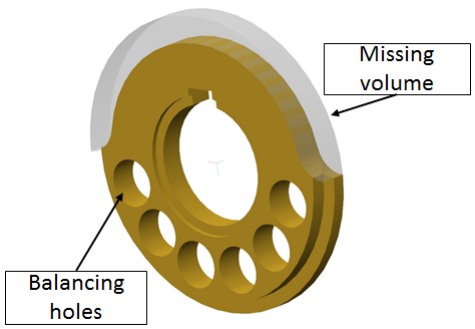


Figure 8. Displacer segment

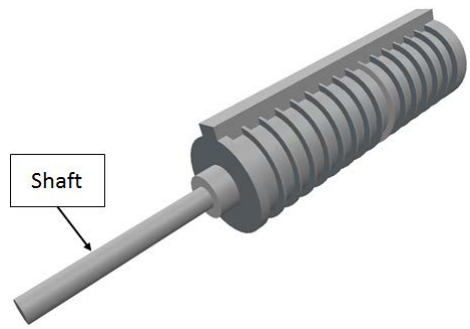


Figure 9. Crank shaft

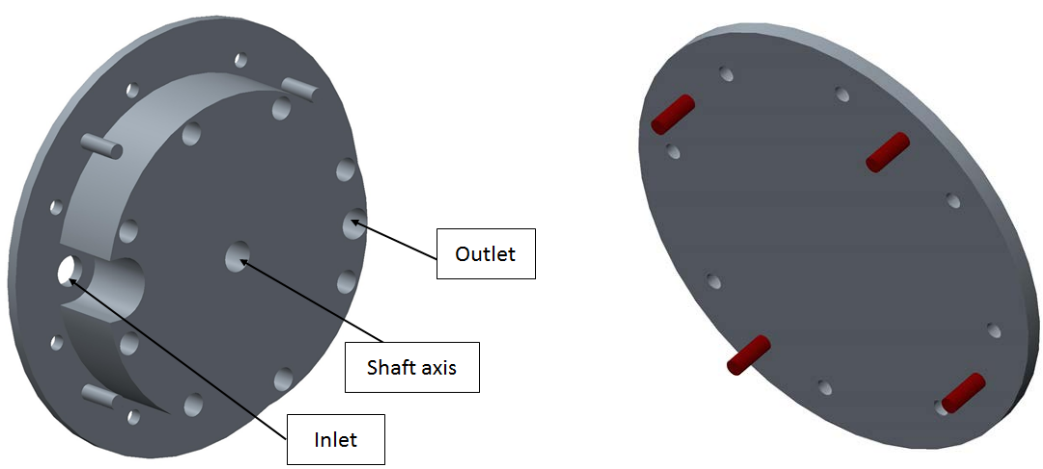


Figure 10. Right/left hand displacer housing end

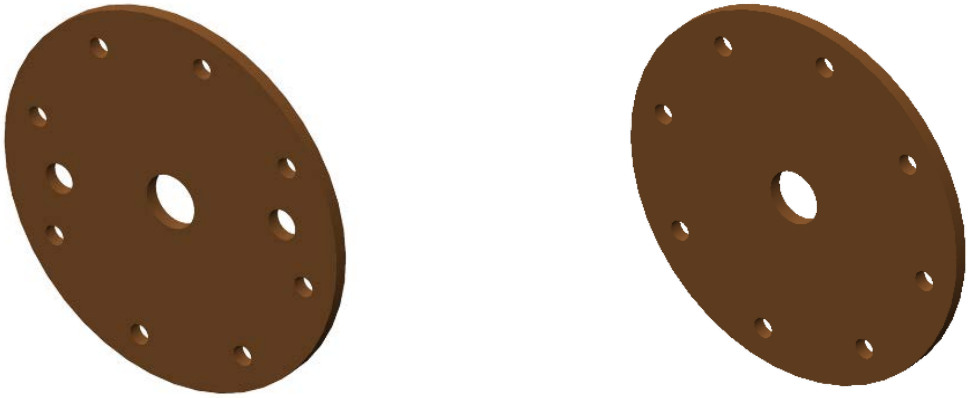


Figure 11. Right/left hand displacer housing end seal

An end cap (Figure 12), in front of the right hand displacer housing end, is used to support the power cylinder and route the working fluid.

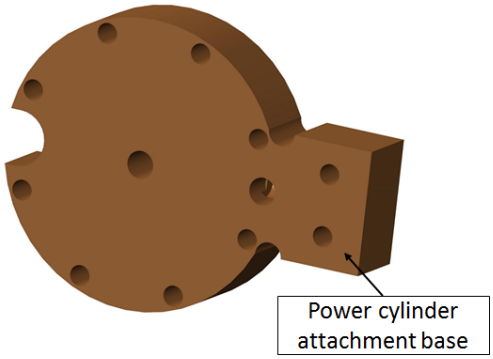


Figure 12. Right hand displacer housing end cap

Finally, a displacer pulley (Figure 13) is added to the end of the crank shaft. The pulley features a center hole for the shaft, and a smaller hole for the connecting rod attachment.

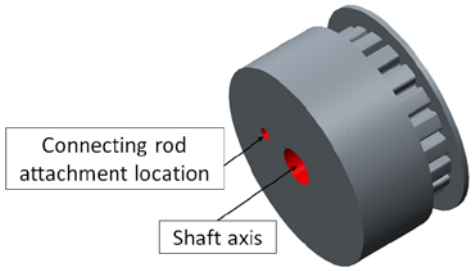


Figure 13. Displacer pulley

3.2 Cylinder-Piston

The cylinder piston assembly has a relatively simple configuration. The power cylinder (Figure 14) has two symmetrical holes that function as inlet and outlet ports. The steel piston (Figure 15) is positioned along the center line of the power cylinder and has a pin on the back side to attach the connecting rod.

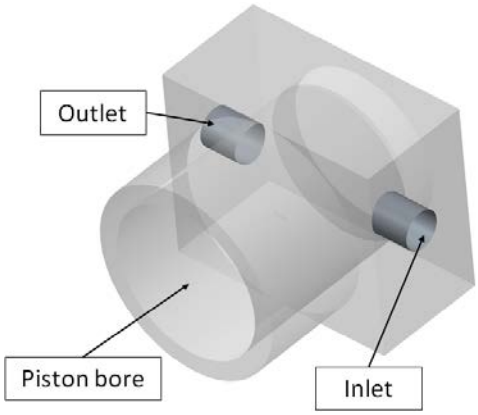


Figure 14. Power cylinder

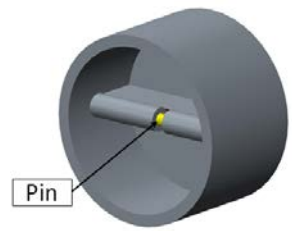


Figure 15. Piston

Another PEEK insulator layer (Figure 16) is placed between the cylinder body and the housing end cap in order to limit conduction heat losses.

Figure 16. Power cylinder insulator

3.3 Connecting Rod and Tube

The connecting rod and tube combines the two major sections of rotary displacer housing and cylinder-piston to complete the system. The connecting rod (Figure 17) links the piston and the displacer pulley (and rotary displacer). The rod is attached to the displacer pulley through a small crank pin (Figure 18).

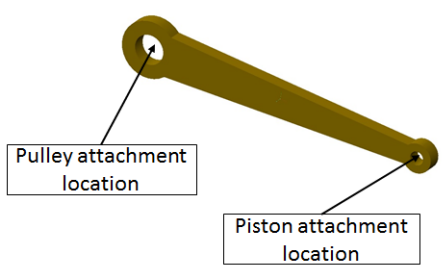


Figure 17. Connecting rod

Figure 18. Crank pin

The hot cross over tube (Figure 19) provides a passageway for hot working fluid that exits the hot side of the displacer housing and enters to the power cylinder. Another passageway between the power cylinder and cold side of the displacer housing is integrated into the housing end and housing end cap. Although these two volumes constitute a dead volume, it cannot be eliminated in practice.

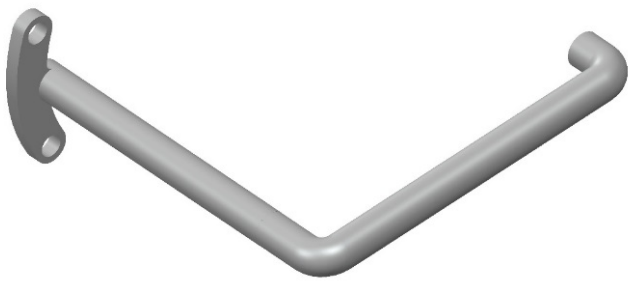


Figure 19. Hot cross over tube

3.4 Assembled Rotary Displacer SE

Once the solid modeling of all the individual components are completed, they are assembled in PTC Creo® as shown in Figure 20. Some parts are made transparent to show the inner parts involved for better understanding of the assembly.

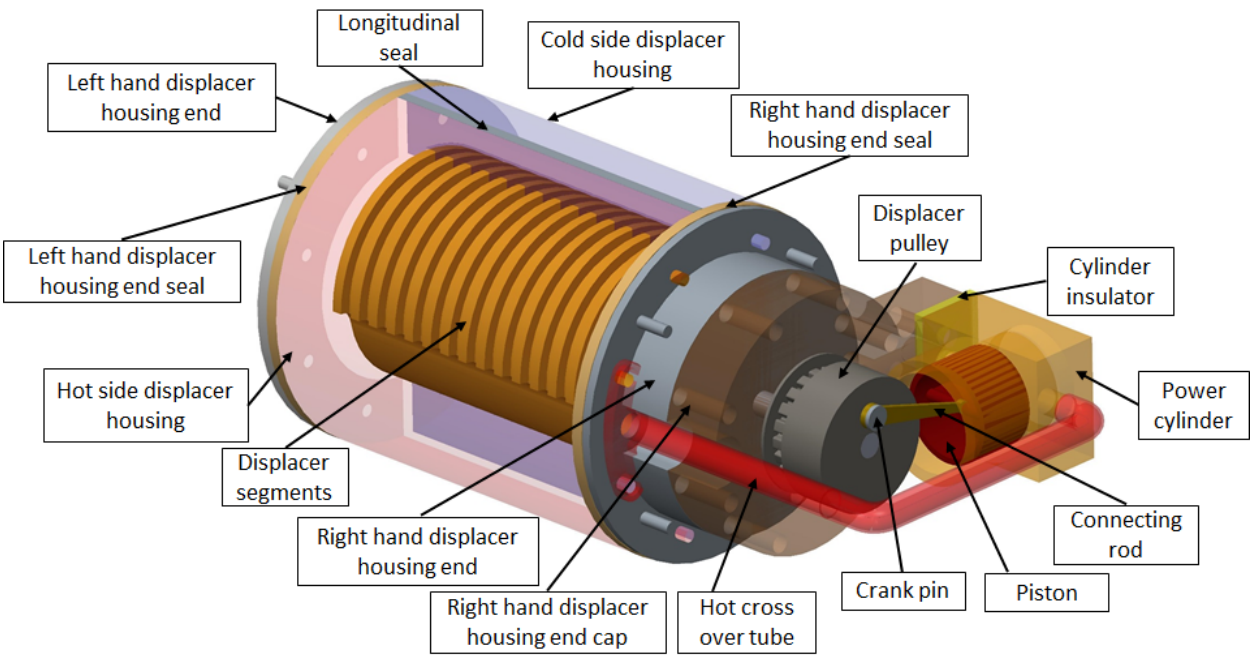


Figure 20. Overall view of the assembled rotary displacer SE

Figure 21 includes a cross sectional view of the assembled rotary displacer SE and highlights the small clearance spaces between the displacer housing and the segments.

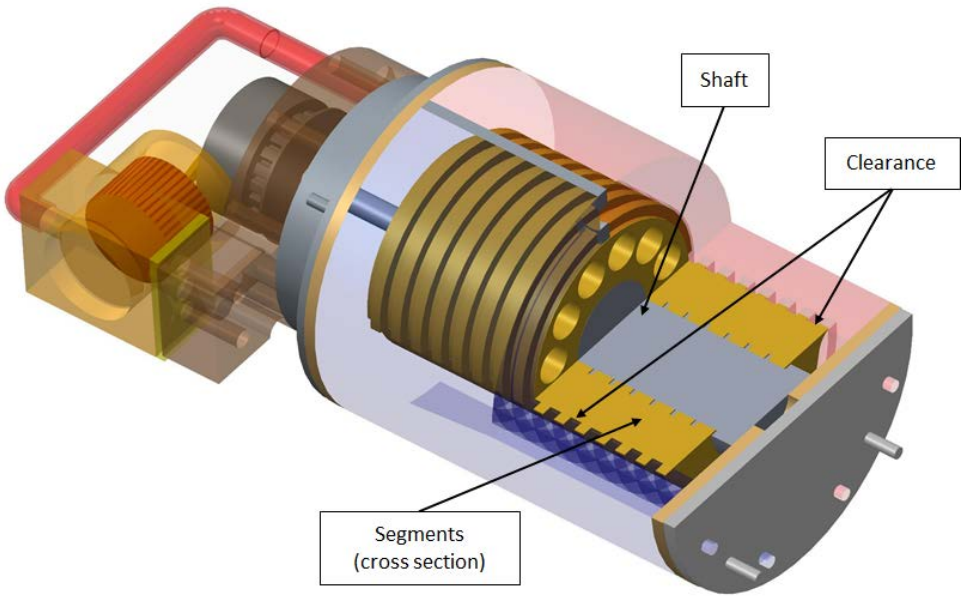


Figure 21. Cross sectional view of the assembled rotary displacer SE

Utilizing the PTC Creo®'s capabilities, an animation is created based on the rotary displacer SE assembly (Figure 20) and it is uploaded to an online video sharing platform. The animation and other rotary displacer SE related videos can be reached at the short URL: goo.gl/HHWvCU.

4. IMPLEMENTATION OF SIMULATION AND VISUALIZATION TOOLS

The developed simulation and visualization tools will be incorporated into the engineering curriculum, as they will be particularly useful for undergraduate mechanical engineering/engineering technology majors in courses, such as Thermodynamics, Applied Thermodynamics, Thermal-Fluid Sciences, Energy Conversion, and Alternative Energy. Additionally, these tools will be important in introducing this emerging energy conversion technology to general public. Use of clear, intuitive visualization tools will be able to engage non-technical individuals including K-12 students, and effectively help increase their overall awareness towards better utilization of the energy. Simulation of specific, well-defined cases will help bridging this technology to its potential applications.

5. CONCLUSIONS

This study discusses the simulation and visualization of an innovative rotary displacer SE operation. An idealized, thermodynamics-based preliminary analysis allowed the consideration of several potential applications of the rotary displacer SE to simulate their performance based on the assumed temperature conditions. Solid modeling of major components of the rotary displacer SE enabled the creation of assemblies and animations to provide a clear, intuitive description of this innovative energy conversion technology. The developed visualization and simulation tools are expected to be instrumental in engaging engineering students and general public in this technology, and in increasing their overall awareness towards better utilization of the energy.

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