Sliding along downhill, uphill, and curved segments: a dynamical simulation exercise for a first course in Physics or Mechanics

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Abstract

In this paper we describe and review a specific simulation exercise in dynamics, developed at Elizabethtown College. It consists of simulating the dynamics of a manned sled along a number of downhill and uphill segments with different slopes, lengths, convexity/concavity and radii of curvature. Students are required to write the free body diagrams and to derive the kinematical equations of motion within each segment. Friction and air resistance are included. The non-uniformly accelerated motion is approximated by uniform acceleration within small path increments. The students (freshmen or sophomores) are requested to update the motion in a spreadsheet, while solutions with other software packages are assigned extra credit. Similarly, a variety of "what if" questions, some of them to be discovered, bring the students to critically analyze the results and find interesting solutions for specific initial or other conditions. We argue that this type of exercise/simulation, that requires students to intermix theoretical analysis with computer simulation and common-sense engineering data analysis, should become very important tools in physics and engineering. Such composite exercises do mimic in some sense "real-life" problems that graduating engineers find in their first job efforts, and do help prepare the students to the multifaceted requirements of graduate research.
We analyze the performances and the attitudes of different classes and students to the sled problem and we report on a survey that reflects some of the students' thoughts on such type of challenges and their usefulness.

Introduction

"The Sledder" is a simulation exercise in dynamics, developed at the Department of Physics and Engineering at Elizabethtown College. It was first offered in 1993 in the framework of a course in Statics, and more recently has been incorporated into first-year Physics courses. Students are asked to simulate the dynamics of a sled along a trajectory of downhill and uphill segments characterized by different slopes, lengths, convexity features and radii of curvature. Friction and air resistance are included. The first task required from the students is to draw the free-body diagrams (FDB) pertinent to each segment of the motion; then the students are required to derive the equations of motion within each segment. Next, the non-uniformly accelerated motion is approximated by uniform acceleration within small path increments. Finally, the students develop a spreadsheet application to update the motion. A reader-friendly summary of results is required, in addition to the full spreadsheet solution. The complete text of the exercise as handed out to students is included at the end of this paper.

Educational goals

The main goal of this exercise/simulation lab is to instruct students in an integrated set of tasks that require an overall understanding of Newton's laws and of kinematics together with basic common sense and the ability to put things together. Additional important goals come from the general teaching philosophy in the Department of Physics and Engineering at Elizabethtown College, where one emphasis is to provide students with strong problem-solving skills, together with experience, growth, and competence in the areas of *teamwork*, *communication*, and *computer proficiency*.

It is in this context that one has to appreciate the creation of this exercise, The Sledder, to help students learn how to use spreadsheets (think "*using computers*"). It is assigned to two- or three-person "teams" (think "*teamwork*") who, as part of the assignment present an interim report (think "*communication*") on the dynamics of the problem.

History and development

The basic premise of the exercise is to use iterative methods to solve non-constant acceleration problems. When first used in 1993, it was assigned to two-person teams in a Statics course. Far fewer students knew how to use spreadsheets in 1993 than at present, so about six weeks was allowed for completion. Since then, the exercise has been used in lower level courses, and the time allowed for completion has been reduced to two weeks. This time frame allows for team deliberations and the formulation of well-structured team presentations. In addition, among a number of "extra-credit" options, students are invited to solve the problem by alternate methods and/or by using additional software packages.

Instructor's support

The Sledder can be integrated as a mini-project in a basic Physics course or even as a simulation-lab in a Physics Laboratory session. In addition to problem-solving skills, computer proficiency, and teamwork, the students are challenged by the fact that they have to carefully plan for the various parts of the exercise and to allot a reasonable amount of time to different stages of the mini-project. The time factor should be carefully pointed out to the students as one of the most relevant ingredients for success. Most students undertaking this exercise at Elizabethtown College are freshmen or sophomores; at this stage of their studies some are still lacking a full maturity in evaluating and managing time needs and resources. Being enrolled in demanding courses with required homework for almost any day of the week, students' routines often lead to approaches that translate to assignments tackled the night before they are due. Not surprisingly, we found that assigning the exercise/lab in class and then having students return it within two weeks without further interactions frequently resulted in poorly formulated solutions with a taste of "last-minute" action that resulted in unsatisfactory submissions by even the most talented students.
To render the exercise most effective and to enhance the students' skills in time management and approach to multi-tasking, we found – over the years – certain actions to be effective:

1. Upon distributing the handout, the instructor reads it in class, explaining all steps, and emphasizing the serial nature of the tasks and the time required for tackling them. This reduces the effects of students bagging the handout and looking at it too close to the submission deadline.

2. Motivation can be added if the exercise is presented as a playground to experience multi-tasking projects very common in "real-life" post graduation jobs or in graduate research projects.

3. The students should be made aware of the enlarged educational goals of this exercise, beyond probing Newton's laws and testing the capabilities of spreadsheet implementation. The need to be alert to time management and to plan a common team strategy and timetable should be stressed. The need to check partial results at all stages has to be emphasized. The need to prepare a clear presentation, discussion and summary of results should be explained to be very important, lest the scientific community (or the future boss or client) would not be able to understand the solution.

4. Meeting with the students about a week after the handout is distributed is effective. It gives the students the opportunity to discuss with the instructor the validity of their free body diagrams and the way to implement the spreadsheet. This checkpoint forces the students to start their work early while at the same time allowing the instructor to help weaker teams out of "stuck" situations ("I do not know where to start or what to do next".)

5. While usually a deadline should be strictly imposed due to both educational and fairness goals, in this exercise we believe these considerations should be overridden by another perspective. In fact, a highly beneficial input is feedback to the students if, instead of grading as is, the work is returned for further improvement, completeness, and/or better presentation. The students have to learn what a well-written, no-compromise final report looks like, and experience through their own repetitive struggle the amount of time and effort needed to achieve a satisfactory output. This approach should be integrated with a well-thought-out grading scheme that, while insuring rewards for on-time submissions, also rewards finalized submissions that were improved by re-work.

Students' evaluations

Students are asked to include an evaluation of "The Sledder" as an integral part – but at the end – of their report. We ask their thoughts, along the lines of: did they enjoy the exercise and did it prove to be useful in building their skills as scientists and engineers. We also solicit suggestions on how to improve the exercise.

In a bird's eye view, the responses show a somewhat even distribution of opinions in three large segments: *the enthusiasts*, strongly supporting and cheering the exercise as a very important step in preparing their needed career skills; *the indifferent ones*, looking at this as just another exercise, albeit longer and more complex than usual; and *the cynics*, openly stating they did not enjoy it and that time might be served better by alternative activities. A large majority of students in the cynic group justify their dislike by stating the exercise is difficult and/or time-consuming. In many cases the instructors can find these same students are not proficient, as yet, in computer skills and/or have not yet come to appreciate the power and beauty of Free Body Diagram analysis techniques. The joy of teaching is to work with all students, but the cynics give us special satisfaction since they, when they see-the-light, are among the most transformed.

Conclusions

We believe that in fast-paced, intense curricula, as students experience in the Physics and Engineering at Elizabethtown College, "The Sledder" and similar integrated exercises play an important role in reinforcing theoretical and computational abilities in first and second year students, while at the same time fostering and enhancing team work, communications, and time-management skills. Overall this exercise can be viewed as a macrocosm of the difficulties and the challenges that students will encounter in later years' projects or even in "real life" professional activities, including engineering jobs and graduate research.

Appendix: "The Sledder" Handout

The Sledder Team members _________________; _________________; _________________

The report for this lab is comprised of your <u>team's</u> Excel file, e-mailed to your instructor before 2:30 p.m., Friday, (date), and your <u>team's</u> FBDs, equation development, narrative, and discussion – all stapled behind this handout and submitted as you walk into class before 2:30 p.m., Friday. (The left-to-right cells on your Excel spreadsheet should be limited so they can all be viewed simultaneously on a normal monitor.) Each team must be certain to involve every team member in developing the logic and in entering data and equations on the spreadsheet.

Lab -- "<u>The Sledder</u>" -- An exercise in using iterative solutions for a non-constant acceleration problem

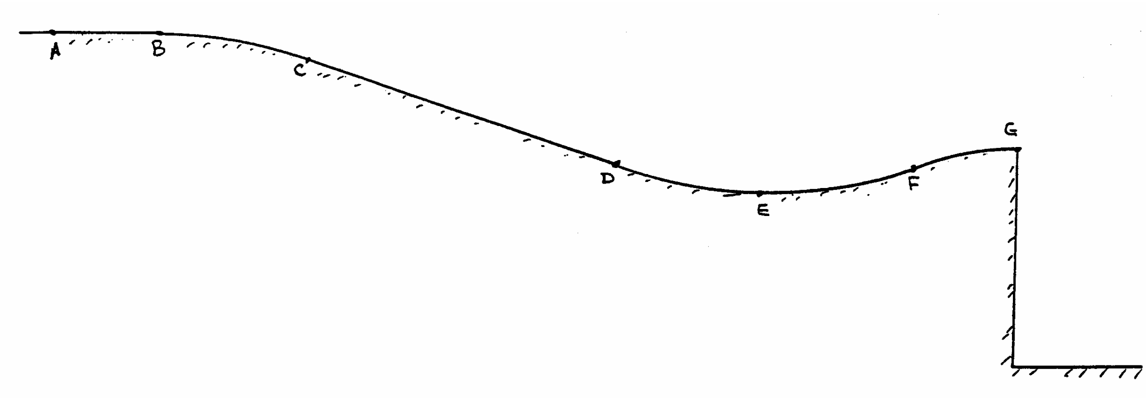
This is a project designed to use knowledge of *Newton's Second Law* and *Kinematics* equations in a physical situation where acceleration may be changing from one instant in time to the next. Up until now, we have principally used $\Sigma \mathbf{F} = m\mathbf{a}$ to solve problems involving constant acceleration -- that is, we found an acceleration using the Free Body Diagram (FBD) procedure and used that constant acceleration to calculate a final position or velocity. In this project, an acceleration is found for one point along the path and that acceleration is assumed constant for a very short time (or displacement); this allows calculations using constant acceleration equations to find conditions at the end of that short time (or displacement). Then the procedure is repeated (iterated) until the final destination is reached.

Here is a case with non-constant acceleration: **see SLED RUN – Figure 1**:

- At point A, sled has speed v_A;
- A to B, level run of length L_{AB};
- B to C, an arc with a radius of R_1 and included angle φ_{BC} ;
- C to D, straight downhill slope with length L_{CD} at angle $-\Theta$ (magnitude of Θ must be the same as magnitude φ_{BC});
- D to E to F, an arc with a radius of R_1 and included angle φ_{BC} from D to E (E is at the bottom of the DEF arc) and included angle φ_{BC} from E to F;

- F to G, an arc with radius R_2 and included angle φ_{FG} (if angle φ_{FG} has the same magnitude as φ_{BC}, then the sled's velocity at G will have no vertical component) (If R_1 is not greatly different than R_2, motion at F can be viewed as "smooth" rather than being discontinuous.)

If the sled makes it to all the way to G, it is then airborne until it impacts on a horizontal surface, at height H_1 below the level at G. (The full exercise starts at Point A and ends at impact. A shortened version starts at A and ends at G (omitting projectile motion); and a greatly shortened version starts at Point C (with a given speed v_C) and ends at Point G.



SLED RUN – Figure 1

Procedure: (This is the procedure for the greatly shortened version (C to G).
At C, speed is v_C (use VC for ease of entry into the spreadsheet).

1. For each segment of the sled run, create a Free Body Diagram (FBD). Forces to be considered include weight (mg), normal force (n), friction force ($f_k = \mu_k n$), and resistive force (R ($=0.5D\rho Av^2$)). The "segments" are CD, DE, EF, and FG. For CD, use an *s, y* coordinate system: *s* along the direction of motion and *y* perpendicularly up from the surface of the sled run. For the curved sections, use an *s, r* coordinate system, with *s* along the direction of motion and *r* radially outward from the center of curvature.

2. Use the FBDs to derive – for each segment - algebraic expressions for n, f_k, R, and centripetal acceleration; and an acceleration equation (for tangential acceleration). [Hint: Point F is an inflection point. Therefore, there is no centripetal acceleration at F.]

3. The assumption in our iterative process is simply this: Acceleration during a very short distance "step" (or a very short time "step") will change so little that the constant acceleration at the beginning of the step can be used to represent the non-constant acceleration during the step. Therefore, with very short steps, the constant-acceleration calculated results (speed, position, time, etc.) at the end of the step will be practically identical to non-constant results. Derive equations to find speed and time at the end of a short distance step. [*An alternate approach would be to use a short time step. However, since this lab calls for results as a function of position (to be stated below), using the short distance step is more direct. Yet another alternate is to use energy considerations rather than Newton's Second Law (see brief discussion at the end of this handout.*]

4. Your equations (from #2 and #3) must now be used in the spreadsheet.

5. On your spreadsheet, use the first and second rows as you wish, but be sure to include "Sledder" and be sure to identify members of your team, and of course include a date.

6. In the third row, write VC=, D=, A=, density=, fric coef =, and rider mass= in cells A3, C3, E3, G3, I3, and K3. The common values (for use by the entire class) will be: VC = 10.0m/s; drag coefficient D = 0.7; cross sectional area of the rider slicing through the air = 1600 square centimeters; air density = 1.25 kg/m^3; coefficient of kinetic friction = 0.1; and mass of the sledder = 60.0kg (we assume the sled is massless). These numbers may be entered in B3, D3, F3, H3, J3, and L3.

7. The variables you are to find - as a function of position - are: time, normal force, friction force, resistive (a.k.a. drag) force, speed, tangential acceleration, and centripetal acceleration. (Note: normal force can be interpreted as "how heavy" the sled rider "feels.")

One possible format is:

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s (m)
or
@ (˚) t(s) n (N) fk (N) R (N) v (m/s) at (m/s^2) ar (m/s^2) (Note: @ means theta)

C 0.00
C+ds (where ds means distance step)
...
D
D + @s (where @s means angle step – we use @ since the Greek theta is tough to find on Excel --the angle at D is –20.0˚)
...
E (the angle at E is 0.0˚)
E + @s
...
F (the angle at F is +20˚, and at F, the point of inflection, centripetal acceleration is momentarily zero)
F + @s
...
G

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8. When you have gotten to this point, your task is to decide on a distance step ("*ds*") and angle step ("*@s*") -- of course, *@s* must be selected so the product of *r* and *@s* is very close to "*ds*" – this logic comes from the definition of angular measure, where the angle equals arc length along a circle divided by radius. Your task will be to experiment with different distance steps and make note of how results vary. While doing this experimentation to decide on your distance step (1.0m?, 0.1m?, 0.01m?), use common data (from #6).

9. Run your program with common data and submit your spreadsheet. Your basic spreadsheet will be on "Sheet 1," but also create a "Sheet 2" with only about 20 rows in the format shown above. Sheet 2 is to include results for: Point C and every fifth meter of travel between C and D; D; every fourth degree of arc between D and E; E; every fourth degree of arc between E and F; F; every fourth degree of arc between F G; and G. *Please understand the previous sentence does not specify your distance step! Your distance step will be much less than 5.0 meters and your full solution will contain hundreds of rows. By printing only about 20 selected rows, your results will be more user-friendly.*

10. Then use common data for all variables except VC, and see if you can find a value of VC that will cause normal force (n) to go to zero somewhere along the sled run. If you find that position, say where it is and what it means.

11. Then use common data for all variables except VC, and see if it is possible for the sled to come to a stop at Point G. (Almost sounds as if we're trying to make a TV commercial showing something stopping just before going off into the abyss.)

12. <u>This ends the formalized procedure</u>, but you are encouraged to experiment further but only after you are sure your results for "common" values are correct. You see, one of the beauties of a spreadsheet is that you can ask many "what if" questions. For clear understanding of your "what if" results, it is best to alter only one variable at a time. (Do not vary air density by more than 5%, otherwise it is unreasonable.) If you do this sort of experimentation, describe what you discovered and comment on whether/why it makes sense.

***This should be a fun project*.**

(The method above uses *Newton's Second Law* and *Kinematics* equations logic. An alternative method is to use *Work-Energy* relations. [Mechanical energy at position 1 plus non-conservative work between positions 1 and 2 equals mechanical energy at position 2. Non-conservative work will be non-linear for the same path segments when acceleration is non-constant, so an iterative technique works equally well with *Work-Energy* as it does for *Newton's Second Law* and *Kinematics*.] It would make for great interest for some teams in the class to use the *Work-Energy* method while others use the *Newton's Second Law* and *Kinematics* method.)

(Extra credit #1: For those of you who have the time and inclination, continue the analysis from Point G until impact. Extra credit #2: Solve the standard problem by using energy considerations. Extra credit #3: Solve the standard problem by using another software package, MATHLAB, or MATHCAD or MAPLE or another one that you know well.)

Suggestions: This lab will be worked out during the first two active lab/class sessions. Efforts and work after class hours are welcome and even encouraged when needed; however all students have to be present in both classes/labs and use the scheduled time to discuss and develop the solutions. Don't be lulled into a sense of false security due to the two weeks span of the project! This project requires different steps, techniques, and a sustained effort. Procrastinators will most probably find themselves struggling with time and ultimately turning in a poor final solution/

report. Plan to have almost everything solved/done before the beginning of the second lab session, and use that second lab session to discuss all your doubts with the instructor and to finalize and polish your solution and your report!

Your feedback is needed -- At the end of the team's report, each student is to append a separate page and answer questions, individually without consultation with other team members:
1. Did you enjoy this exercise? Why or why not?
2. Was this a worthwhile exercise? Why or why not?
3. Regarding the work that was done in lab and the effort put into preparing the Team Report, assess how much effort each team member – including yourself -- made. (Use percentages that add up to 100%: e.g., Manny Smith, 20%; Jo Doe, 30%; Jack Jones, 50%)
4. What suggestions do you have to improve this exercise if it is used in future? (not necessary to answer this question.)

Acknowledgments

Among the first students to do this exercise were Sue Niezgoda '96 and Jodi Raffensberger '96. It is they who named it "The Sledder" and that name – although routinely rejected by Spell-Check – has a certain appeal. Sue L. Niezgoda (Ph.D. Civil Engineering, Penn State University, 2004) is now an Assistant Professor of Civil Engineering at University of Wyoming College of Engineering/ Laramie; and Jodi Raffensberger Sammarco (Ph.D. Chemical Engineering, Rutgers University, 2004) is a Research Chemical Engineer with Merck Pharmaceutical Chemical Engineering Research and Development, Rahway NJ.

Biographies

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ILAN GRAVÉ is an Associate Professor of Physics and Engineering. He joined Elizabethtown College in 2002, having previously taught at the University of Pittsburgh. His varied physics and engineering background includes research and industrial experience in Italy, Israel, and the USA.