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Solving Problem-Solving Problems: Solution Step Discipline

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Abstract

Engineering and engineering technology are nothing if they are not problem solving. Yet after more than a decade of schooling, college freshmen typically arrive with insufficient expertise in assessing problems and producing orderly, mathematical solutions. Whether at an academic department level or by individual professor, college students are guided toward some structured problem solving method. Many problem solving methodologies deal well with the crucial aspects of problem assessment, analysis and solution planning. Yet even if students successfully evaluate the problems, they still struggle with executing and professionally presenting the mathematical steps of their solutions. The author has developed an elegantly concise, yet focused, approach to understanding and presenting these mathematical steps. Termed “Solution Step Discipline” (SSD), faculty for all engineering technology courses at Purdue University School of Technology’s Richmond location incorporated it in the Fall 2002 semester. Surprisingly, the straightforward approach has challenged the students—indicating that the focus remains on the truly key elements of structured thinking. With instructor feedback, students do master SSD, which in turn can enhance the effectiveness of most any overarching problem solving structure. Without a cumbersome number of students at the Richmond location, faculty members were able to uniformly implement Solution Step Discipline and better prepare students for academic success throughout their curricula.

Introduction (Given):

College and university faculty are charged with transforming incoming freshmen into graduate engineers and technologists. To this end, the faculty are traditionally provided students with over a decade of structured academic experience. Over those years, their teachers have intentionally prepared these students for the next level of their academic adventure. Their professors expect prepared freshmen to arrive with basic facts, foundational concepts, and critical skills.

However, freshmen usually lack the problem solving expertise needed for success in technical academics. Even after twelve years of preparation, students struggle to assess and solve problems that differ from elementary text examples. Memorization, rote procedures and calculator gymnastics have triumphed over conceptual understanding. College-level exercises that require application of concepts or extended calculations overwhelm their ability to scout out and stay on a solution path.

There are a number of issues associated with this disconnect. Teachers deal mostly with brief, similar problems, where the information is clearly provided for a two or three-step solution. "Word problems" with new circumstances and sometimes unneeded data are often the greatest fear of students and teachers alike, as they require conceptual understanding and solution expertise. (Such problems also introduce the opportunity to emphasize the importance of units of measurement—another remedial college topic.)

To further thwart college-bound students, teachers rarely require a neat presentation of the students' solutions. Even if their work does not entirely disappear with calculator power-down, the teacher is apt to ask them to encircle their answers, so it can be located for "right-or-wrong" assessment from among the amorphous markings. There is little accountability for detailed solution steps, much less a clear representation of them. Students soon realize that the only deliverables are their final answers. As their demeanor often indicates, students' paradigms are shattered in college when detailed solutions—rather than simply answers—become important.

Most K-12 school system administrators and teachers would profess that they stress problem solving throughout their mathematical curricula. Yet whatever is being taught under this banner is clearly uncoordinated, to the wrong level and/or in the wrong direction.

The challenge then has been to find approaches that to some meaningful degree improve the ability of engineering and engineering technology students to solve increasingly complex and open-ended technical problems. Two general approaches are recognized for the post-secondary institutions:

First, colleges can seek effective ways to collaborate with the K-12 systems to improve preparation of students. Some initiatives are taking this arguably long-term, yet foundationally wise, approach.[1,2,3,4] Better problem solving abilities can help all students too, not just those who might enter science, engineering or technology fields.

Second, colleges can seek better approaches to effectively and efficiently assist their students in learning how to solve increasingly complex, technical problems. Most post-secondary institutions and their faculty do address this critical shortfall with their students, sometimes at the individual course level, sometimes as a coordinated curriculum focus.

This is education aptly termed "remedial problem solving"—not because teaching problem solving is easy, but because the rudiments have been so delayed. One can never compensate quickly for what should have been years of progressive learning and practice. In *Teaching Engineering*, Wankat and Oreovicz noted that a person will generally take ten years to accumulate the linked knowledge needed for problem solving mastery.[5]

Yet colleges are expected to close this gap. Proficiency must be expeditiously brought to the students' work. The professor, or perhaps the academic department, works to inject structure into the students' set ways. Rote mathematical procedures must be set aside; effective problem solving strategies with conceptual thinking adopted. Self-defeating habits must be recognized and discarded; productive ones learned and incorporated. This is the world of remedial problem solving melded into the early sessions of so many college curricula. This is the path to higher order problem solving required in upper level coursework.

Fortunately, the importance and challenge of the task have resulted in much scholarly and helpful work. Formal problem solving methodologies abound.[6,7] Many of these are comprehensive, involving detailed heuristics. The approaches usually share some basic components, such as: problem definition, assumptions, concepts/approaches evaluation, solution path development, plan execution, answer review, and report. The author concurs with the value of these basic methodology components.

Within Purdue University, engineering technology faculty have utilized a rather common framework for organizing student problem solving. This "GFSA" approach stands for: Given–Find–Solution–Answer. The student is required to format his or her work into these four components. Briefly, the Given section calls for distilling the relevant information into concise form. The Find section is to clearly define the goal of the work. The Solution portion usually represents the bulk of the written submission. The Answer portion formally labels the result of this effort as meeting the Find section's identified goal. With some modification, this rather simple, overarching structure brings some order and continuity to the students and faculty on many other campuses, as well.

However, even students who are endeavoring to follow their assigned methodology can find themselves mired in the calculation steps. As it turns out, their lack of discipline in the details of such steps soon introduces errors, sends them astray and hinders their recovery.

Perhaps it is because students' problems are so often the result of poor, higher-order skills that we do not recognize calculation details as having a potential for thwarting the entire process. This paper directly addresses this component of the problem solving methodology.

The Challenge (Find):

The challenge is to find a means of helping students efficiently solve their technical problems, by minimizing confusion and error in their mathematical solution steps.

The Approach (Solution):

Efforts to bring order to the situation sometimes serve the professors and sometimes the students. For instance, a professor might require students to define terms at the beginning of a problem solution, to help the student's learning process. Another professor might require students to write only on one side of the paper, primarily to aid the grader's task. The structures we layer onto the student's problem-solving world can have purpose, but they can also distract the student and hinder the learning process.

Many consider the most important aspects of student problem solving to be understanding the problem itself and then planning the path to the solution. These could be treated as part of problem set-up, the Given–Find sections above. This author would concur in the importance of problem set-up. Professors encourage students to slow down, to ask about concepts at work, to look for relationships between the information given and that sought, and to plan the solution path to the answer before charging forward. The author usually tells his students to invest 30-50% of their homework time in problem set-up.

It is instructive to note how various problem solving methodologies speak to the task of executing the mathematical steps. There is such emphasis on the unquestionably critical set-up and solution plan development that the least attention is paid to how the plan itself is executed. Most of the literature in the field simply ignores this aspect of the problem solving. The others typically provide little more than "Show all your work.", as though knowing how to execute the plan mathematically is obvious, elementary or does not significantly relate to problem solving success. Table 1 shows examples of summarized, mathematical step guidance provided the student once the solution plan is developed:

Specific guidance in executing solution plan.	Author(s)
Create and solve mathematical model.	Kremer[8]
Carry out the plan. • "To carry out the plan is much easier; what we need is mainly patience." • details perfectly clear, not hiding an error • Check each step for correctness	Polya[9]
Clear presentation. Show units.	Burghardt[10]
Do It (Analysis) • carefulness • systematic • attention to detail	Woods[11]
Take Action / Implement the plan. • Perform calculations • Carry and cancel units	Elger, *et.al*[12]
Solve the Problem. Show completely all steps.	Eide, *et.al*[13]

Table 1. (Mathematical) Solution Step Guidance

Wankat and Oreovicz improve on the above by adding to "neat" and "understandable" the need to show all steps to obtain an algebraic solution before substituting values.[5] This begins to add structure to the portion of the solution consistently ignored.

Over the years, the author had addressed many student errors through the development of rules for use within the overarching GFSA system. As the students demonstrated some new, creative type of error, yet another rule was added to help future students avoid that pitfall. While the list grew, it was difficult to discern any improvement in the students' learning. First, students seemed to have an expanding pallet of solution pitfalls, ensuring a long list of do's and don'ts. Second, the students seemed to find themselves overwhelmed and frustrated, trying to comply with the multitude of "enshrined wisdom".

The students clearly needed more than the four letters of the GFSA problem solving structure. However, burying them under a multitude of legalism proved no help. At this point, the author began to look for ways to simplify.

The accumulated list of rules was a valuable fossil bed of student errors. In analyzing them, the large portion related to the logistics of working through the calculation steps stood out. The lack of discipline in the solution step calculations was indeed hindering the students.

Upon further examination, the author was able to simplify the needed discipline into some very basic, yet crucial, concepts. These distilled and clarified ideas, termed "Solution Step Discipline" ("SSD"), are:

1. Solution Steps form a logical chain.
2. Solution Steps proceed down the page.
3. Solution Steps are to be understandable to a knowledgeable reader.
4. Each Solution Step must be one and only one of:
   A. Starting Equation
   B. Substitution
   C. Calculation

At first these might seem elementary or obvious, but it is the conciseness of the list that gives it real utility. While the first three concepts form the discipline framework, number four embodies the first three and becomes the focus for the students. That is, the ABC step categories closely relate to the first three principles.

A solution step may be a (A) "Starting Equation". If the student decides to use a Starting Equation step, it must be a valid, general relationship. It could have been established in the Given portion of the solution, in the text, or earlier in the solution. A reader must recognize this as a valid starting or anchor point, on which to attach a chain of logic. Beginning with an A step (only) ensures the mathematical portion of the solution will utilize equations. It prevents problem-specific values from appearing before the general relationship appears as an equation.

A solution step may be a (B) "Substitution". This step proceeds down the page, and its logic is founded upon the previous equation. If the student chooses to utilize a Substitution step (only), all mathematics must halt and only replacement with valid equivalents (e.g., mathematical expressions or numerical values) takes place. These can be from the Given section, a solution diagram, a textbook or earlier solution results. In any case, the student understands that a reader must be able to recognize the validity of the substitution through discerning the source of the terms. Further, by requiring that all substitutions be made with units, the student can better recognize units-related errors at the point they are made.

A solution step may be a (C) "Calculation". Should the student select a Calculation step (only), it flows logically from the previous equation into a new one. The author prohibits more than one equals sign in any step, ensuring that the student continues to work down the page. Because only mathematical manipulations are allowed, the student is free to focus on mathematical procedure and accuracy. Mathematics may be separated into as many individual C steps as the student desires, without violating concept 3, above. By excluding distracting substitutions, there is less likelihood of errors, whether the C-step operations are units conversions, algebra or calculus. Students should be more likely to recognize and resolve unreasonable answers.

The distillation of concepts into SSD has many designed benefits. Students must see each of their solutions as a series of logical steps, rather than random pieces of work. They must exclude "gut feelings" or "leaps of faith". Their logic must be discernable to others. This firms-up the solution path or plan beforehand—and helps them stay on it during their solutions. By segregating the ABC steps, students can better focus on the task at hand and review progress, as well as easily stop and resume their work. Students retain the freedom to order problem steps according to their own preference, for example: calculation–substitution or substitution–calculation. SSD also facilitates their finding and correcting errors. Beyond these benefits, the presentation of the students' work is such that it is easier to interpret and grade, especially for awarding partial credit. All of these benefits can be had by incorporated SSD into existing problem solving methodologies.

An elementary example of a problem solved with and without SSD is provided in Figure 1, to demonstrate the effects upon solution presentation. SSD violations are flagged.

SSD was implemented in the fall of 2002, at Purdue University's School of Technology location in Richmond, Indiana. The Technology student body of about 150 students is sufficiently small to enable full implementation (about 60% are in technical curricula). All instructors of Purdue technical courses reviewed the SSD concepts and agreed to incorporate them into course expectations. All students in each technical course receive SSD reference information, on three-hole-punched card stock (Figure 2). Instructors make it a point to review the SSD criteria with the students and confirm their understanding at the beginning of each semester. Students now recognize the approach as a curriculum expectation.

Sample Problem: A projectile is launched horizontally at 10.0 ft/s and vertically upward at 30.0 ft/s. Determine the projectile's height, after traveling 15.0 ft horizontally.

Given: Projectile motion. $a_x = 0\,ft/s^2$, $a_y = -32.2\,ft/s^2$, $v_x = 10.0\,ft/s$, $(v_y)_0 = 30.0\,ft/s$, $(s_x)_0 = 0\,ft$, $(s_y)_0 = 0\,ft$
Find: s_y (at $s_x = 15.0\,ft$)

Example Solution <u>With SSD</u>:

w/ $a_x = 0$	$s_x = (v_x)\,t + (s_x)_0$	"A" Step (Starting Eq.)
	$t = \dfrac{s_x - (s_x)_0}{v_x}$ s	"C" Step (Calculation)
w/ $s_x = 15.0\,ft$	$t = \dfrac{15.0\,ft - 0\,ft}{10.0\,ft/s}$	"B" Step (Substitution)
	$t = 1.50\,s$	"C" Step (Calculation)
	$s_y = \frac{1}{2}(a_y)\,t^2 + (v_y)_0 t + (s_y)_0$	"A" Step (Starting Eq.)
w/ $t = 1.50\ s$	$s_y = \frac{1}{2}(-32.2\,ft/s^2)(1.50\,s)^2 + (30.0\,ft/s)(1.50\,s) + (0\,ft)$	"B" Step (Substitution)
	$s_y = 8.78\,ft$ ANSWER	"C" Step (Calculation)

Example Solution <u>Without SSD</u>:

	$s_y = \frac{1}{2}(a_y)\,t^2 + (v_y)_0 t + (s_y)_0$	"A" Step (Starting Eq.)
w/ $t = 1.50\ s$	$s_y = (-16.1\,ft/s^2)(1.50\,s)^2 + (30.0\,ft/s)(1.50\,s) + (0\,ft)$	SSD Violations: 1. Combined calculation and substitution steps. 2. Substituted time value not validated *above* in solution.
	$s_y = 8.78\,ft$ ANSWER	"C" Step (Calculation)
	$s_x = (10.0\,ft/s)\,t + 0\,ft$	SSD Violation: Not a valid, general starting equation.
w/ $s_x = 15.0\,ft$	$t = \dfrac{15.0\,ft}{10.0\,ft/s}$	SSD Violation: Combined calculation and substitution steps.
	$t = 1.50\,s$	"C" Step (Calculation)

Figure 1. Sample Solutions with and without Solution Step Discipline

GFSA Method—Focus on Solution Step Discipline

- **G**iven: Distillation of problem information
- **F**ind: Identification of Solution goal
- **Solution**: Logical presentation of path from "Given" to "Find"
- **A**nswer: Reporting of Solution results

Solution Steps form a logical chain of actions down the page.
A knowledgeable reader should be able to understand each step.

Each and every Solution step must be from one of the following three categories:

A. Starting Equation	An equation, recognized as valid, that begins a series of steps.	Examples: • Equation accepted as generally valid (for example, from the textbook) • Equation determined earlier in the Solution • Relationship from the above "Given"
B. Substitution	A step where equation unknowns/ expressions are replaced by equivalent values/ expressions	Examples; substituting (with units): • Values from the above "Given" • Values determined earlier in the "Solution" • Values from a Solution diagram • Expressions accepted as generally valid (for example, from the textbook) • Expressions determined earlier in the Solution
C. Calculation	A step where the equation is manipulated with standard mathematical operation(s) into another equation.	Examples: • Algebra operations • Units conversions • Calculus operations

Make sure that each of your Solution steps fits one of these categories—and only one!

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Figure 2. Solution Step Discipline Reference Card

While solution discipline was universally foreign to the freshmen, they generally seemed to grasp the concepts and, sometimes, even the need for SSD. As students proceeded with their solutions, each step was to be examined for compliance with the ABC-only criterion. Students perceived no real challenge to implementing SSD—until their work was assessed and returned.

At that point, students recognized the disconnect between their solution thinking and even this simple ABC-only requirement. Students truly struggled. The mantra for the freshmen became: "Is this an A, B or C step?". However, through this struggle and subsequent consultations, the students addressed the lack of structure in not only their presentation, but in their thinking.

Conclusions (Answer):

For professors, problem solving has typically become second-nature. Our acquisition of these skills is often years-distant from the similar learning process that today's engineering and technology students undergo. We can struggle to even understand their struggle.

Some college students may naturally grasp and adopt an efficient problem solving methodology; however, most will struggle. Any well designed methodology can be undercut by the students' unstructured approach to working through the solution details. SSD offers some assistance. And yet, the author was truly amazed by the difficulty students had with the very elementary concepts embodied in SSD. The conclusion was that peripheral issues had been successfully stripped away and core concepts indeed distilled for focus.

College faculty stress problem solving to a K-12 community already touting such emphasis at every grade level. Students continue to arrive at college with muddled presentation of their thinking. It is not difficult to recognize that organizing one's solution presentation helps organize one's solution thinking.

Yet college and university faculty treat this solution presentation as largely outside the realm of problem solving education. A new awareness is needed. It is also time for engineering and technology faculty to promote incorporation of concise SSD elements in at least high school mathematics, science and technical coursework. It is probably more appropriate to incorporate and teach SSD fundamentals than the higher level, problem solving components.

The ABC's were shown above to reinforce the overall SSD. Likewise, SSD supports overarching problem solving methodologies. To that end, SSD contributes to the students' awareness of solution flow: how a mathematical process begins, how information is deemed valid and used, and where they are along the path to the answer. It frees the student to concentrate on accurately performing each step. And SSD enhances finding and fixing any errors. It can be incorporated into and improve other problem solving approaches, as well. Additional important emphases, such as units and significant digits, are easily meshed with SSD. Where the problem solving methodology is overly complex or detailed, SSD may provide a means of simplifying the procedure and improving the effectiveness of the students' efforts.

By summer 2004, SSD will have been implemented for two academic years. Every student receives an SSD reference card in every Purdue University technical course, every semester. "ABC-Only" has become the currency of the technical problem solving. Students enter their sophomore coursework with SSD ingrained, within a GFSA methodology. With each course, the SSD becomes more habitual and students are better able to focus on course content. While the enrollment at Purdue's Richmond location is not large, we are seeing the results of SSD integrated across the curriculum.

By uncovering the key components of problem solving, we can develop approaches to efficiently focus our students' efforts. The important components are many and documented, such as problem defining, solution path brainstorming and development, assumption listing, calculating answers and assessing results. With an unassuming simplicity, Solution Step Discipline is one effective approach to meeting the key need for students to efficiently stay on a solution path.

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