

## **State-Space Analysis of Linear, Time-Invariant Control Systems Using Virtual Instruments**

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### Abstract

This paper describes an innovative and cost-effective method of modernizing undergraduate technology and sciences laboratory and education so that our graduates can be well trained with the latest technology. This will also help the technology and science programs to satisfy the TAC of ABET 2000 accreditation criteria and other accreditation needs to maintain accreditation of their programs. This paper discusses the hardware/software used in each group of the Computer Based Virtual Engineering Laboratory (CBVEL), and its application to design Virtual Instruments (VI) modules for the analysis of Linear, Time-Invariant (LTI) control systems.

### I. Introduction

The fields of engineering, science, and technology at present are very dynamic, due to recent advances in computer and other technologies. These advances resulted in a number of computer programs that solve traditional as well as novel problems. These programs, using the computer's increased computational capabilities, are key to the explosion and success of the Internet and the Web. This also led to the revolution of the Internet-enabled instrumentation using programs like Laboratory Virtual Instrument Engineering Workbench (LabVIEW). LabVIEW is a graphical programming environment and is based on the concept of data flow programming; a programming concept different from the sequential nature of traditional programming languages, that cuts down the design and development time of an application. It is widely accepted by industry, academia, and research laboratories around the world as a standard for data acquisition and instrument control software. At South Carolina State University (SCSU), the PC and VI based system concept is used to design and develop a laboratory called Computer Based Virtual Engineering Laboratory (CBVEL). This CBVEL will help us in providing an interdisciplinary Integrated Teaching and Learning experiences that integrates team-oriented, hands-on learning experiences throughout the engineering technology and sciences curriculum, and engages students in the design and analysis process beginning with their first year. This will modify our existing laboratories, and help us better educate and train our graduates to serve the needs of the technological and engineering community. Students can use this CBVEL along with other software and test equipment in engineering technology hall and in other buildings. This CBVEL can also be accessed from remote sites using Internet.

The CBVEL consists of IBM compatible computers with appropriate software and hardware from National Instruments (NI), and is connected to School of Engineering Technology and Sciences (SETS) network and existing equipment<sup>1,2</sup>. Virtual Instrument (VI) modules for different courses and research areas are currently developed. Examples of some of these VIs are Circuit Analysis, Electronics, Communications, Digital Signal processing, Power Quality Analysis, Digital Electronics, Control Systems, and Digital Filters<sup>3,4,5,6,7</sup>. These modules will be used to better train the engineering technology, sciences, and information technology graduates. Some of the research areas that will be benefited by this laboratory are Non Destructive Testing, Power Quality Analysis, Control and Robotics, Intelligent Sensors and Distributed Control, Fuzzy Logic, and Space Science. This paper is arranged as follows: Section II discusses various components used in CBVEL, and LabVIEW application areas. Section III discusses some of the VIs for the State Space Analysis of Linear Time Invariant Control Systems. Section IV deals with conclusion and discussion. Section V deals with acknowledgement and Section VI presents the references.

## II. Components used in CBVEL and application areas of LabVIEW

### a. Components in CBVEL

The following are some of the components of CBVEL. The major components of this CBVEL are 12 PXI, 166 Mhz computers from NI and 20, 350 Mhz INTEL Pentium computers from Gateway. Besides these the PXI systems are equipped with NI components such as Function Generator Cards, Multimeter Cards, Oscilloscope Cards, Training Modules, Stepper and Servo Motion Control Cards, Virtual Bench Software, IMAQ Vision Cards, Field Point Dist. Systems, and Switches.

The PXI systems are industry standard systems with high-speed data acquisition cards. It has number slots with Windows 95 operating system. The multimeter, function generator, and oscilloscope cards are primarily for student and faculty use in different technology and science courses. Virtual Bench software is used to use these cards for different simulation and laboratory exercises. The motion control, vision, and distributed control cards are primarily for research. All these cards are installed in different ports of this PXI system. The Gateway computers are equipped with LabVIEW, HI-Q, and other software. The Hi-Q software can be used to perform advanced mathematical analysis. The student and faculty use these computers for simulation purposes.

### b. Application Areas of LabVIEW Software

The following are some of the application areas of LabVIEW<sup>8,9</sup>:

Simulation; Data Acquisition; Data Processing - built in analysis library that includes signal generation, measurement, filters, windows, curve fitting, probability and statistics, linear algebra, numerical methods; Instrument and Control; Object oriented/graphical programming; Fuzzy Logic; Genetic Algorithm; and Joint Time and Frequency Analysis. Fuzzy Logic, Genetic Algorithm, and Joint Time and Frequency Analysis can be used to analyze and design complex control systems. Linear Algebra (matrices) is an important tool and used extensively in the state space analysis of LTI control systems.

### III. Control Systems VIs

We have developed VIs for circuit analysis, electronics, communication, signal processing and filters, power and control systems, and information technology courses<sup>3, 4, 5, 6, 7</sup>. The faculty and students use these VIs in various lecture and laboratory courses. The students build systems in breadboard and develop appropriate vi modules to test their design. They connect the design to the equipment in the PXI system module and verify their design. The control systems VIs cover both classical control (frequency domain analysis) and modern control (state space analysis). These VIs use different built in LabVIEW functions such as digital controls for input, formula node, case structures, loops and matrices for mathematical equations, and digital indicators, graphs and charts for output. It also uses add on LabVIEW simulation toolkit **GSim** to perform frequency domain (Bode Plot, Root Locus, Nyquist Plot, etc.) and time domain analysis (pole placement, controller, observer, etc.) of LTI control systems. This paper discusses the use of VI concepts to perform time domain analysis of LTI control systems. Two examples VIs of time domain analysis (pole zero plot with different system parameters and pole placement) of LTI control systems presented below demonstrate the use of these functions. The theory behind frequency domain and time domain analysis is well founded. There are number of books that discusses these topics and the intent of this paper is not to discuss these principles. The interested reader may refer to books by Ogata<sup>10</sup> and Kuo<sup>11</sup>.

#### Example 1 - Pole-Zero Plot<sup>12</sup>

The objective of this example is to demonstrate the following:

1. Conversion from state space form to zero-pole form
2. Plot the zeros and poles of the system
3. Compute the system parameters such as settling time, percent overshoot, natural frequency, and damping factor

The original system representation is in state space form and is as follows:

$$\dot{x} = Ax + Bu, \dots y = Cx + Du$$

Where

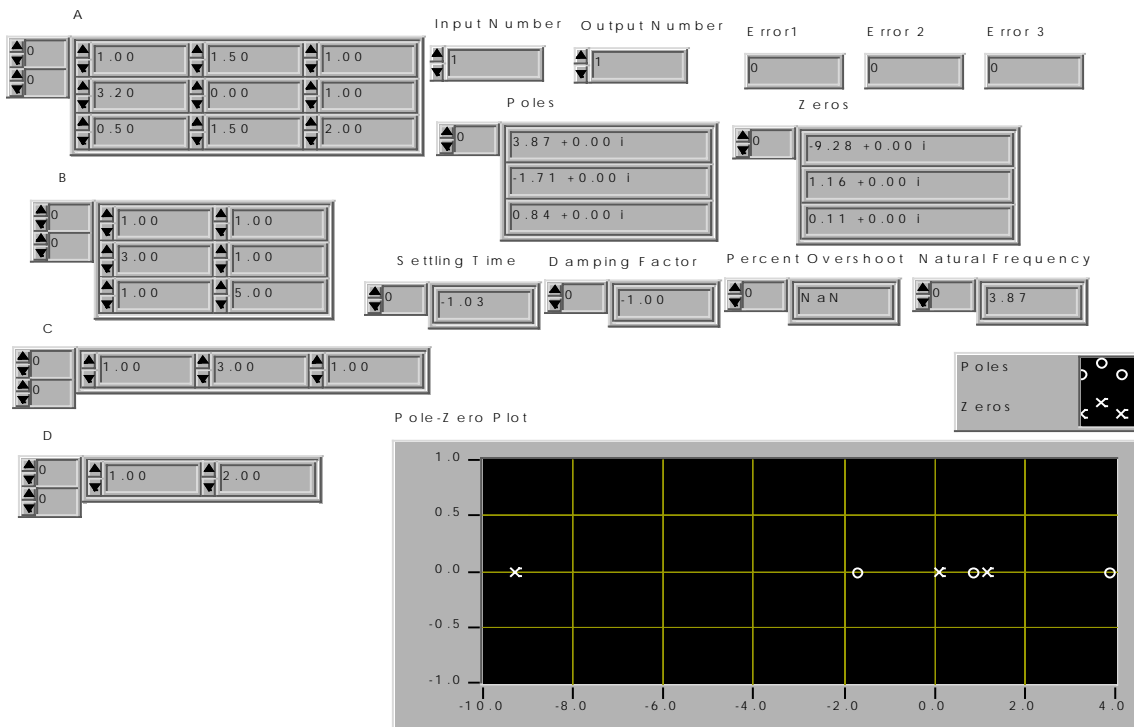
$$\dot{x} = dx/dt, x = \text{statevector} = \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix}, \dots A = \begin{bmatrix} 1 & 1.5 & 1 \\ 3.2 & 0 & 1 \\ 0.5 & 1.5 & 2.0 \end{bmatrix}, \dots B = \begin{bmatrix} 1 & 1 \\ 3 & 1 \\ 1 & 5 \end{bmatrix}$$
$$y = \text{output}, \text{and } \dots u = \text{input} = \begin{bmatrix} u1 \\ u2 \end{bmatrix}, \dots C = [ 1 \ 3 \ 1 ], \dots D = [ 1 \ 2 ]$$

We have used the following built in VIs in the **GSim** to complete this example:

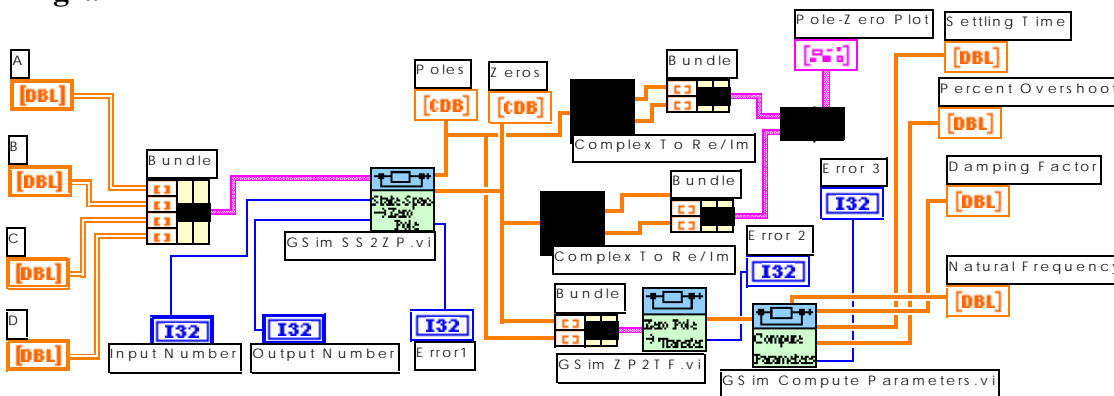
1. Gsim SS2ZP.vi - converts the state space representation to zero-pole form
2. Gsim ZP2TF.vi - convert the system representation from zero pole form to the transfer function form

3. Gsim Compute Parameters.vi - computes parameters of the system.
- Besides the Gsim VIs we have also used the following functions in LabVIEW:
1. Array digital controls - to input A, B, C, and D matrices
  2. Digital controls - to select inputs and outputs
  3. X-Y graph - to plot poles and zeros
  4. Array digital indicators - to represent system parameters
  5. Digital indicators - to represent error codes, if any (0, means no error)
  6. Complex array digital indicators - to indicate poles and zeros of the system

### VI Front Panel



### VI Digram



The front panel shows the zeros and poles of the system with the first input only. The simulation can be performed to show the poles and zeros of the system for second input by changing the input number to 2.

## Example 2 - Pole Placement and Stability <sup>10</sup>

This example deals with the state feedback control and pole placement. The problem statement for this example is as follows:

Consider the system defined by

$$\dot{x} = Ax + Bu, \dots y = Cx$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix}, \dots B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \dots C = [ 1 \quad 0 ]$$

By using the state feedback control  $u = -Kx$ , it is desired to have the closed loop poles closed-loop poles at  $s = -1.8 \pm j2.4$ . Determine the state feedback gain matrix  $K$ .

### **Solution**

The theoretical solution to this example is in pages 784-786 of reference 10. The system poles are at  $s = \pm 4.539$ . The system is unstable. The value of  $K$  matrix is found to be 29.6 and 3.6. The vi module for this system is shown below. The  $s$  value and the magnitude of the  $K$  matrix from the VI module are same as theoretical solution.

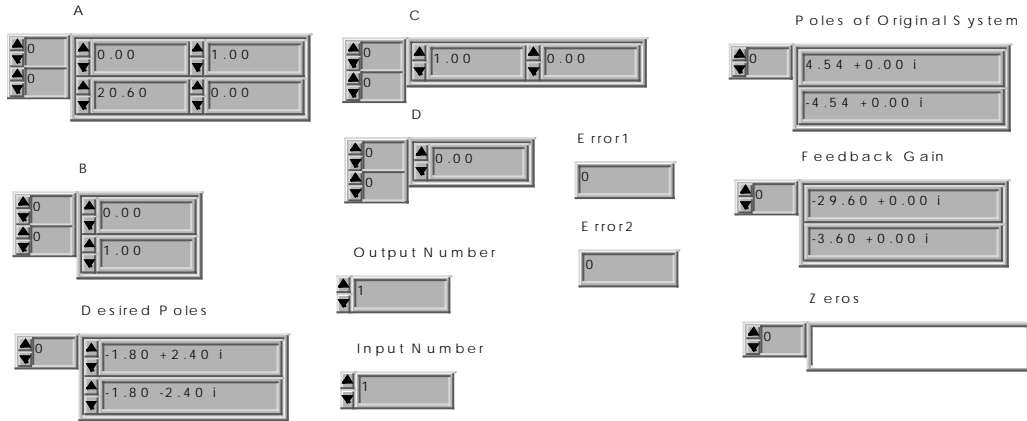
We have used the following built in VIs in the **GSim** to complete this example:

1. Gsim SS2ZP.vi - converts the state space representation to zero-pole form
2. Gsim Design from SS.vi - to design linear state feed back

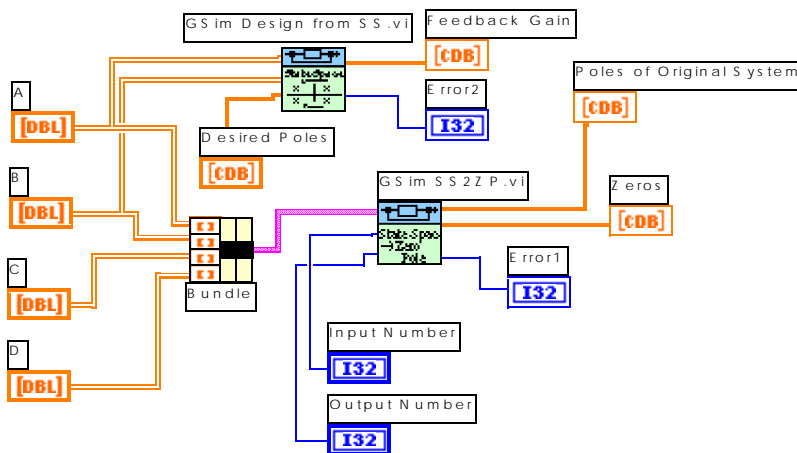
Besides the Gsim VIs we have also used the following functions in LabVIEW:

1. Array digital controls - to input A, B, C, and D matrices
2. Digital controls - to select inputs and outputs
3. Digital indicators - to represent error codes, if any (0, means no error)
4. Complex array digital indicators - to indicate poles and zeros of the system and the  $K$  matrix

## VI Front Panel



## VI Diagram



## IV. Conclusion and Discussion

We have developed a number of other VIs for other courses. The remote access component of the CBVEL is under development. The Computer based Virtual Engineering at South Carolina State University, once fully developed, will be one of its kind in the State of South Carolina. It will provide a truly modern environment in which students and faculty members can study engineering, technology, and sciences at a level of detail, and this will be possible because of the versatility of LabVIEW and other products from NI. A number of software packages are currently being used in engineering, technology, and sciences curriculum. LabVIEW has features and VIs identical to most of the features found in all these software packages. Therefore, one can use LabVIEW only to address the needs of various courses. This will be beneficial for students and faculty and introduce standardization across the curriculum. The control and simulation toolkit is LabVIEW add-on software for simulating, designing and analyzing linear and nonlinear control systems. This toolkit can be used to accelerate the system design process since it uses VIs modeled after frequently used system symbols and blocks used in control system.

## V. Acknowledgement

We are thankful to DOD/AirForce for providing us the funding to design and develop this laboratory, and NI representatives for their outstanding support and constant encouragement to complete this work. Also, this work was supported in part by a grant to SCSU through the NASA-PAIR program under NCC 5-454.

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