AC 2008-1675: STATISTICAL PROCESS CONTROL LABORATORY EXERCISES FOR ALL ENGINEERING DISCIPLINES

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Statistical Process Control Laboratory Exercises for all Engineering Disciplines

Abstract

Despite its importance in industry, statistical process control (SPC) is rarely taught in undergraduate controls courses. However, one or two lectures, coupled with the hands-on assignment in this paper, are sufficient to give a good introduction to the topic. This paper presents a case for why all engineers should study SPC, gives a brief tutorial on SPC, and provides some simple exercises for students that would be appropriate for laboratory or homework use.

Introduction

Process control is inherently multidisciplinary. It is used in a wide variety of industries, including automotive and consumer products manufacturing, aerospace, semiconductor device manufacturing, bulk chemical manufacturing, and refining. Industrially, a process control project draws on economics and software engineering in addition to the various engineering disciplines that may be involved. Generally, industrial controls projects (large or small) require multidisciplinary teams to be successful. Control engineers come from a variety of fields including mechanical, aerospace, electrical, computer, chemical, and agricultural engineering[1].

This breadth of entry points is reflected in undergraduate engineering curricula. Most ABET accredited undergraduate engineering programs have at least one course in controls systems analysis and design. The exception is civil engineering which generally does not require a controls course. (However, even there the environmental engineering faculty at the first author's institution have been strongly encouraging students with an interest in environmental engineering to take a controls course as it has a high degree of relevance for waste water treatment.) Thus, it should be clear that control systems engineering is a basic engineering fundamental that is relevant for most (if not all) engineering disciplines. As engineering educators it is vital that we give our students the best possible education in controls systems that we can.

Most undergraduate controls courses are taught from the classical point of view, which assumes that measurements are perfect, deviations from the desired set-point are a common occurrence, and that the energy necessary to bring the system back to the desired setpoint is small (or equivalently, has little cost)[2]. This allows the education to focus on the basics such as open and closed loop stability, and controller design.

In contrast, Statistical Process Control (SPC) assumes that real deviations from the setpoint are relatively infrequent and that the cost associated with returning the process to the setpoint is not negligible[3]. Various statistical measures such as Shewart Charts, Western Electric Rules, CUSUM charts (CUSUM stands for cumulative sum), and the Exponentially Weighted Moving Average are employed to determine what is a "real" deviation (or alternatively, what deviations are significant enough to attempt to correct). SPC is extremely important in industrial practice, and its assumptions provide an educationally useful counterpoint to the classical point of view. However, despite its importance in industry and pedagogical value, SPC is rarely taught in undergraduate controls courses[1]. Reasons for this vary from program to program, but probably include lack of awareness by faculty, lack of resources to teach the topic, and lack of time in a crowded course.

Our objectives for this paper are to encourage engineering faculty to include SPC in the introductory controls class, to provide a basic tutorial on SPC, and to give a hands-on student problem that would be appropriate as a home work assignment or laboratory exercise. One (or at most two) one-hour lectures are all that are needed to give students a good introduction to SPC so lack of time in a crowded course should not be a reason to skip the topic, given its industrial importance and pedagogical value.

A Brief Tutorial on SPC

SPC assumes that measurement error may be significant and that not every deviation in the measured variable is worth correcting. Only those deviations that are significantly different (in a statistical sense) from normal variation in the measurements are worth correcting. Thus we need a criterion to determine what constitutes a 'real' deviation and what is just random variability in the measurement.

A Shewart chart (see Figure 1) is the most basic tool for determining which variations are due to a fundamental shift in the process variable and which are merely measurement

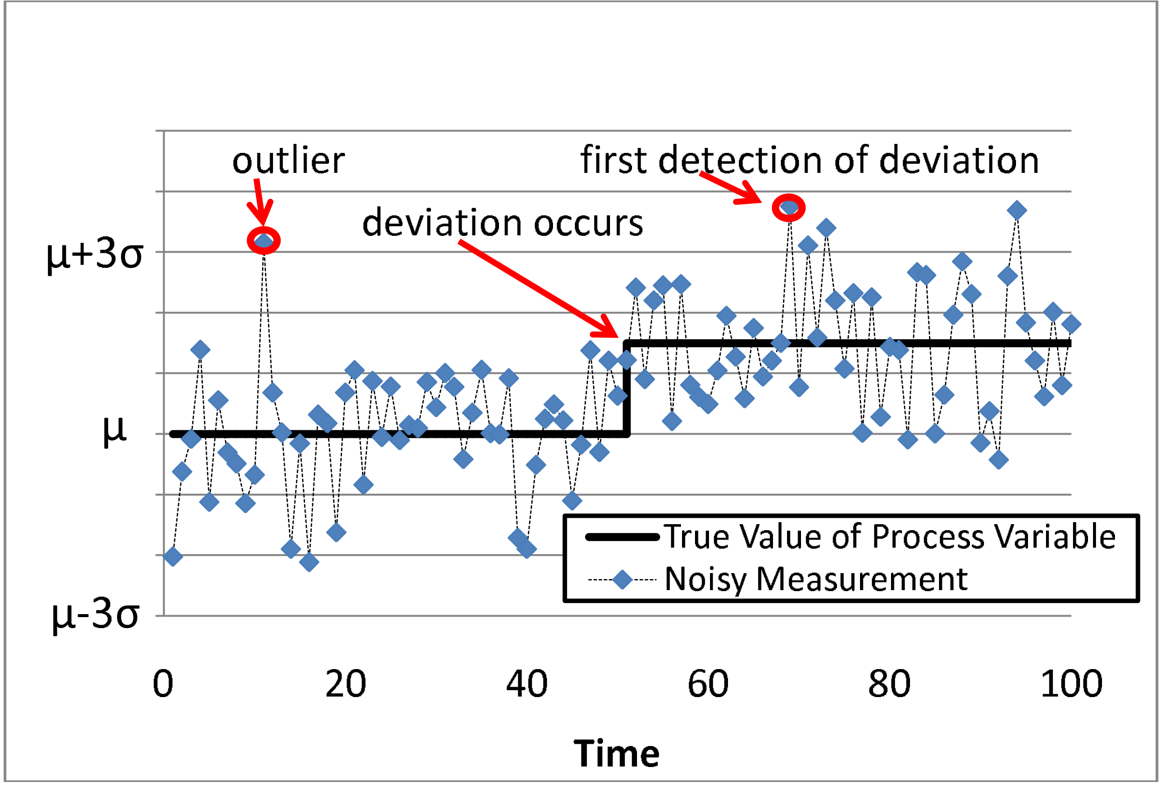

Figure 1. Basic Shewart Chart

noise. A Shewart chart is a plot of the measurement versus time, with plus (minus) three standard deviations as the upper (lower) control limits. Assuming that the discrepancy between the true value of the process variable and the measurement of that variable is a random variable drawn from a normal or Gaussian distribution (white noise) then the probability that the measurement will be more than 3 standard deviations from the mean is 0.0027. Thus, if a measurement is more than 3 standard deviations from the mean it is likely due to something other than purely random variations in the measurement (under the assumptions of SPC theory). The Shewart chart shows that when a measurement is outside of the control limits that the process is 'out of control' and that corrective action should be taken.

A Shewart chart is a useful tool, and a useful starting point to begin study of SPC, but there are some issues with using a Shewart chart is as well. As shown in Figure 1, with a random variable from a normal distribution there are occasional outliers. The rate of false positives is 0.27%, which is low but not zero. One such false alarm is shown at time equal 11 in Figure 1. A second problem with Shewart charts is that they are not necessarily rapid in detecting real deviations from the desired process operating condition that should be corrected. In Figure 1 the deviation occurs at time 51 but is not detected until time 69. In other words, 18 samples are taken before the Shewart chart detects that there is a significant problem. A third problem is that the Shewart chart is not good at picking up small offsets from the desired process operating point. At time 51 there is 1.5σ shift in the mean of the measurements in Figure 1. If there were a 0.3σ shift it would be barely noticeable in the random variations of the measurement (see Figure 3).

These shortcomings motivate the slightly more sophisticated SPC methods known as the Western Electric rules and the CUSUM[4] method. The Western Electric rules are as follows. A statistically significant process deviation is deemed to have occurred when one or more of the following is true:

1. A single measurement is more than three standard deviations from the mean.
2. 2 out of 3 consecutive measurements are more than 2 standard deviations from the mean.
3. 4 out of 5 consecutive measurements are more than 1 standard deviation from the mean.
4. 8 consecutive measurements are on the same side of the mean.

The Western Electric rules are much faster at detecting the actual deviation for the example data shown in Figure 1. Rule 2 is true at time 54, rule 3 at time 55, and rule 4 at time 56. Thus the Western Electric rules detect the deviation after only 3 samples (as opposed the 18 samples required by the Shewart chart).

The increased sensitivity of the Western Electric rules allows more rapid detection of errors (as in the above example), however, it also means more false alarms. Consider Figure 2 where there is no deviation in the true value of the process variable. However, the Western Electric rules (namely rule 3) indicate that there are two deviations.

The CUSUM method is a way to detect small (but sustained) deviations that the Shewart chart and the Western Electric rules do not detect. Figure 3 shows an example of this. A

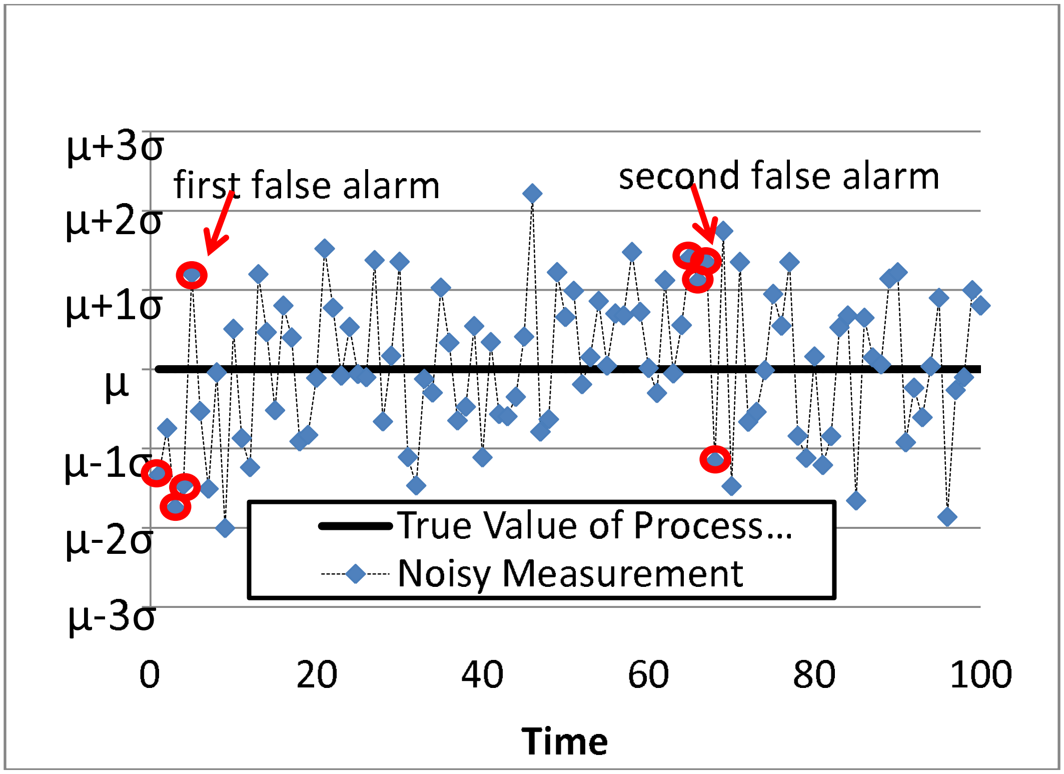

Figure 2. Increased sensitivity of the Western Electric rules gives more false alarms.

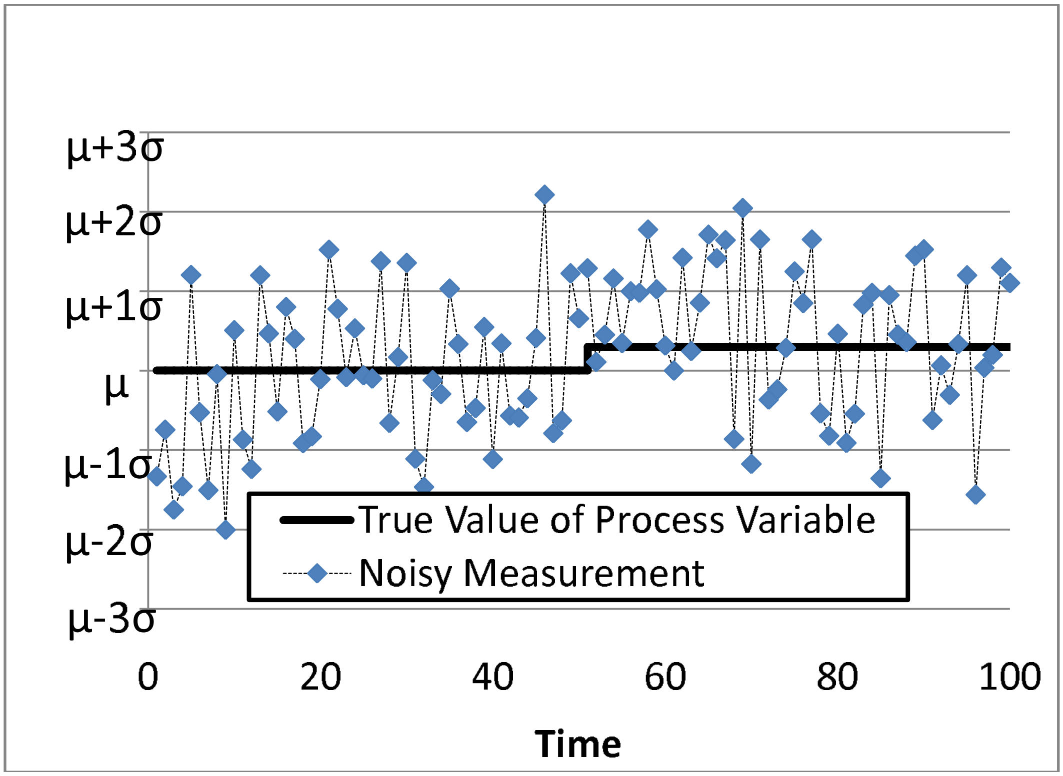

Figure 3. A small offset is introduced at time 51. Using the same noise as in Figure 2 the Western Electric rules do not detect the deviation, with the exception of the two false positives noted in Figure 2 already.

deviation of +0.3 is introduced at time 51 using the same noise sequence as in Figure 2. The Western Electric rules only detect errors at the same two places that they detected them in Figure 2. In other words, to the Western Electric rules Figures 2 and 3 look identical.

The CUSUM is a summation or running total of the measurement. In the absence of a real deviation from the desired operating point the difference between the measurement and the mean will be a normally distributed random variable with zero mean (white noise). The sum of this white noise will be a random walk about zero. However, if there is a small (but real) deviation from the desired operating point then the sum of the difference between the measurement and the mean will increase linearly with time. This accretion of error is very noticeable when the sum is plotted versus time (see Figure 4).

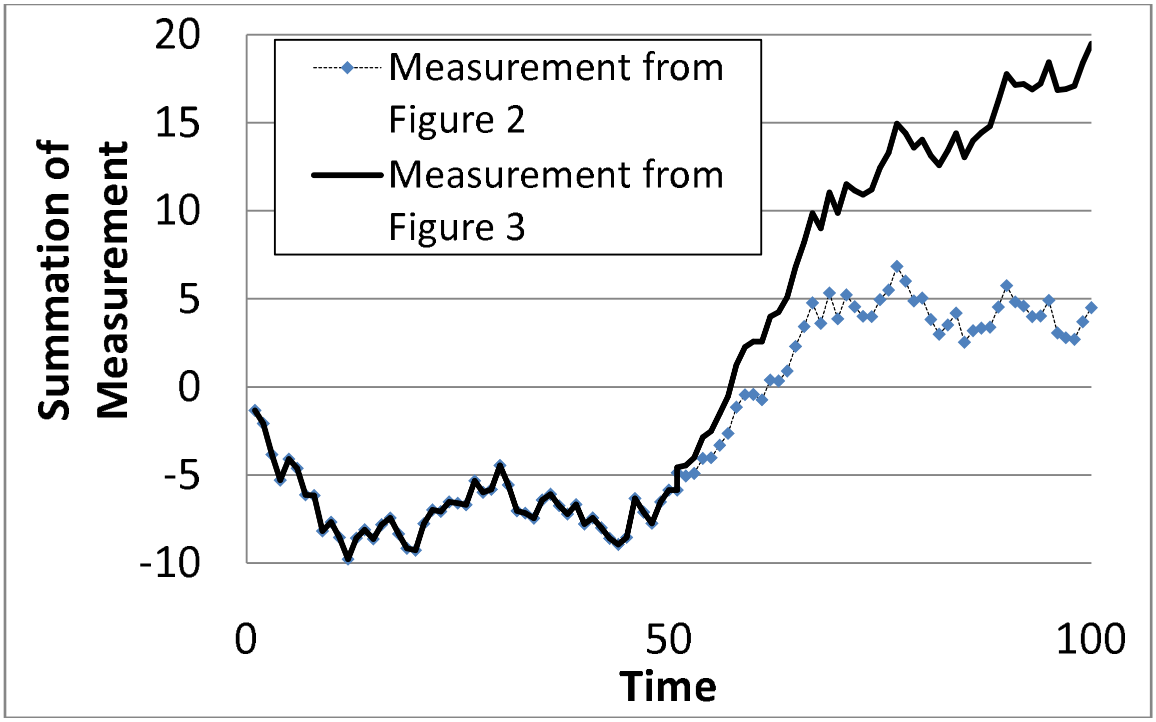

Figure 4. CUSUM chart shows a clear difference between case of no deviation in the process variable (from Figure 2) and the case of a small offset (from Figure 3). The CUSUM chart can detect small but lasting deviations.

Pedagogical Value

Although it only takes one or two one-hour lectures to give students a solid grounding in the basics of SPC it allows the introduction of many valuable concepts. As has already been mentioned, it allows students to re-examine the (implicit) assumptions that measurements are perfect, deviations are frequent, and that correcting deviations costs zero energy. After having completed the lab exercise/homework assignment (see below) students will also appreciate the importance (and difficulty!) of knowing what the 'real' mean and variance of the measurement are. Students will observe the inherent tradeoff between sensitivity to real deviations and false alarms. By giving students context for real implementations of process control, they will understand not only SPC, but also gain a better understanding of classical feedback control as well. The concepts of SPC are generalizable to other areas of engineering and are useful for building a general framework for understanding how the world works. These are insights that undergraduates will not get any other way. Thus, in terms of the return on (time) investment, one or two hours of lecture on SPC offer a huge payback.

Laboratory/Homework Exercise

Although one to two lectures is sufficient to cover the basics of SPC students do not fully comprehend SPC until they do it. Student understanding increases dramatically after completing the following hands-on exercise.

> Using Matlab or Excel, generate test data sequence of 100 data points for this SPC exercise.
>
> $$\text{test data} = \text{`true signal'} + M_1 \times \text{measurement noise}$$
> $$\text{true signal} = M_2 \times \text{unit step}(t_{step})$$
>
> In Matlab, normally distributed random numbers are generated with the `randn` command, uniformly distributed random numbers are generated with the `rand` command. In Excel, normally distributed random numbers may be generated with the NtRand Add-in[5], uniformly distributed random numbers are generated with the '=`rand`()' function.
>
> Evaluate the test data with a Shewart chart, the Western Electric Rules, and a CUSUM chart for different magnitudes of measurement noise (e.g., M_2 = 0.5, 1.0, 2.0) and for steps occurring at different points in time (e.g., t_{step} = 5, 25, 50).
>
> Answer the following questions:
>
> 1. Which method performs best, and why?
> 2. Does it matter in which direction the step goes? (M_2 positive or negative)
> 3. What happens if the measurement noise is uniformly (vs. normally) distributed?
> 4. How does changing the magnitude of the measurement noise (M_1) and the time at which the step occurs (t_{step}) affect the performance of each method?
> 5. Some would say that the important variable is the step-to-noise ratio $\left(\frac{M_2}{M_1}\right)$. Comment on this.
> 6. How many false positives (and false negatives) did you get for each method, and when did they occur?
> 7. For extra credit try adding a sinusoidal signal instead of a step function. How does each SPC method perform with this type of fault?

Students get to evaluate each of the SPC methods presented in lecture. They can see how critical it is to establish good “baseline” data (by varying the time before the step occurs), the effect of uniform vs. normally distributed noise, and that signal to noise ratio is the important variable for determining which faults can or cannot be detected. (Faults that are on the level of the noise in the measurement cannot be detected.)

Figures 5 and 6 show examples of work from two different students. In both cases the students created and presented test data in a way that demonstrates that they understand not only the SPC statistical tests but also the interplay between the signal-to-noise ratio.

The effectiveness of the exercises was assessed by before-and-after student surveys. Students were asked to respond twice to the following question,

> “On a scale of 1 (do not understand at all) to 10 (I totally know this stuff) how well did you understand statistical process control (Shewart charts, Western Electric rules, etc)?”

After SPC was covered in lecture, but before doing the laboratory exercise, the average student response was 5.7 with a high of 7 and a low of 4. After completing the exercise the average student response was 8.3 with all students responding either 8 or 9. Students also responded to open ended questions before and after the lab. One illustrative student comment on the post-lab survey was “in hindsight I thought SPC was harder than it really was.”

Conclusions

SPC has broad application across the engineering disciplines. It is feasible to add to a standard class on control, even in an already crowded curriculum. The pay-back for the time spent is large, in that students not only get an introduction to a new topic, it allows them to better understand the context for classical feedback control as well. Since control engineers come from a wide variety of engineering disciplines[1] SPC is a topic that all engineers should be exposed to at some point in their undergraduate education. We presented a simple hands-on exercise suitable for a laboratory or homework assignment that illustrates all of the basic SPC concepts and has demonstrably improved student comprehension of SPC topics.

Software Notes

Students may have an easier time implementing the Western Electric rules in Excel, particularly if they know how to use Excel’s IF function. Students using Excel will need to download an Add-in[5] for generating noise with a normal distribution as there is not native support for this in Microsoft Excel. Alternatively, the data may be generated in Matlab and exported to Excel.

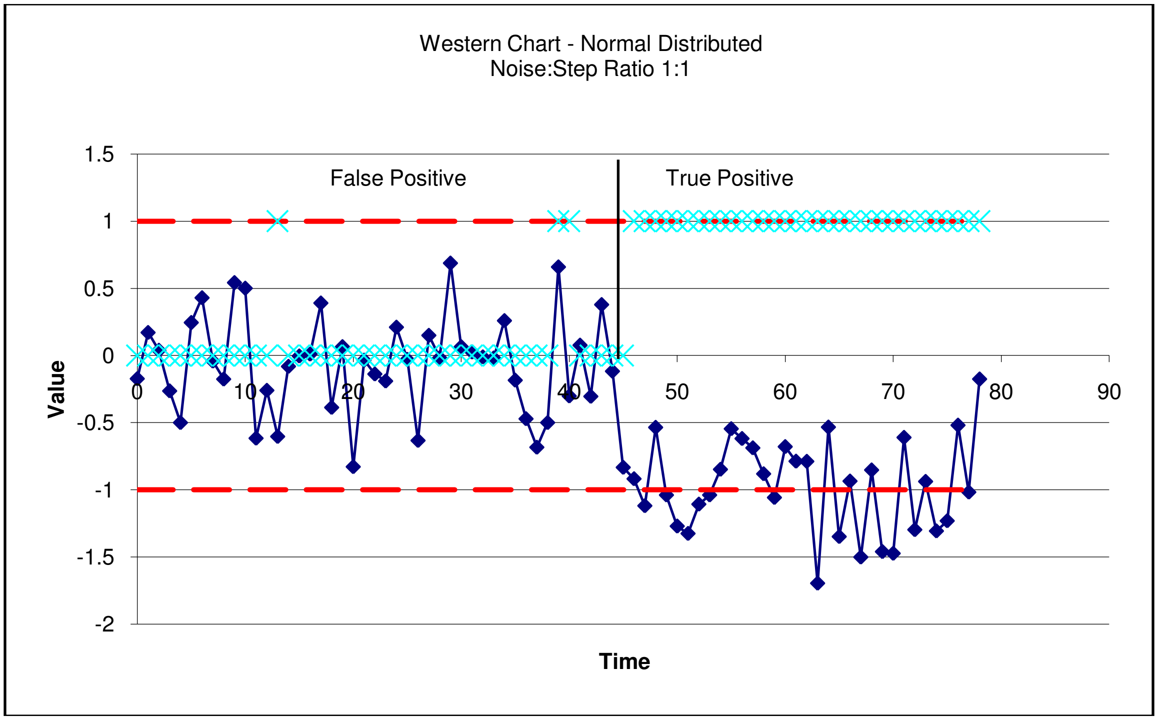

Figure 5. Student work showing Western Electric rules for the lab exercise.

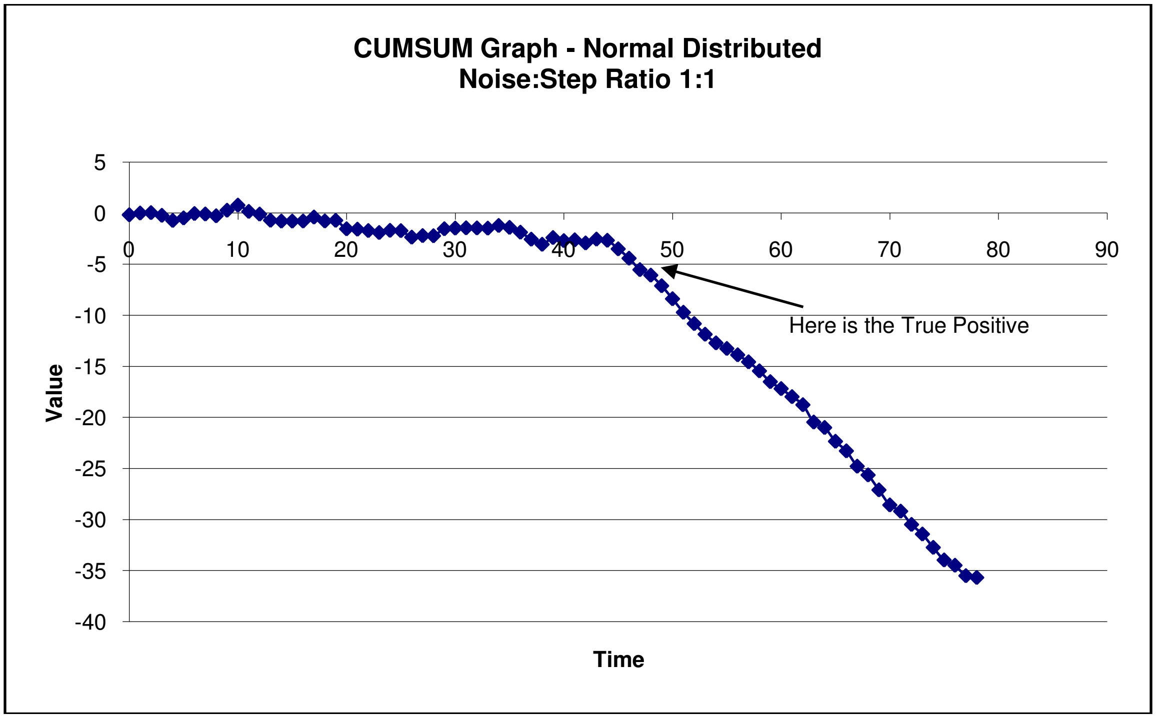

Figure 6. Student work showing CUSUM chart for lab exercise.

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